DIVAKARA.B.C

NETWORK ANALYSIS
[As per Choice Based Credit System (CBCS) scheme]
SEMESTER – III (EC/TC)

Subject Code	15EC34	IA Marks	20
Number	04	Exam Marks	80
Total Number of Lecture Hours	50	Exam Hours	03

CREDITS – 04

Course objectives: This course enables students to:

- Describe, Apply and Analyze basic network concepts emphasizing Series and Parallel Combination of Passive Components, Source Transformation and Shifting.
- Describe, Apply and Analyze use of mesh and nodal techniques for Formulating the Transfer Function of Networks.
- Apply and Analyze various network theorems in solving the problems related to Electrical Circuits.
- Describe and Analyze two port networks and methods of analyzing the Electrical Networks.

Modules	**Teaching Hours**	**Revised Bloom's Taxonomy (RBT) Level**
Module -1		
Basic Concepts: Practical sources, Source transformations, Network reduction using Star – Delta transformation, Loop and node analysis With linearly dependent and independent sources for DC and AC networks, Concepts of super node and super mesh.	**10 Hours**	**L1, L2,L3,L4**
Module -2		
Network Theorems: Superposition, Reciprocity, Millman's theorems, Thevinin's and Norton's theorems, Maximum Power transfer theorem and Millers Theorem.	**10 Hours**	**L1, L2, L3,L4**
Module -3		
Transient behavior and initial conditions: Behavior of circuit elements under switching condition and their Representation, evaluation of initial and final conditions in RL, RC and RLC circuits for AC and DC excitations. **Laplace Transformation & Applications** : Solution of networks, step, ramp and impulse responses, waveform Synthesis.	**10 Hours**	**L1, L2, L3,L4**

Module -4		
Resonant Circuits: Series and parallel resonance, frequency- response of series and Parallel circuits, Q–Factor, Bandwidth.	**10 Hours**	**L1, L2, L3,L4**
Module -5		
Two port network parameters: Definition of z, y, h and transmission parameters, modeling with these parameters, relationship between parameters sets.	**10 Hours**	**L1, L2, L3,L4**

Course outcomes:

Acquire knowledge for solving problems related to

- Series and Parallel combination of Passive Components, Source Transformation and Source Shifting.
- Network Theorems and Electrical laws to reduce circuit complexities and to arrive at feasible solutions.
- Various Two port Parameters and their Relationship for finding Network Solutions.
- Analyze the Performance of various Types of Networks Using different concepts and principles.

Graduate Attributes (as per NBA)

- Engineering Knowledge.
- Problem Analysis.
- Design / development of solutions.

Question paper pattern:

- The question paper will have ten questions.
- Each full question consists of 16 marks.
- There will be 2 full questions (with a maximum of four sub questions) from each module.
- Each full question will have sub questions covering all the topics under a module.
- The students will have to answer 5 full questions, selecting one full question from each module.

Text Books:

1. M.E. Van Valkenberg (2000), "Network analysis", Prentice Hall of India, 3rd edition, 2000, ISBN: 9780136110958.

2. Roy Choudhury, "Networks and systems", 2nd edition, New Age International Publications, 2006, ISBN: 9788122427677.

Reference Books:

1. Hayt, Kemmerly and Durbin "Engineering Circuit Analysis", TMH 7th Edition, 2010.
2. J. David Irwin /R. Mark Nelms, "Basic Engineering Circuit Analysis", John Wiley, 8th ed, 2006.
3. Charles K Alexander and Mathew N O Sadiku, " Fundamentals of Electric Circuits", Tata McGraw-Hill, 3rd Ed, 2009.

B.E ELECTRICAL AND ELECTRONICS ENGINEERING (EEE) CHOICE BASED CREDIT SYSTEM (CBCS) SEMESTER - III			
ELECTRIC CIRCUIT ANALYSIS (Core Subject)			
Subject Code	15EE32	IA Marks	20
Number of Lecture Hours/Week	04	Exam Hours	03
Total Number of Lecture Hours	50	Exam Marks	80
Credits - 04			

Course objectives:

- To familiarize the basic laws, theorems and the methods of analysing electrical circuits.
- To explain the concept of coupling in electric circuits and resonance.
- To familiarize the analysis of three-phase circuits
- To analyze the transient response of circuits with dc and sinusoidal ac input
- To impart basic knowledge on network analysis using Laplace transforms.

Module-1	**Teaching Hours**
Basic Concepts: Active and passive elements, Concept of ideal and practical sources. Source transformation and Source shifting, Concept of Super Mesh and Super node analysis. Analysis of networks by (i) Network reduction method including star – delta transformation (ii) Mesh and Node voltage methods for ac and dc circuits with independent and dependent sources. Equilibrium equations using KCL and KVL, Duality. **Resonant Circuits:** Analysis of simple series RLC and parallel RLC circuits under resonances. Resonant frequency, Bandwidth and Quality factor at resonance. Practical RL-RC circuits. ■	**10**
Revised Bloom's Taxonomy Level: L_1 – Remembering, L_2 – Understanding, L_3 – Applying, L_4 – Analysing.	
Module-2	
Network Theorems: Analysis of networks with and without dependent ac and dc sources by Thevenin's and Norton's theorems. Analysis of ac and dc circuits for maximum power transfer to resistive and complex loads. Application of Millman's theorem and Super Position theorem to multisource networks. Reciprocity theorem and its application. ■	**10**
Revised Bloom's Taxonomy Level: L_1 – Remembering, L_2 – Understanding, L_3 – Applying, L_4 – Analysing.	
Module-3	
Transient Analysis: Review of ordinary linear nonhomogeneous first and second order differential equations with constant coefficients. Transient analysis of dc circuits by classical method for unit step input only. Behaviour of circuit elements under switching action ($t = 0$ and $t = \infty$). Evaluation of initial conditions. ■	**10**
Revised Bloom's Taxonomy Level: L_2 – Understanding, L_3 – Applying, L_4 – Analysing, L_5 – Evaluating.	
Module-4	
Laplace Transformation: Laplace transformation (LT), LT of Impulse, Step, Ramp, Sinusoidal signals and shifted functions. Waveform synthesis. Initial and Final value theorems. Laplace Transform of network and time domain solution for RL, RC and RLC networks for ac and dc excitations. ■	**10**
Revised Bloom's Taxonomy Level: L_1 – Remembering, L_2 – Understanding, L_3 – Applying, L_4 – Analysing.	

15EE32 ELECTRIC CIRCUIT ANALYSIS (Core Subject) (continued)

CHOICE BASED CREDIT SYSTEM (CBCS)		
Module-5		**Teaching Hours**
Unbalanced Three phase systems: Analysis of three phase systems, calculation of real and reactive powers. **Two Port networks:** Definition, Open circuit impedance, Short circuit admittance and Transmission parameters and their evaluation for simple circuits. Network functions of one port and two port networks, Properties of poles and zeros of network functions. **Complex Wave analysis:** Analysis of simple circuits with non-sinusoidal excitation. ■		**10**
Revised Bloom's Taxonomy Level	L_1 – Remembering, L_2 – Understanding, L_3 – Applying, L_4 – Analysing.	

Course outcomes:

At the end of the course the student will be able to:

- Apply knowledge of mathematics, science, and engineering to the analysis and design of electrical circuits.
- Identify, formulate, and solve engineering problems in the area circuits and systems.
- Analyze the solution and infer the authenticity of it.

Graduate Attributes (As per NBA)

Engineering Knowledge

Problem analysis

Question paper pattern:

- The question paper will have ten questions.
- Each full question is for 16 marks.
- There will be 2 full questions (with a maximum of four sub questions in one full question) from each module.
- Each full question with sub questions will cover the contents under a module.
- Students will have to answer 5 full questions, selecting one full question from each module.

Text/Reference Books

1	Engineering Circuit Analysis	William H Hayt et al	Mc Graw Hill	8th Edition,2014
2	Engineering Circuit Analysis	J David Irwin et al	Wiley India	10th Edition,2014
3	Fundamentals of Electric Circuits	Charles K Alexander Matthew N O Sadiku	Mc Graw Hill	5th Edition,2013
4	Network Analysis	M.E. Vanvalkenburg	Pearson	3rd Edition,2014
5	Electric Circuits	Mahmood Nahvi	Mc Graw Hill	5th Edition,2009
6	Introduction to Electric Circuits	Richard C Dorf and James A Svoboda	Wiley	9th Edition,2015
7	Circuit Analysis; Theory and Practice	Allan H Robbins Wilhelm C Miller	Cengage	5th Edition,2013

CBCS Scheme

USN

15EC34

Third Semester B.E. Degree Examination, June/July 2017
Network Analysis

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Calculate the current through 2Ω resistor for the circuit shown in Fig.Q1(a) using source transformation. **(08 Marks)**

b. Use mesh analysis to determine the three mesh currents I_1, I_2 and I_3 in the circuit shown in Fig.Q1(b). **(08 Marks)**

Fig.Q1(a)

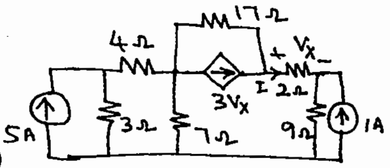
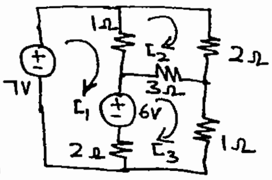


Fig.Q1(b)



OR

2 a. Find the equivalent resistance R_{AB} using star and delta transformation for network shown in Fig.Q2(a). **(08 Marks)**

Fig.Q2(a)

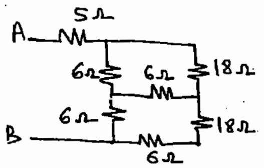
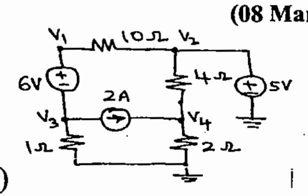


Fig.Q2(b)



b. For the circuit shown in Fig.Q2(b), determine all node voltages. **(08 Marks)**

Module-2

3 a. Using Millman's theorem, find the current through load resistance R_L for the circuit shown in Fig.Q3(a). **(08 Marks)**

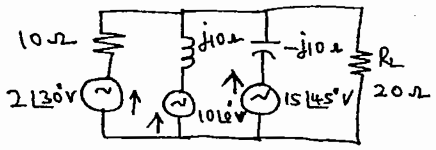


Fig.Q3(a)

b. State the maximum power transfer theorem and also prove that $P_{max} = \frac{V_{th}^2}{4R_L}$, where

V_{th} = thevenins voltage. **(08 Marks)**

OR

4 a. Obtain the Thevenin's equivalent of the circuit shown in Fig.Q4(a). **(08 Marks)**

b. Using superposition theorem, find the current in 6Ω resistor in the network shown in Fig.Q4(b). **(08 Marks)**

Fig.Q4(a)

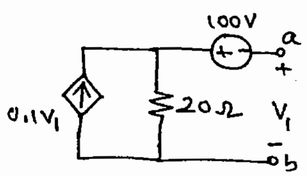
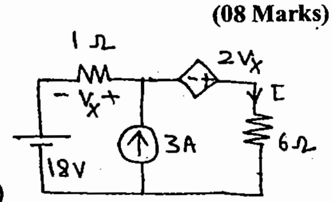


Fig.Q4(b)



Module-3

5 a. In the network shown in Fig.Q5(a), the switch is closed at t = 0, determine $i, \frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$. **(08 Marks)**

b. For the network shown in Fig.Q5(b), the switch 's' is opened at t = 0 solve for V, DV and D^2V at $t = 0^+$. **(08 Marks)**

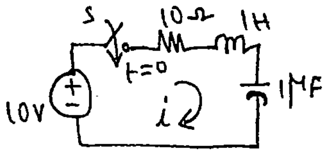


Fig.Q5(a)

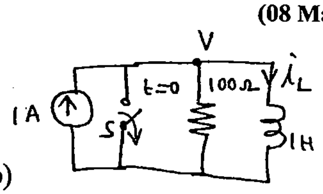


Fig.Q5(b)

OR

6 a. Find the Laplace transform of the periodic signal x(t) shown in Fig.Q6(a). **(08 Marks)**

b. Given the signal $x(t) = \begin{cases} 3, & t < 0 \\ -2 & 0 < t < 1 \\ 2t - 4 & t > 1 \end{cases}$

Express x(t) interms of singularity functions. Also find the Laplace transform of x(t). **(08 Marks)**

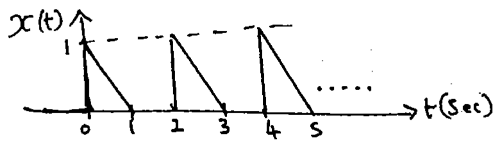


Fig.Q6(a)

Module-4

7 a. Derive the expressions of half power frequencies W_1 and W_2 and also bandwidth of a series resonance circuit. **(09 Marks)**

b. Find the values of L at which the circuit shown in Fig.Q7(b) resonates at a frequency of 500 r/s. **(07 Marks)**

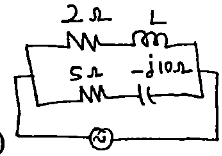


Fig.Q7(b)

OR

8 a. Derive the expressions of a resonance frequency and dynamic impedance of a parallel resonance circuit. **(09 Marks)**

b. A coil has a R = 20Ω, L = 80mH and C = 100pF are connected in series. Determine : i) impedance at resonance ii) resonance frequency iii) quality factor iv) circuit current if supply voltage is 50V. **(07 Marks)**

Module-5

9 a. Derive the expression of Z-parameters in term of h-parameter. **(07 Marks)**

b. Find the ABCD – parameters for the network shown in Fig.Q9(b). **(09 Marks)**

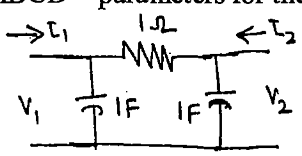


Fig.Q9(b)

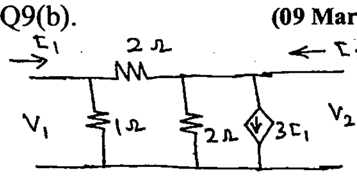


Fig.Q10(a)

OR

10 a. Find the Y-parameter for the two port network shown in Fig.Q10(a). **(08 Marks)**

b. Obtain the expression of h-parameters interms of Y-parameters. **(08 Marks)**

CBCS Scheme

USN 1GA15EC095

B-C-DIWAKARA

15EC34

Third Semester B.E. Degree Examination, Dec.2016/Jan.2017
Network Analysis

Time: 3 hrs. Max. Marks: 80

Note: Answer FIVE full questions, choosing one full question from each module.

Module-1

1 a. Derive the expression for i) Δ to Y transformation ii) Y to Δ transformation. **(10 Marks)**

b. Using source Transformation, find power delivered by 50V source. Shown in Fig Q1(b). **(06 Marks)**

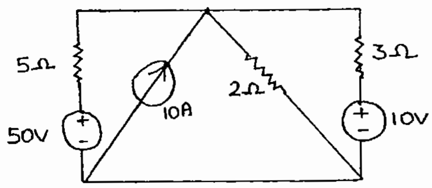


Fig Q1(b)

OR

2 a. Find the voltage across 20Ω resistor in the Network. Shown in Fig Q2(a) by Mesh analysis. **(08 Marks)**

b. Find i, using nodal analysis for the circuit shown in Fig Q2(b). **(08 Marks)**

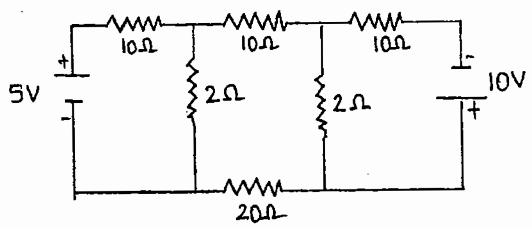


Fig Q2(a)

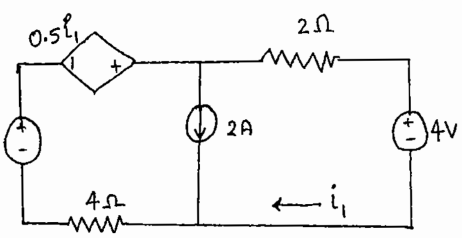


Fig Q2(b)

Module-2

3 a. State and prove maximum power transfer Theorem for AC circuits. **(08 Marks)**

b. For the network shown in Fig Q3(b), obtain the Thevenin's equivalent as seen from terminals p and q. **(08 Marks)**

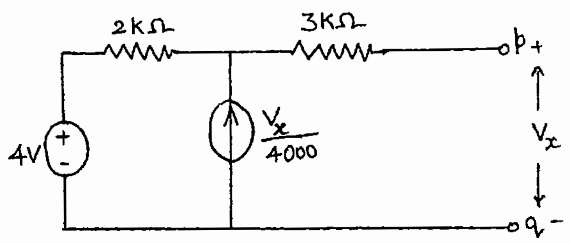


Fig Q3(b)

OR

4 a. State and explain Millman's theorem. **(08 Marks)**

b. Verify reciprocity theorem for the circuit shown in Fig Q4(b). **(08 Marks)**

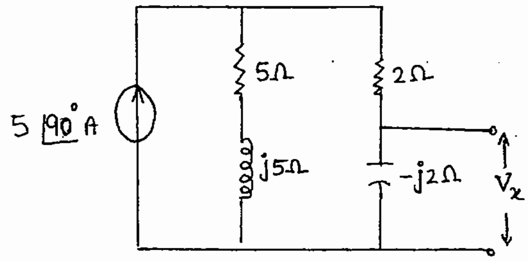


Fig Q4(b)

Module-3

5 a. Stat and prove initial value Theorem and final value theorem. **(08 Marks)**

b. In the circuit shown in Fig Q5(b) V = 10V, R = 10Ω, L = 1H, C = 10μF and $V_c = 0$. Find $i(0^+)$, $\frac{di}{dt}(0^+)$ and $\frac{d^2i}{dt^2}(0^+)$, it switch K is closed at t = 0. **(08 Marks)**

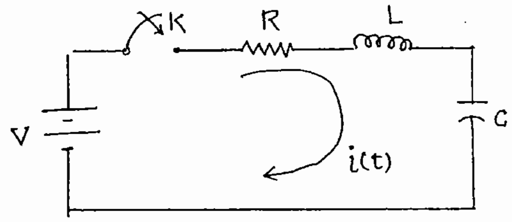


Fig Q5(b)

OR

6 a. In the network shown in Fig Q6(a), a steady state is reached with the switch K open. At t = 0, the switch is closed. For the element values given, determine the values of $V_a(0^-)$ and $V_a(0^+)$. **(08 Marks)**

b. Obtain the Laplace Transform of saw tooth waveform shown in Fig Q6(b). **(08 Marks)**

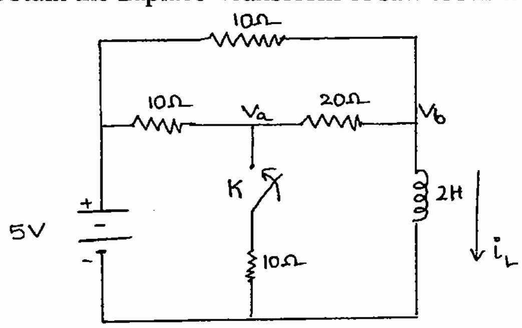


Fig Q6(a)

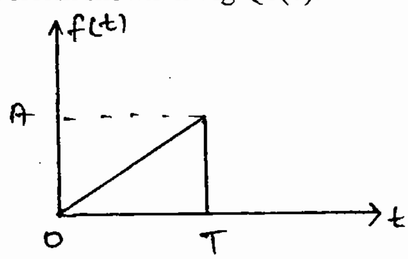


Fig Q6(b)

Module-4

7 a. Prove that $f_0 = \sqrt{f_1 f_2}$ where f_1 and f_2 are the two half power frequencies of a resonant circuits. **(08 Marks)**

b. A series RLC circuit consists of R = 10Ω, L = 0.01H and C = 0.01μF is connected across a supply of 10mV. Determine, i) f_0 ii) Q-factor iii) BW iv) f_1 and f_2 and v) I_0. **(08 Marks)**

OR

8 a. Obtain the expression for the resonant frequency for the circuit shown in Fig Q8(a) **(08 Marks)**

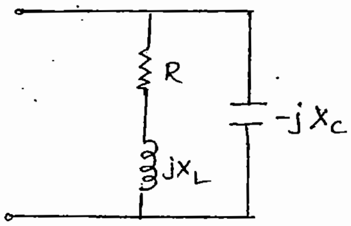


Fig Q8(a)

b. An RLC series circuit has an inductive coil of 'R' Ω resistance and inductance of 'L' H is in series with a capacitor 'C' F. The circuit draws a maximum current of 15A when connected to 230V, 50Hz supply. If the Q-factor is 5, find the parameter of the circuit. **(08 Marks)**

Module-5

9 a. Derive the z-parameters in terms of Y parameters. **(08 Marks)**

b. Determine Y parameter of the two – port network shown in Fig Q9(b). **(08 Marks)**

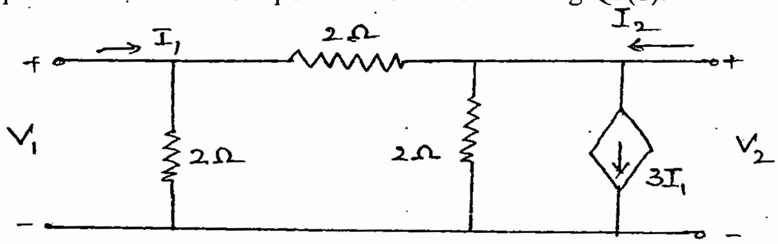


Fig Q9(b)

OR

10 a. Obtain hybrid parameters (h) in terms of impedance parameters (z). **(08 Marks)**

b. Find the Y parameters for the circuit shown in Fig Q10 (b). Then use the parameter relationship to find ABCD parameters. **(08 Marks)**

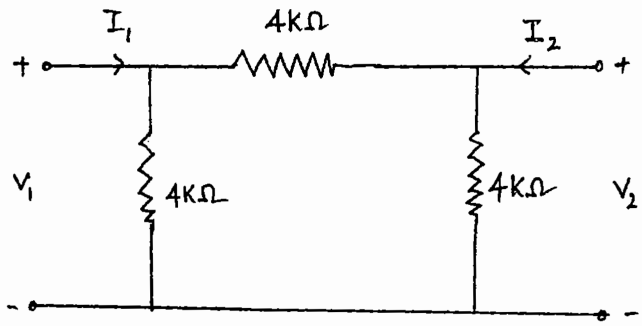


Fig Q10(b)

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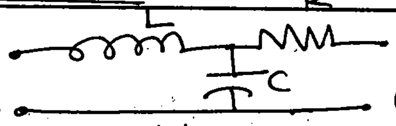
Network analysis
[15EC34]
module - 1
Basic concepts

DIVAKAR B C

Introduction ⇒



(a) network

Network ⇒ Interconnection of various electrical elements connected in any manner is called network

There are 2 types of electrical elements namely passive & active

Active element is one which provides energy to an electrical network/circuit
Ex ⇒ voltage source / current source

Passive elements are those which stores energy (or) dissipates energy in the form of heat & it does not contain any source of Emf in it.

Network analysis ⇒ Finding the voltage or current in the given network by adopting fundamental laws and other simplification techniques is known as network analysis

Classification of network ⇒

Based on the behaviour of the network, electrical networks are classified as

a) Linear and Nonlinear networks
b) unilateral and bilateral networks

Ⓒ Active and passive networks
Ⓓ Lumped and distributed networks

Linear network ⇒ If the Resistance inductance or capacitance offered by an element does not change linearly with the change in the applied voltage or circuit current (I) element is termed as linear element

Non Linear network ⇒ A network whose parameters change their values with change in voltage, current, time

Unilateral network

properties or characteristics changes with its direction of operation
Ex:- Diode [which allows flow of current only in one direction]

Bilateral network ⇒ properties or characteristics remains same in either direction
Ex ⇒ Resistor

Active & passive network

Active network is one which contains at least one energy source
Energy source may be either voltage or current
Ex:- Batteries, opamp, BJT

Passive network ⇒ It is one which does not contain any source of Emf in it
Ex:- Resistor, Inductor, capacitor

Lumped network ⇒ A network in which all the network elements are physically separable.

Distributed network ⇒ A network in which the circuit elements cannot be physically separable
Ex ⇒ Transmission Line where R, L, C are distributed along the Length & cannot be shown as separate elements

Current [I] ⇒ Rate of flow of electrons/charges is called electric current
unit is amperes

Voltage | Emf | potential difference
The force that tend to cause a free electron to move from a negative terminal of a battery to a positive terminal of a battery is called Voltage
unit is volts

Branch ⇒ circuit elements connected between 2 terminals usually in series [End to end connection]

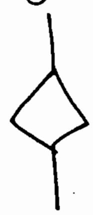
Node or junction ⇒ It is a common point where two or more branches meet.

Loop ⇒ It is a closed path

Mesh ⇒ It is also a loop which does not contain any other loop inside it

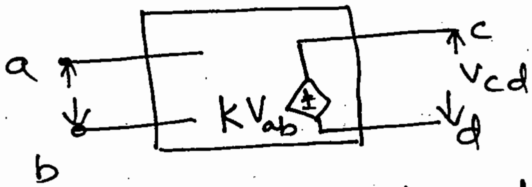
Energy sources ⇒ Depending upon voltage current characteristics, the energy sources are classified in to
a) Dependent voltage | current sources
b) Independent voltage | current sources

Dependent sources ⇒ A Dependent voltage or current source is one which depends on some other quantity i.e, it may be either voltage / current in the circuit.
They are also called as controlled sources & it is represented by Rhombus | Diamond shape symbol



There are four types
a) Voltage controlled voltage source
b) Voltage controlled current source
c) Current controlled current source
d) Current controlled voltage source

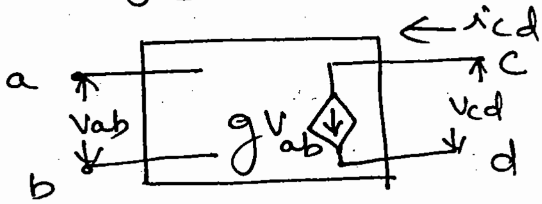
a) Voltage controlled voltage source [VCVS]



A voltage controlled voltage source is a four terminal network component that establishes a voltage V_{cd} between 2 points c & d in the circuit that is proportional to a voltage V_{ab} between 2 points a and b.

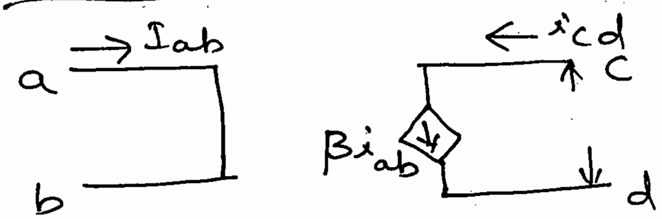
$$V_{cd} = K V_{ab}$$

b) Voltage controlled current source [VCCS]



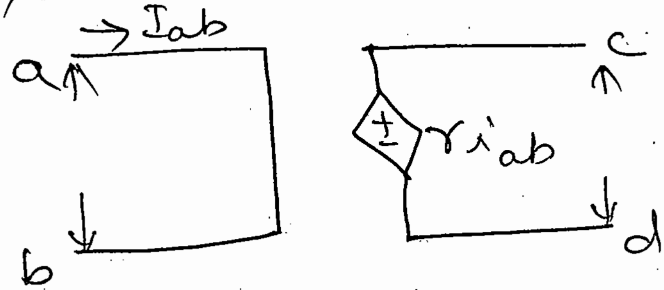
$$i_{cd} = g V_{ab}$$

c) current controlled current source [CCCS]



$$i_{cd} = \beta i_{ab}$$

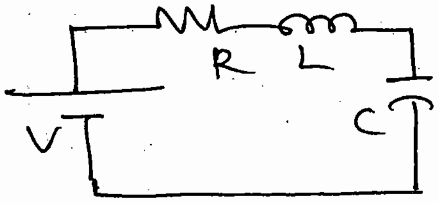
d) current controlled voltage source [CCVS]



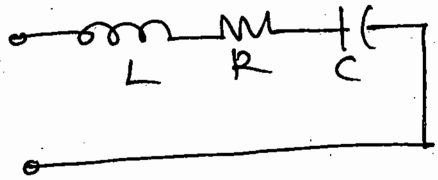
$$V_{cd} = r i_{ab}$$

circuit ⇒ A closed conducting path through which electric current intended to flow.

Every circuit is a network, but all networks are not circuits



(a) circuit



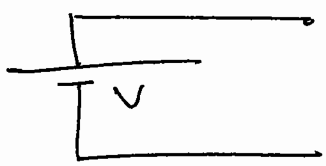
(b) network

Independent sources ⇒
A source in which voltage or current does not dependent on any other quantity in the given circuit is called Independent sources

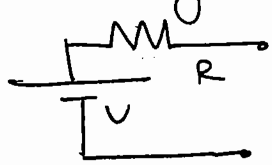
Independent sources may be
i) Ideal | practical voltage sources and current sources

An Ideal voltage source is the energy source which delivers constant voltage irrespective of network configuration

practical voltage source is the energy source which has small internal resistance with voltage source.



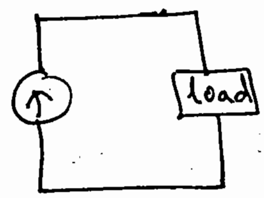
Ideal voltagesource



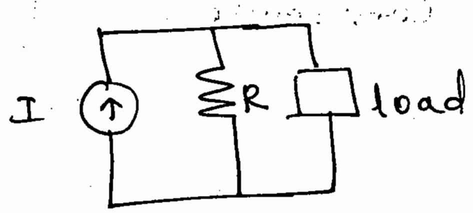
Practical voltagesource

ii) Ideal current source is the energy source which gives constant current across its terminals irrespective of the voltage connected across its terminals

practical current source is the energy source which has small internal resistance connected in parallel with the current source



Ideal current source



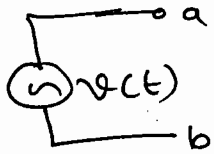
practical current source

The voltage source can be classified as

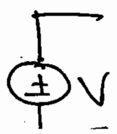
a) Time variant source | ac source

b) Time Invariant source | DC source

Time variant source ⇒ It is the source in which the voltage is varying with time (or) ac sources, represented by Lower case letters

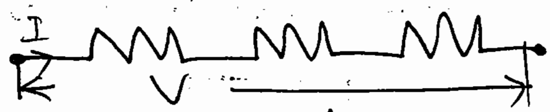


Time Invariant sources ⇒ It is the source in which the voltage is not varying with time (or) DC source represented by capital letters



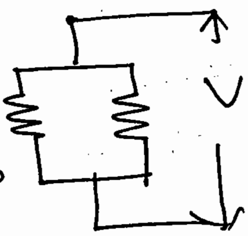
Ohm's Law ⇒ current flowing through a conductor is directly proportional to the potential difference between its ends provided that the temperature remains constant.

$$V \propto I \quad \text{or} \quad V = IR$$

* End to end connection is called series connection [

In series connection current remains same where as the voltage divides

* Any number of elements connected between the 2 terminals is called parallel connection



In parallel connection current divides where as the voltage remains same

Always voltage sources should be short circuited

In short circuit $R_{sc} = 0$

I = maximum $\quad \therefore V = I \times R_{sc}$

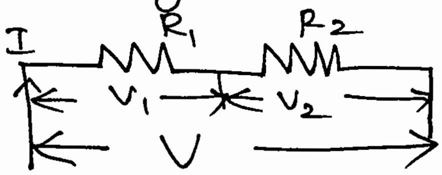
$$\therefore V = 0$$

current sources should be open circuited

$R_{oc} = \infty$, $I = 0$, $I = \frac{V}{R} = \frac{V}{\infty} = 0$

$\therefore V_{oc} = \infty$

Voltage divider rule [series connection]



when the resistances are connected in series the current remains the same where as the voltage divides.

The equivalent resistance $R_{eq} = R_1 + R_2$

The voltage drop across $V_1 = I \cdot R_1$

but by ohm's Law $I = \frac{V}{R_{eq}}$

$\therefore V_1 = \frac{V}{R_{eq}} \cdot R_1 = \frac{V \cdot R_1}{R_1 + R_2}$

Similarly the voltage drop across R_2 i.e, V_2

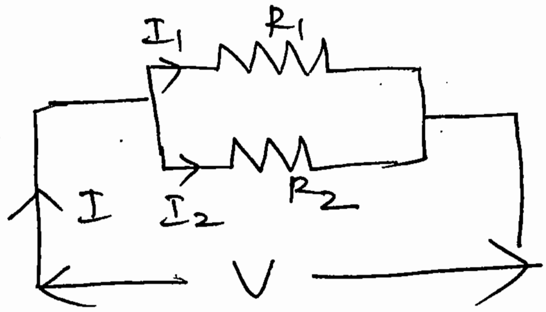
$V_2 = I \cdot R_2 = \frac{V}{R_{eq}} \cdot R_2 = \frac{V \cdot R_2}{R_1 + R_2}$

In general "The voltage divider rule states that the voltage across that branch is equal to the applied voltage multiplied by the resistance of the same branch divided by the sum of the resistances"

Current divider rule [Parallel connection]

When the resistances are connected in parallel

The potential drop remains the same, the current divides & $R_S = \frac{R_1 R_2}{R_1 + R_2}$



The current for the first branch

$I_1 = \frac{V}{R_1}$ but $V = I \cdot R_S = \frac{I \cdot R_1 R_2}{R_1 + R_2}$

$\therefore \quad I_1 = \frac{I \cdot \cancel{R_1} R_2}{\cancel{R_1}(R_1 + R_2)} \Rightarrow \frac{I \cdot R_2}{R_1 + R_2}$

The current for the second branch

$I_2 = \frac{V}{R_2}$ but $V = I \cdot R_S = \frac{I \cdot R_1 R_2}{R_1 + R_2}$

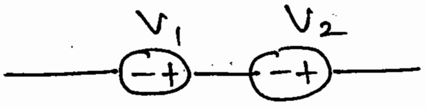
$\therefore \quad I_2 = \frac{I \cdot R_1 \cancel{R_2}}{\cancel{R_2}(R_1 + R_2)} = \frac{I \cdot R_1}{R_1 + R_2}$

Current divider rule states that

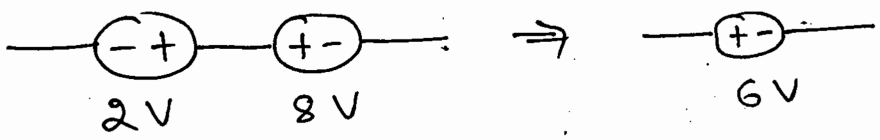
The current across that branch is equal to the main current multiplied by the resistance connected to the opposite branch divided by sum of all the resistances.

Combination of sources

Ideal voltage sources in series

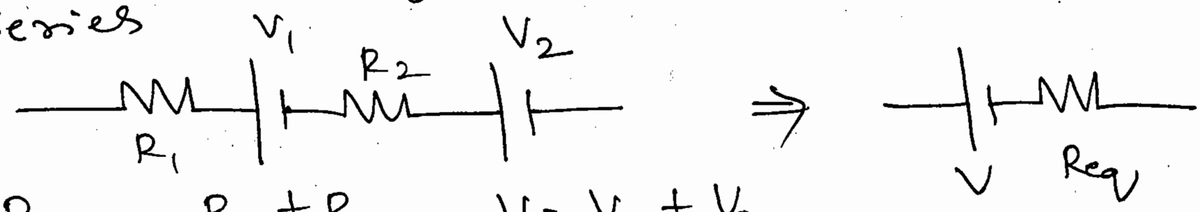


$\Rightarrow \quad V = V_1 + V_2$



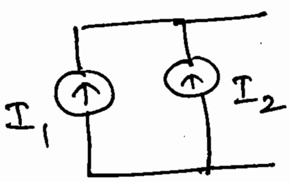
Same polarity $\Rightarrow$ subtraction
opposite polarity $\Rightarrow$ addition

practical voltage sources connected in series



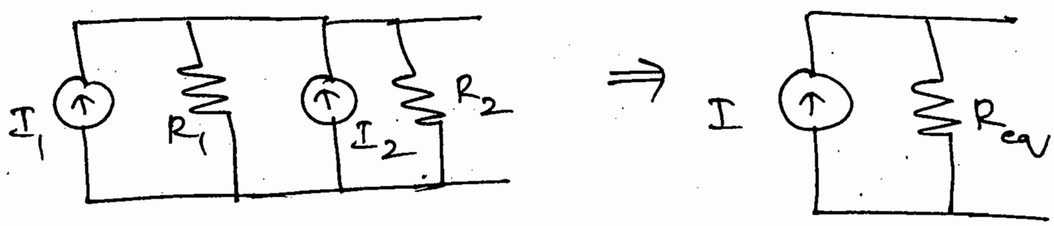
$R_{eq} = R_1 + R_2 , \quad V = V_1 + V_2$

Practical current sources in parallel



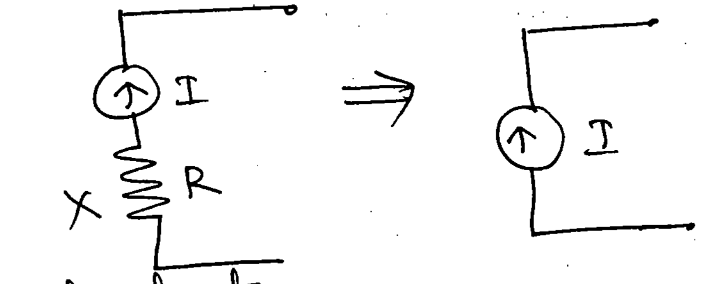
$\Rightarrow \quad I = I_1 + I_2$

practical current sources in parallel



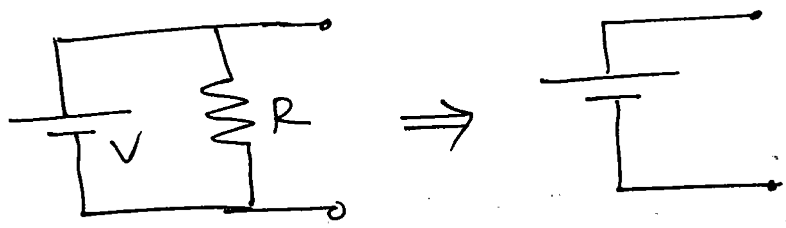
$I = I_1 + I_2 \qquad R_{eq} = \dfrac{R_1 R_2}{R_1 + R_2}$

Resistance connected in series with the current source should always be neglected [redundant] because the current remains same in series connection



Redundant

Resistance connected in parallel with the voltage source is Redundant because the voltage remains same in parallel connection



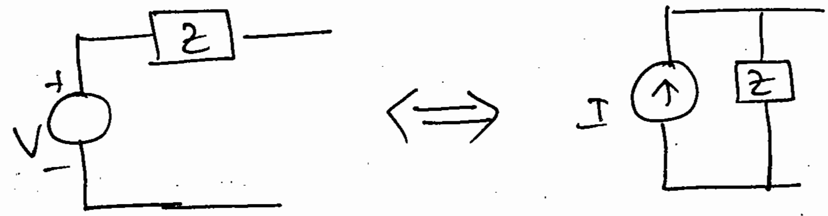
Source transformation ⇒

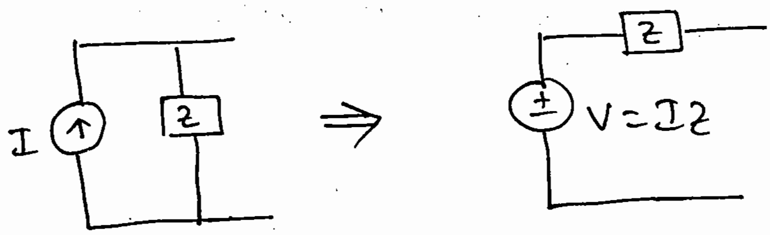
In source transformation a voltage source is transformed in to a current source (or) vice versa. Transformation of source is done retaining the terminal characteristics of the original source.

A voltage source in series with an Impedance can be transformed in to a current source in parallel with an impedance. If V is the source voltage & Z is the series impedance

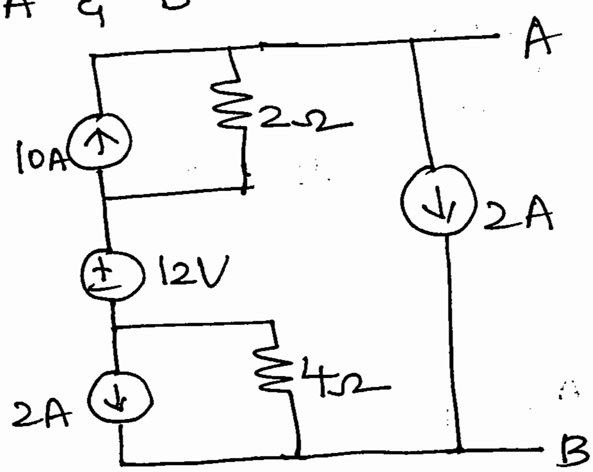
$$I = \frac{V}{Z}$$

Conversely, a current source with an impedance in parallel can be transformed in to a voltage source in series with an impedance & source voltage is $V = IZ$

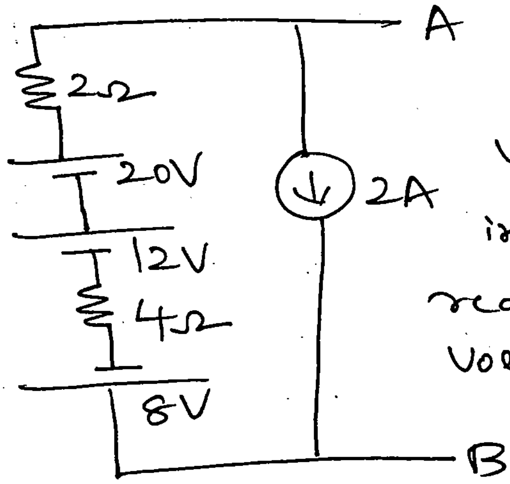




① Reduce the network shown in figure in to a single voltage source between A & B

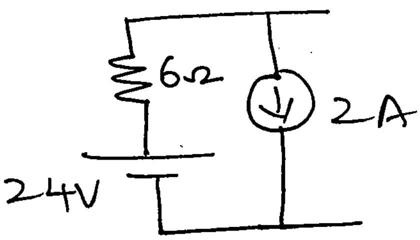


Current sources 10 A & 2A are converted to voltage sources & the circuit can be redrawn as shown

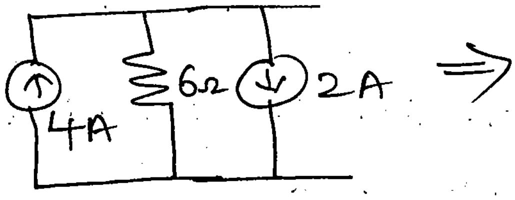


20V, 12V & 8V voltage sources are in series & they can be reduced into a single voltage source

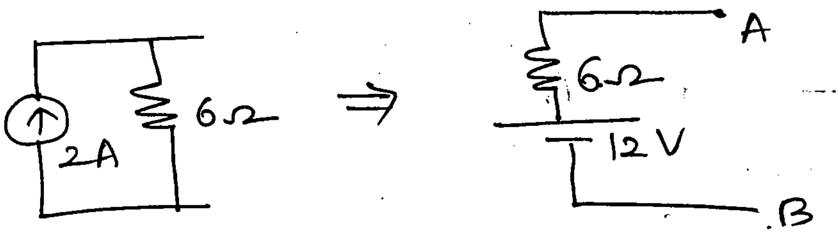
$32 - 8 = 24V$; $2 + 4 = 6\Omega$



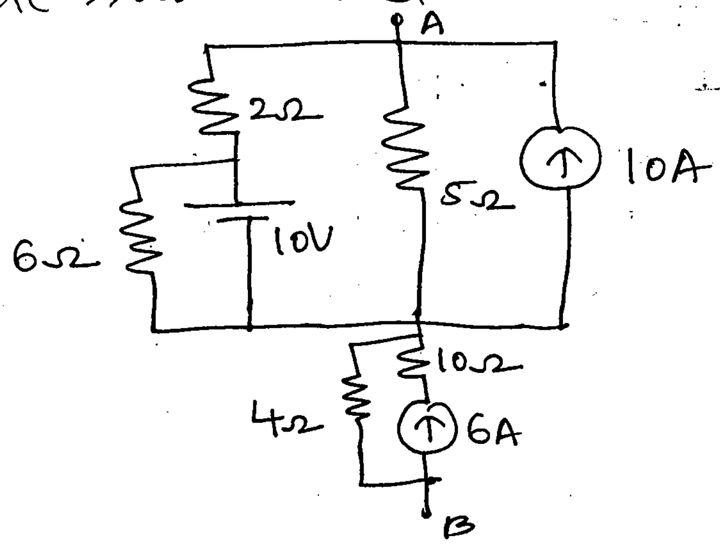
24V is converted into current source



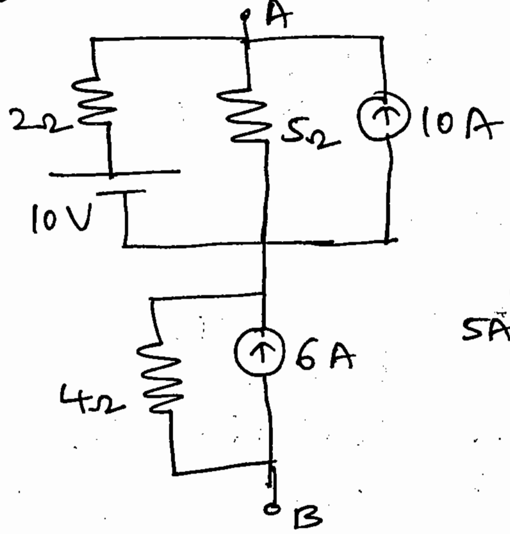
⇒ current source direction is opposite hence subtract.



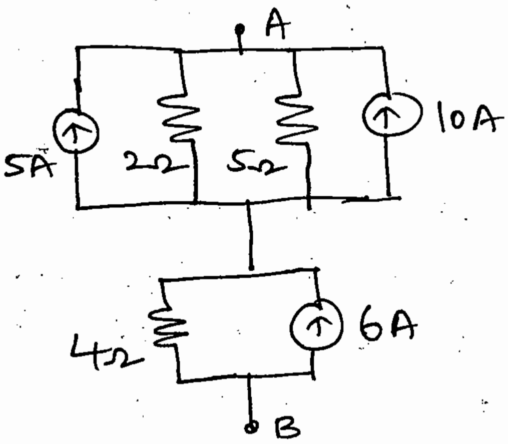
(2) Reduce the network shown into a single source network

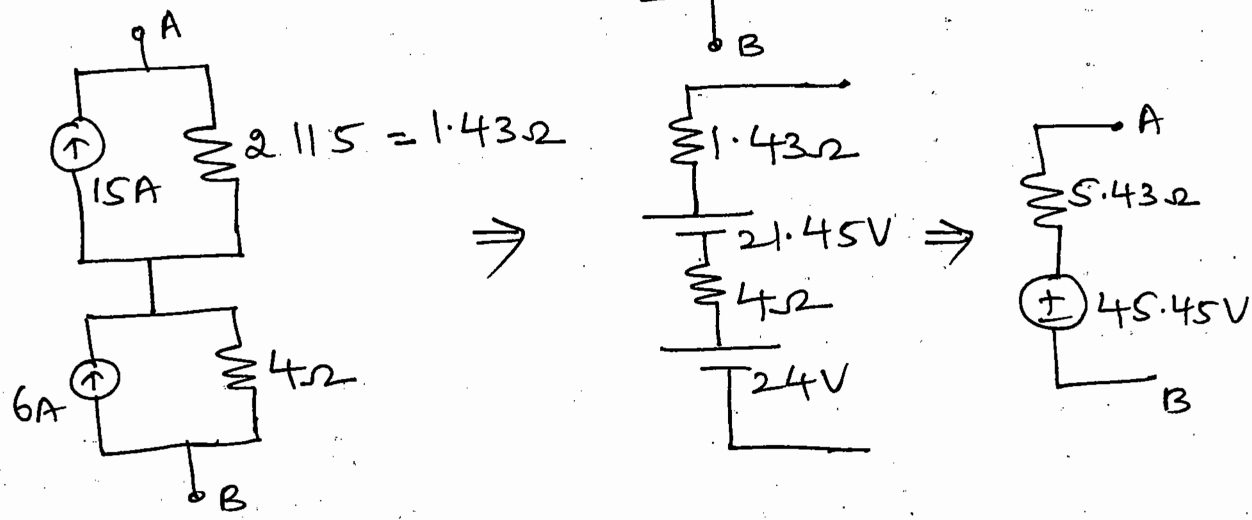


6 Ω resistor in parallel with 10 V & 10 Ω resistor is in series with 6 A current source are redundant hence neglected.

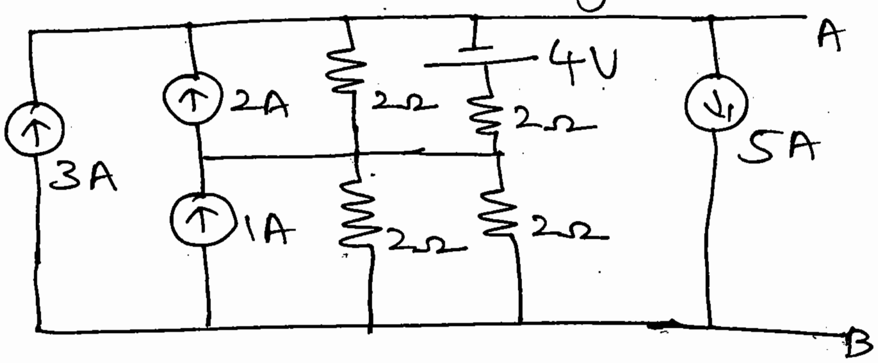


10 V source can be converted to currentsource

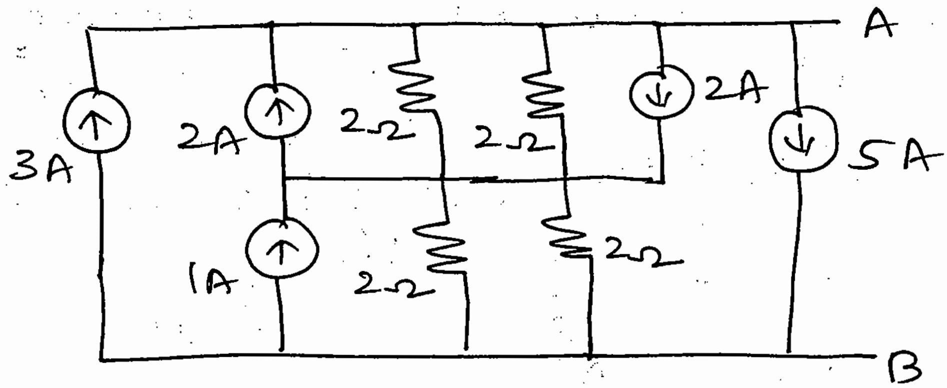




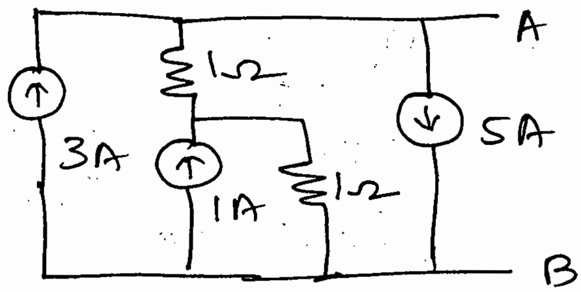
③ using sourcetransformation technique reduce it in to a single source network



converting 4 V source to a current source

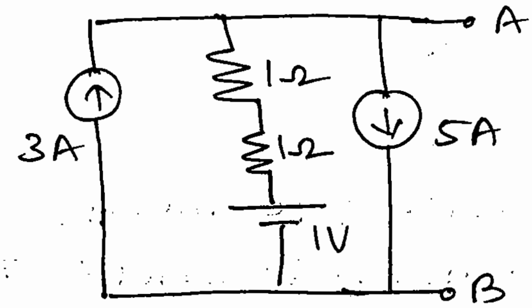


since the 2 2A current sources are acting in opposite direction the resultant current will be zero.

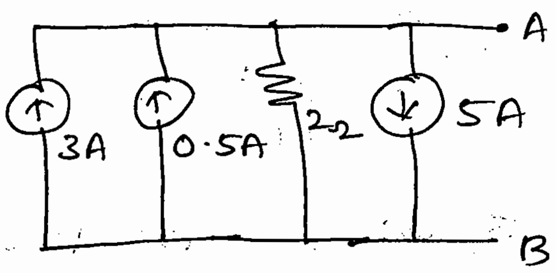


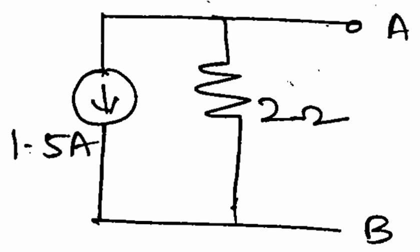
$2\Omega \parallel 2\Omega = 1\Omega$

converting 1 A current source to voltage source

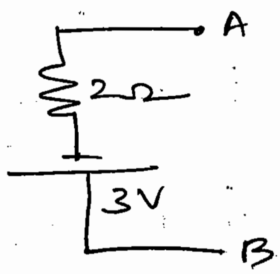


$\Rightarrow$

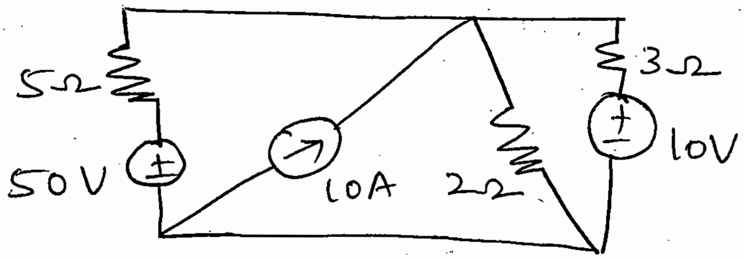




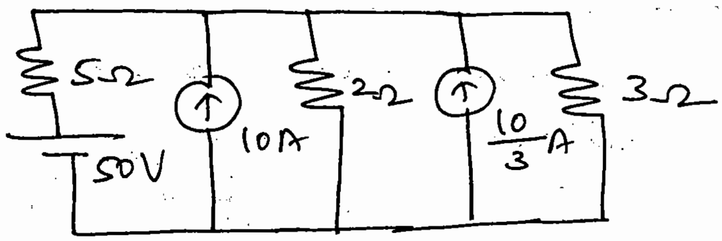
$\Rightarrow$

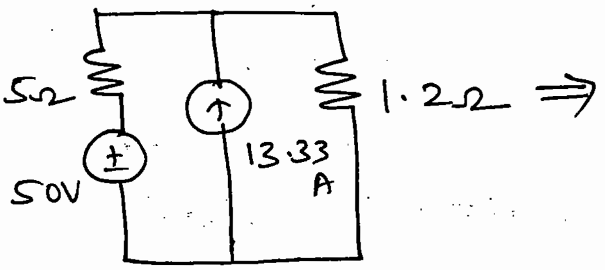


using source transformation technique find the power delivered by 50V source for the figure shown

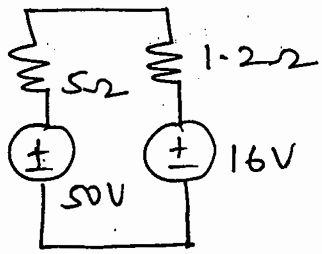


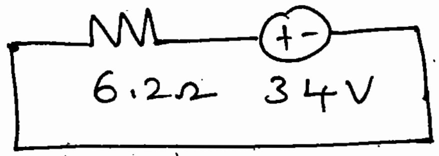
convert 10V in series with 3Ω to current source





⇒





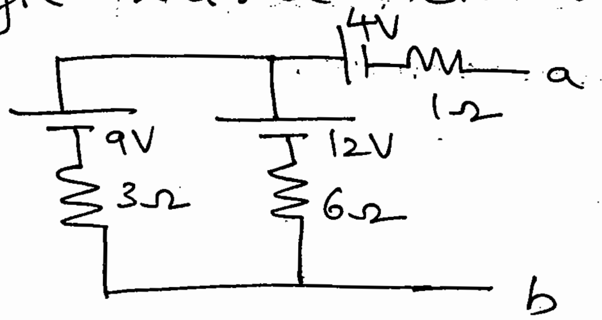
$I = \frac{34}{6.2} = 5.48A$

power delivered by 50V source is

$P = VI = 50 \times 5.48$

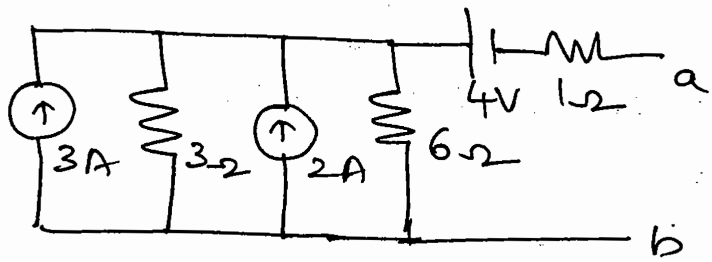
$\boxed{P = 274.19 \text{ watts}}$

Reduce the network shown in to a single source network



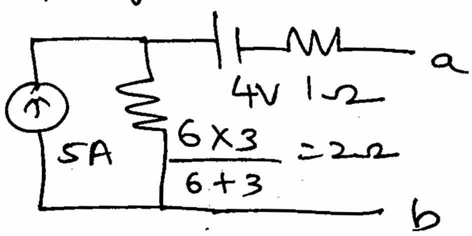
$\frac{9}{3} = 3A$

$\frac{12}{6} = 2A$

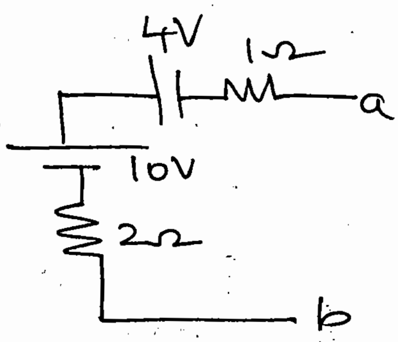


Direction of current source is same

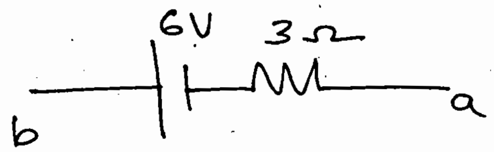
∴ perform addition



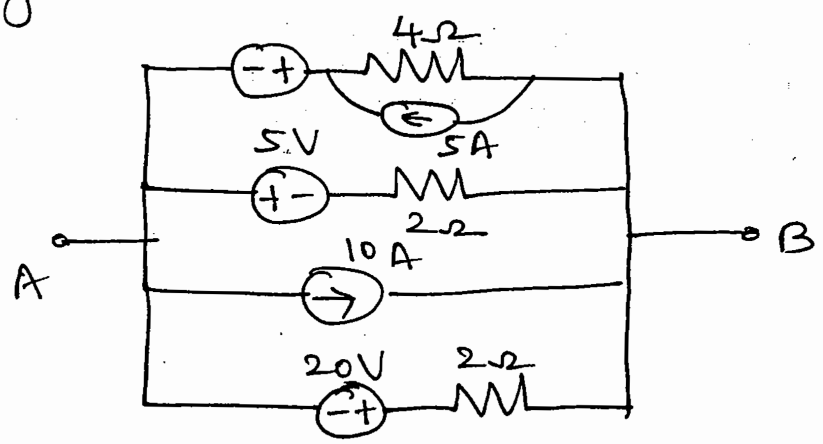
⇒



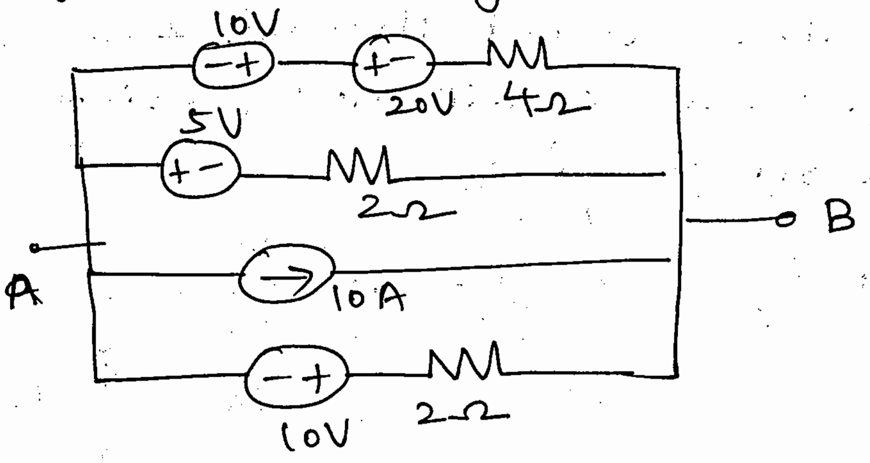
Voltage source polarity is same hence subtract

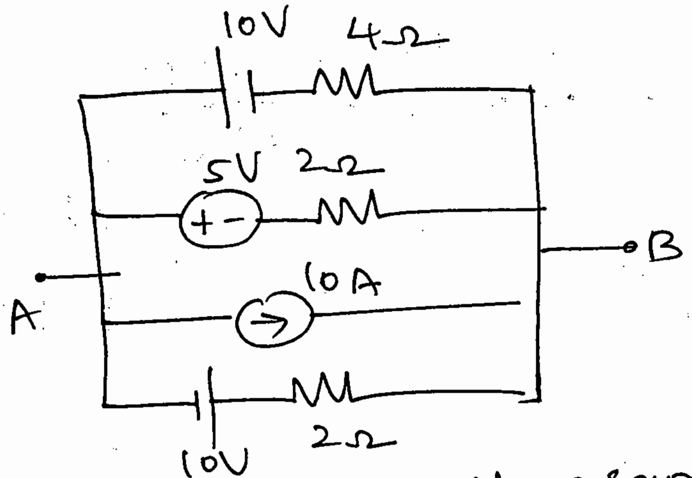


Reduce the network shown in to a single source network



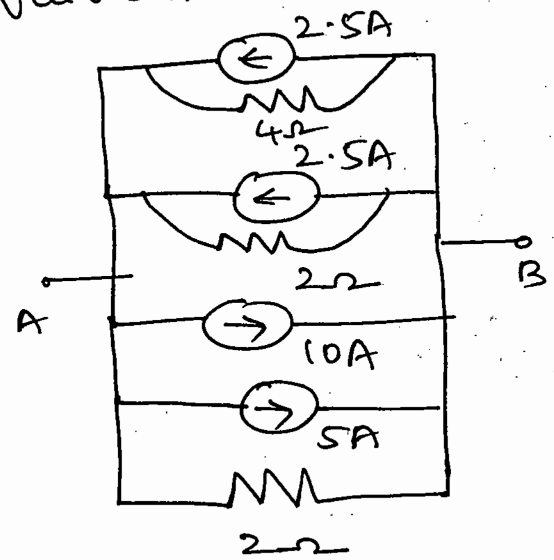
converting 5A || with 4Ω to its equivalent voltage source



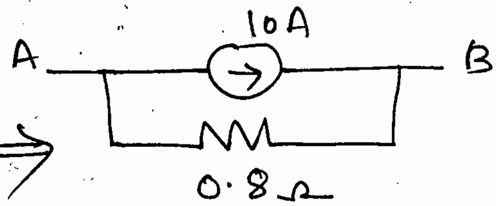


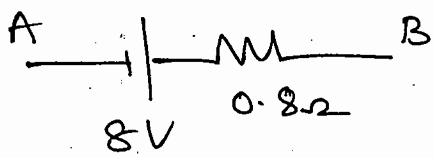
converting all voltage sources in to its equivalent current sources

$15-5=10\,A$



$\Rightarrow$

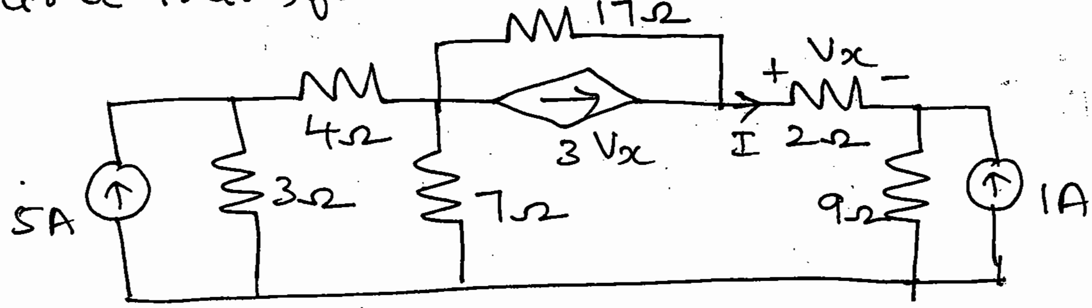


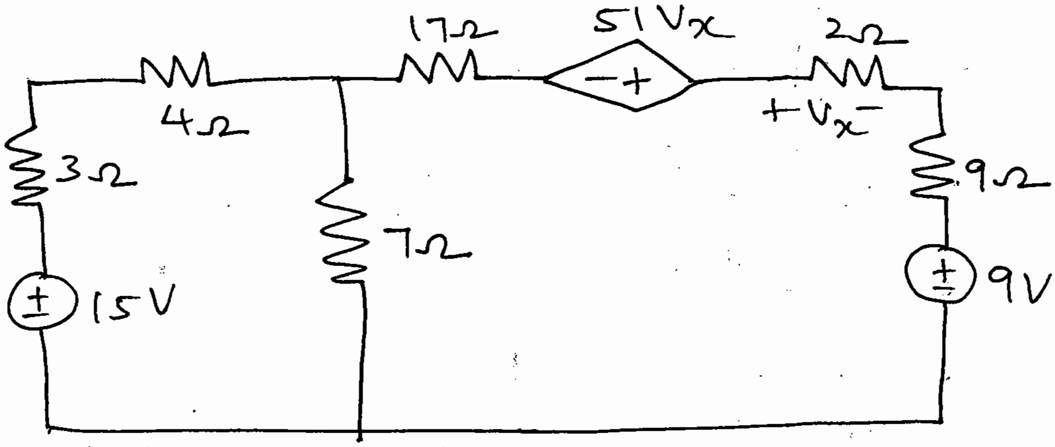


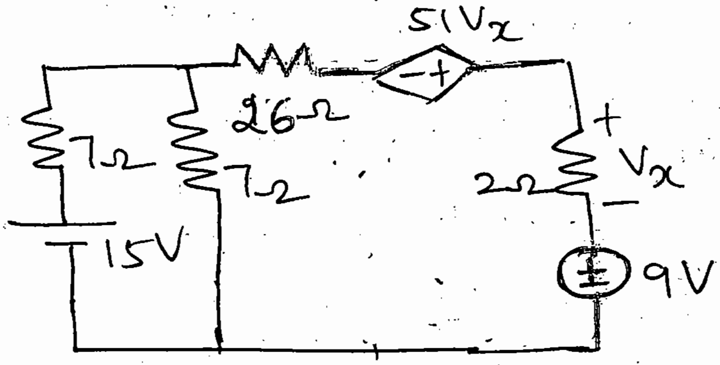
July 2017

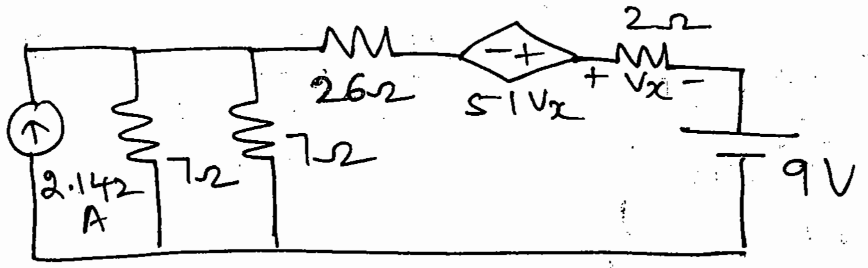
(08m)

Calculate the current through 2Ω resistor for the circuit shown using source transformation







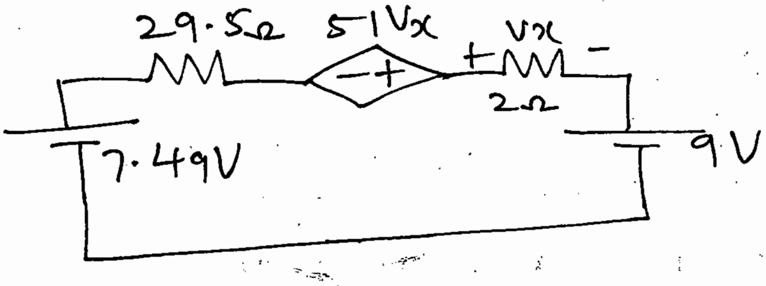


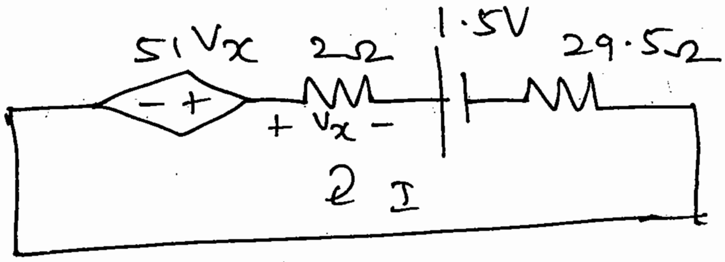
$7 \| 7$

$= 3.5\Omega$

3.5×2.142

$= 7.49V$





$V_x = 2I$

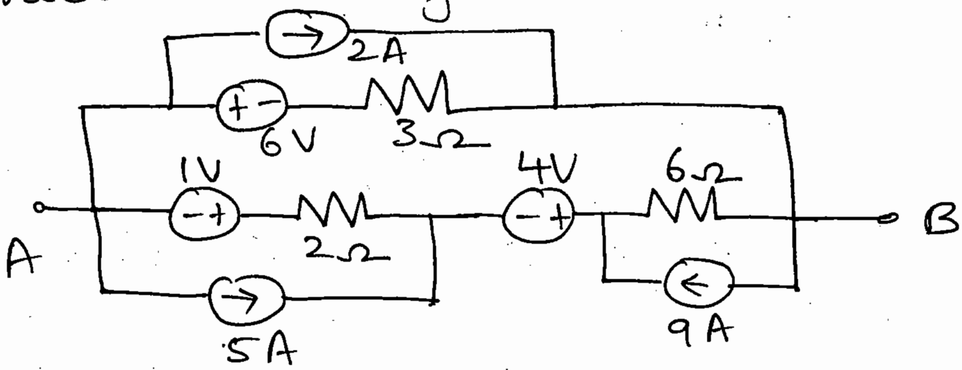
$$51V_x - 29.5I - 2(I) - 1.5 = 0$$

$$51(2I) - 29.5I - 2I - 1.5 = 0$$

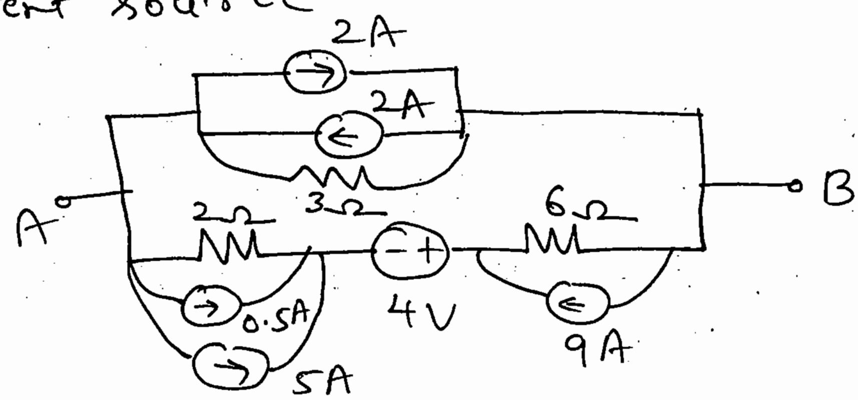
$$70.5I = 1.5$$

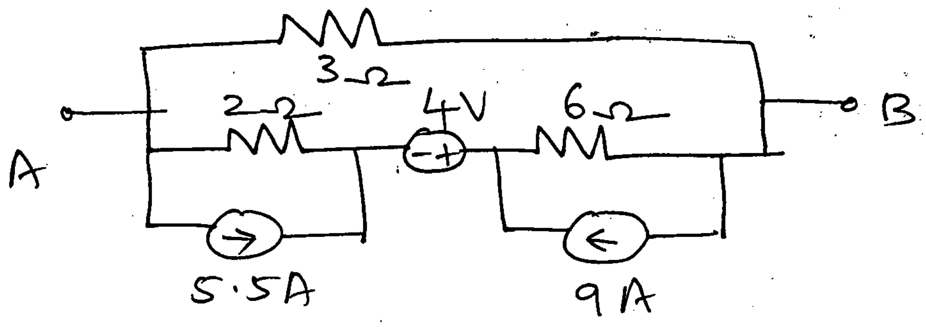
$$\boxed{I = 21.27\ mA}$$

Reduce the network shown in to a practical voltage source

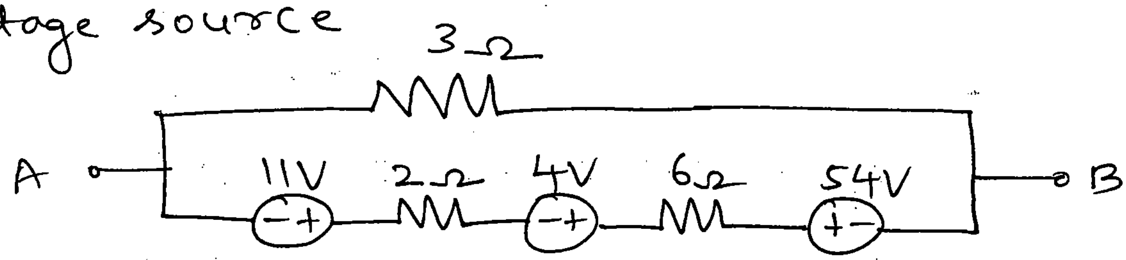


convert 6V & 1V voltage source to current source

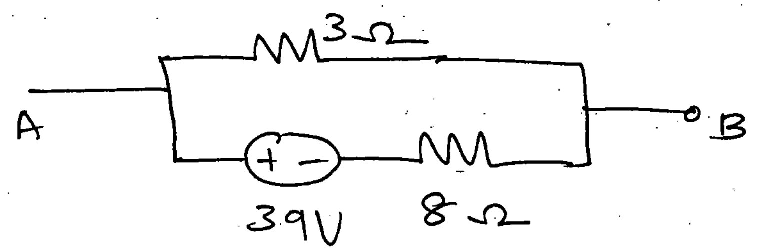


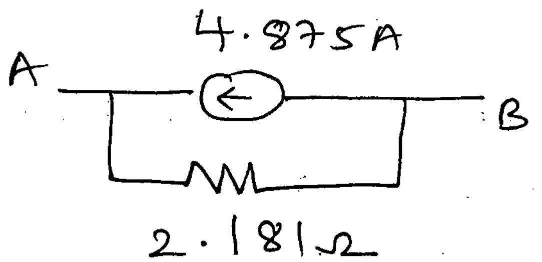


Convert 5.5A & 9A current sources to voltage source

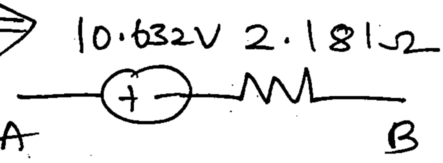


$$\begin{array}{r} 54V \\ -15V \\ \hline 39V \end{array}$$



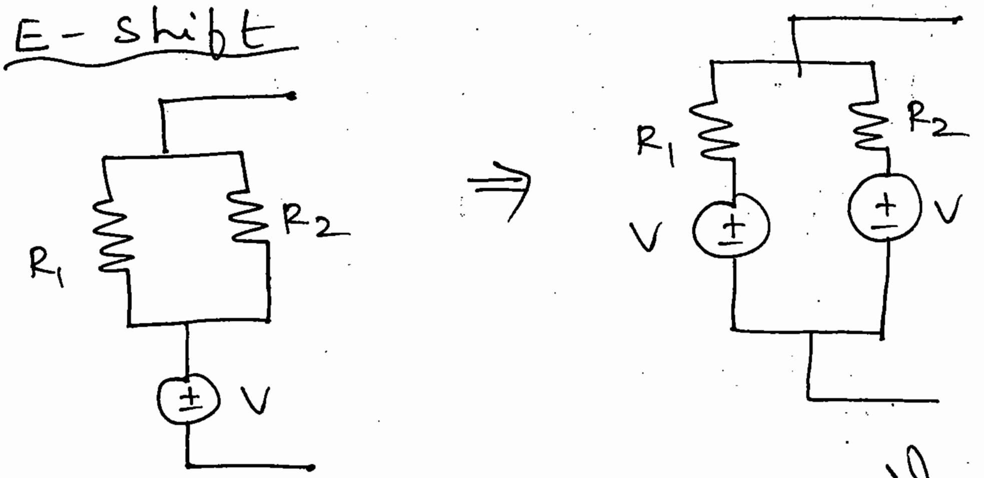


⇒

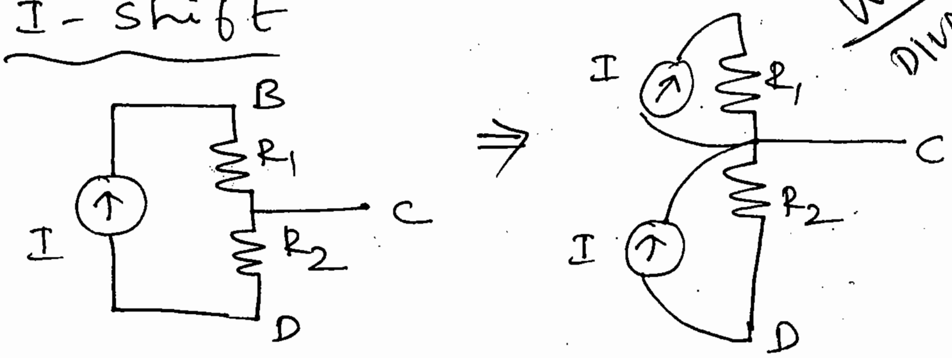


Source shifting ⇒ If in a network there is no impedance / Resistance in series with voltage source (or) If there is no Impedance in parallel with current source, then the source transformation cannot be applied directly. In such cases we use source shifting i.e, E-shift [Voltage source shift] and I-shift [current source shift]

E- shift

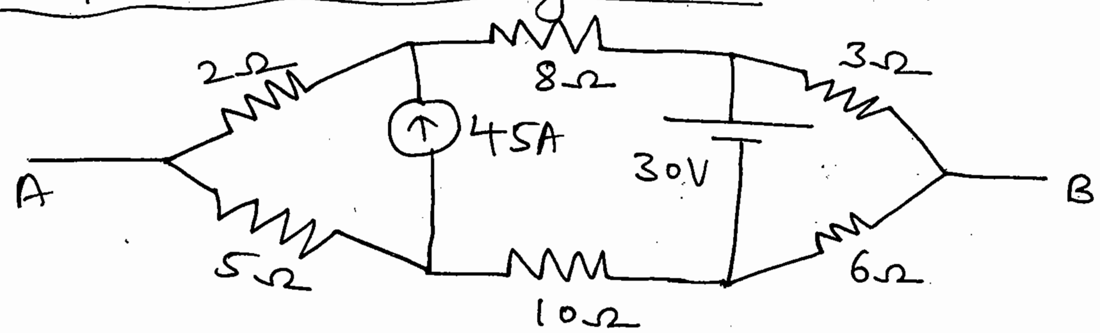


I- shift

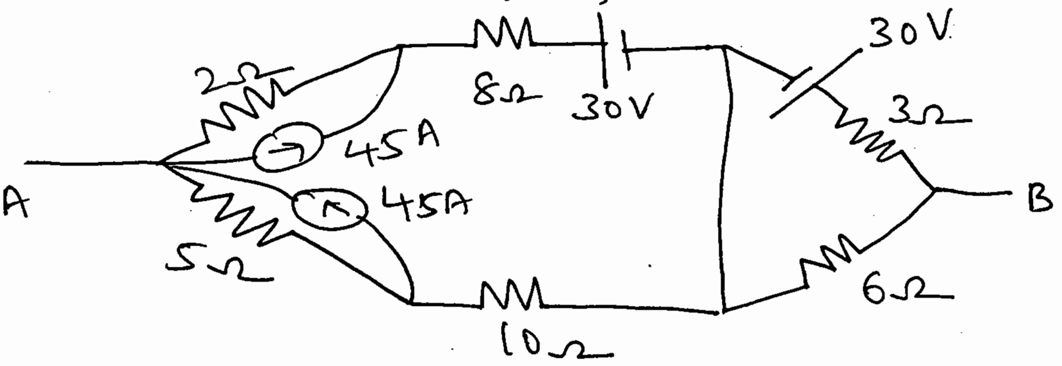


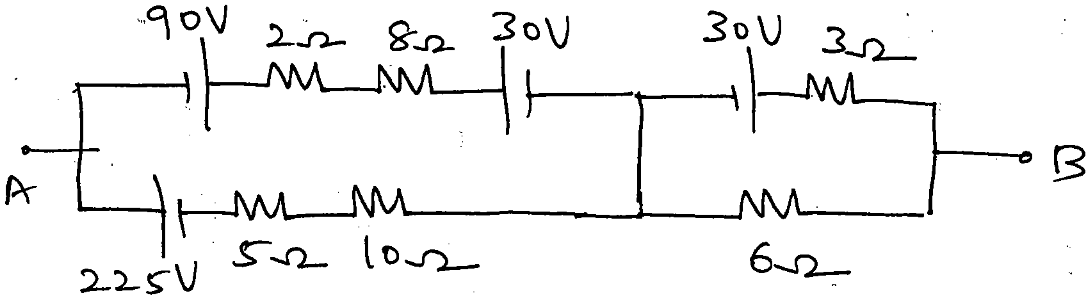
DIVAKAR-B-C

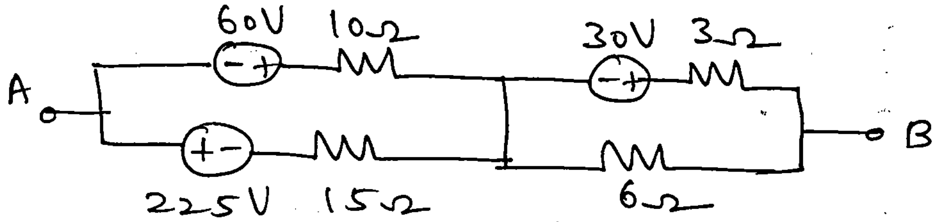
Reduce the network shown in fig in to a practical voltagesource

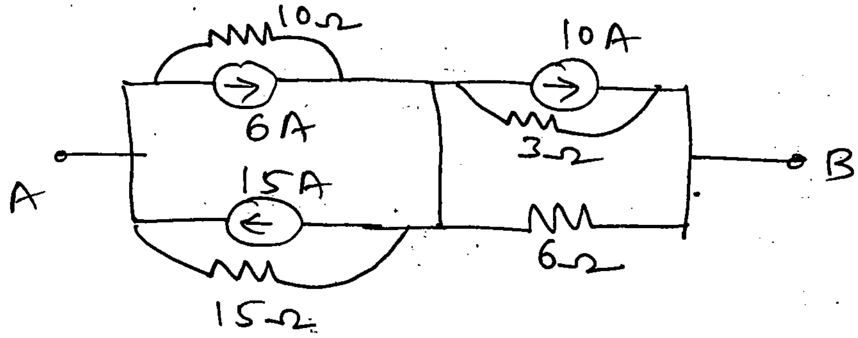


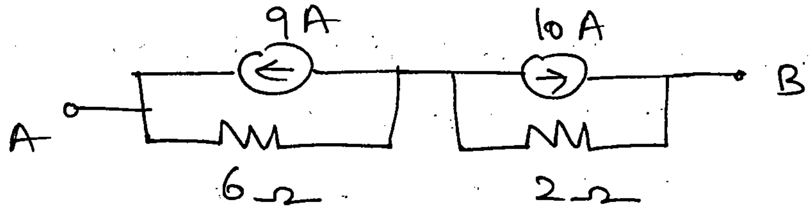
Perform I-shift & E-shift

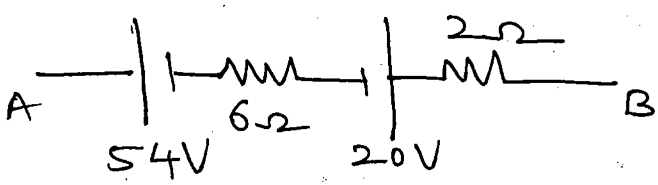


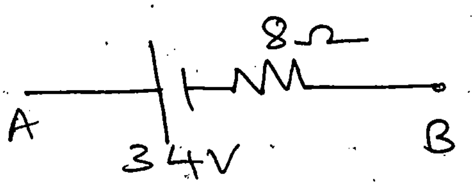

90V
2Ω
8Ω
30V
30V
3Ω
A
B
225V
5Ω
10Ω
6Ω


60V
10Ω
30V
3Ω
A
B
225V
15Ω
6Ω

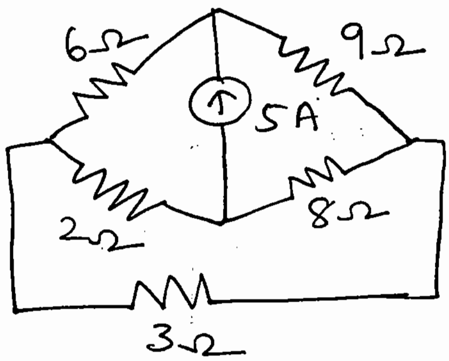

10Ω
10A
6A
3Ω
A
B
15A
6Ω
15Ω


9A
10A
A
B
6Ω
2Ω

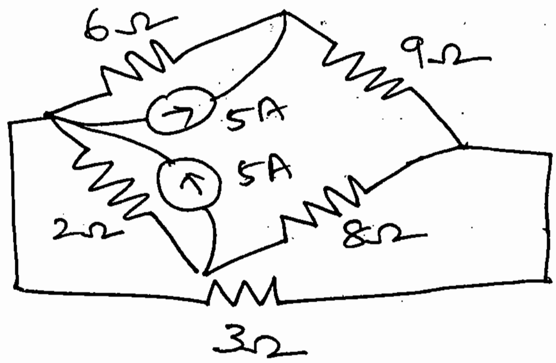

2Ω
A
6Ω
B
54V
20V

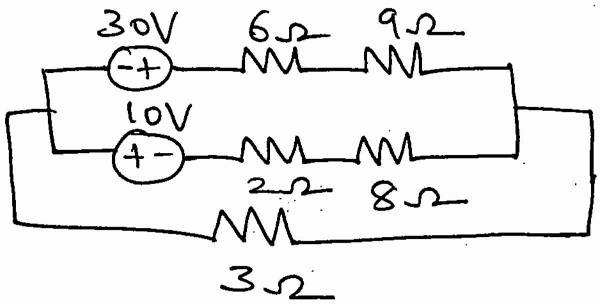

8Ω
A
B
34V

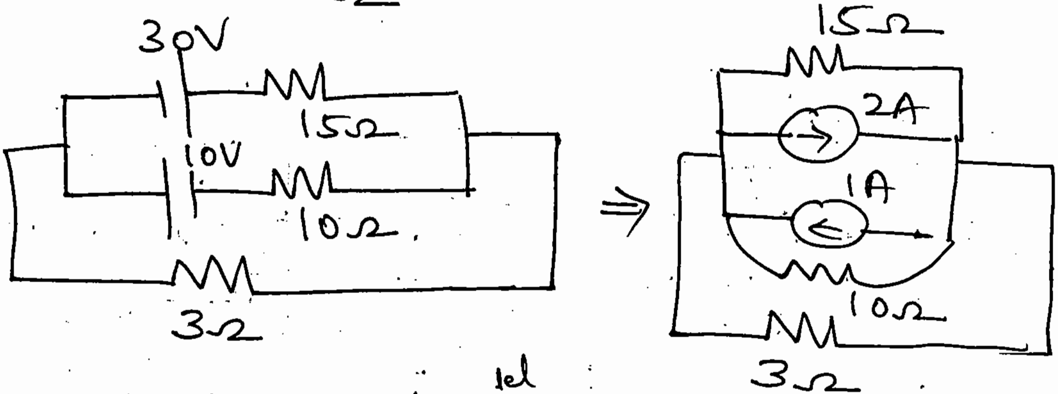
Reduce the network shown in to a single source network using source shifting technique



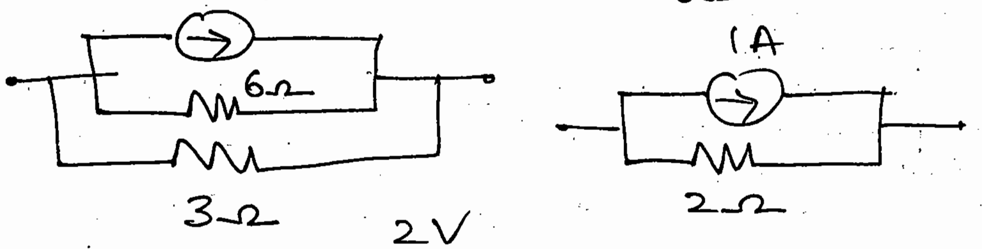
perform I-shift

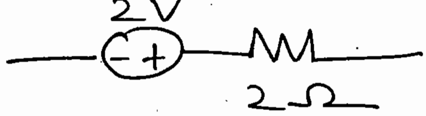




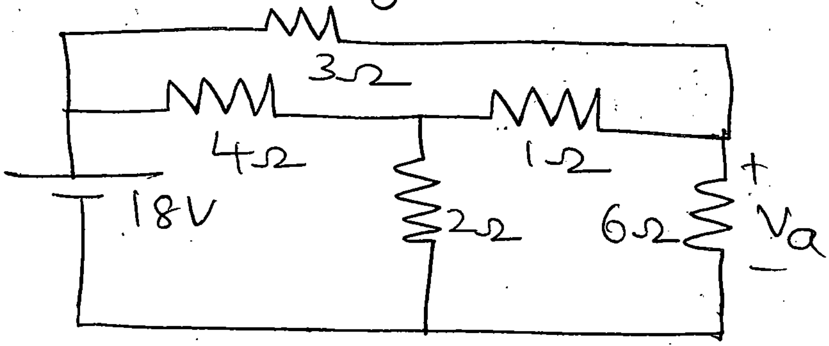


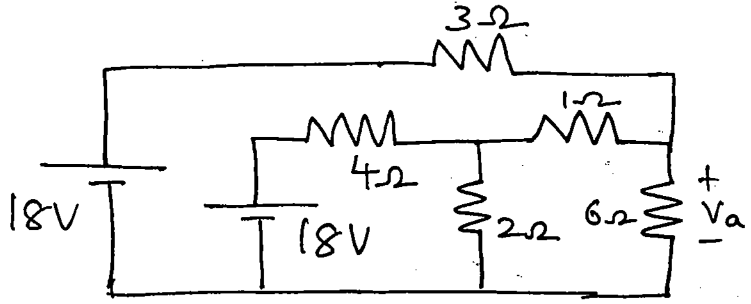
15 || 10 = 6Ω



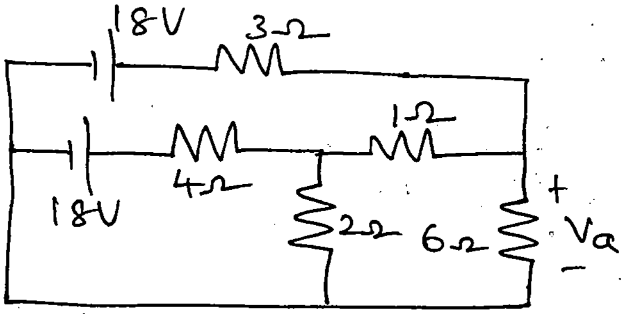


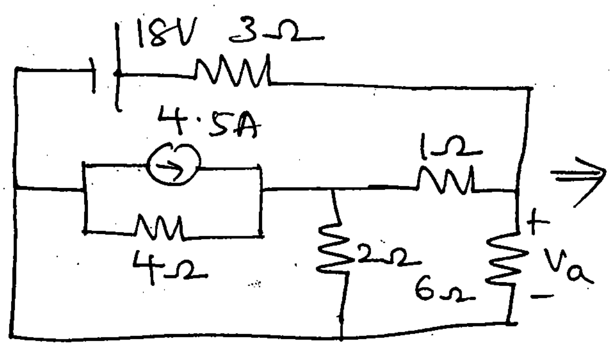
Calculate the voltage across 6Ω resistor using source shifting technique



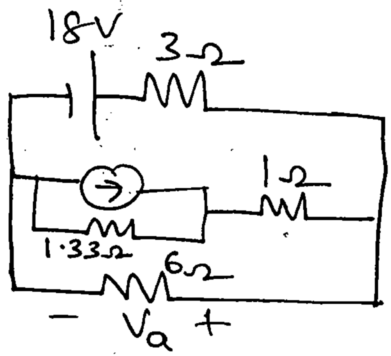


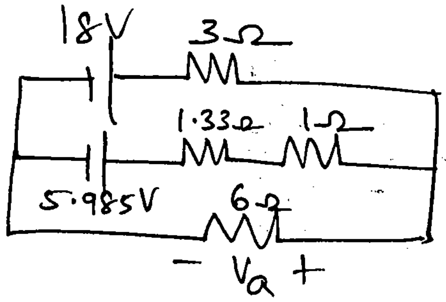
now the 2 voltage sources have resistors in series and source transformation can be applied



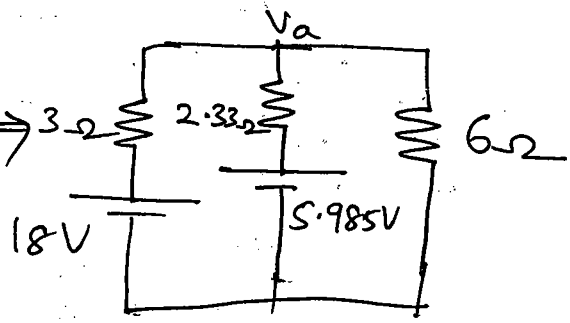


$\Rightarrow$





$\Rightarrow$

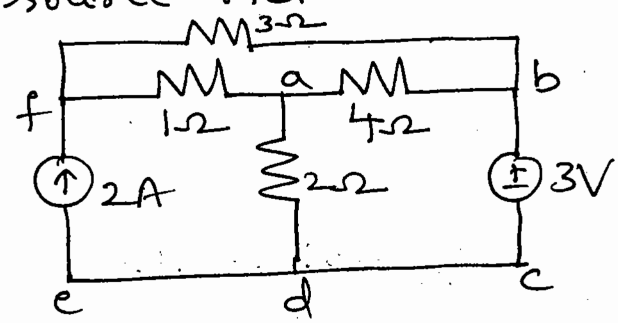


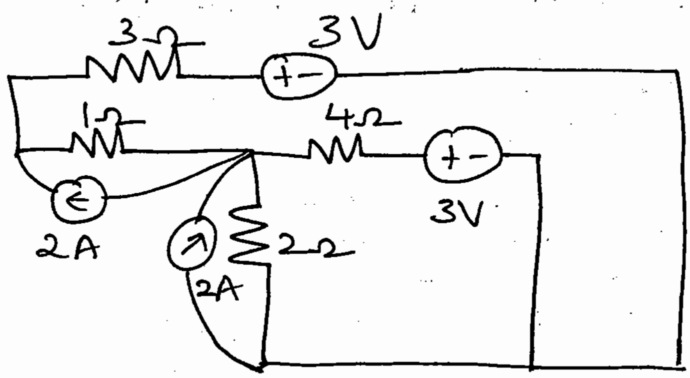
KCL @ node

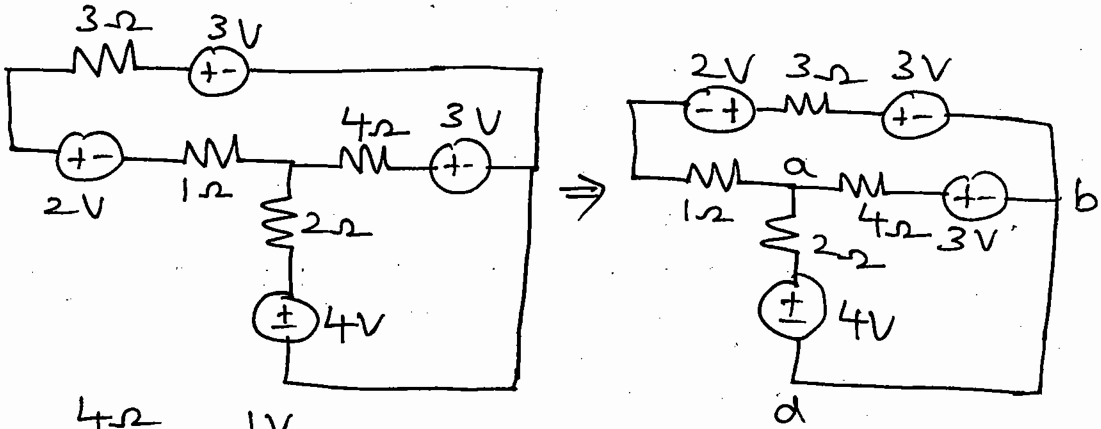
$$\frac{V_a - 18}{3} + \frac{V_a - 5.985}{2.33} + \frac{V_a}{6} = 0$$

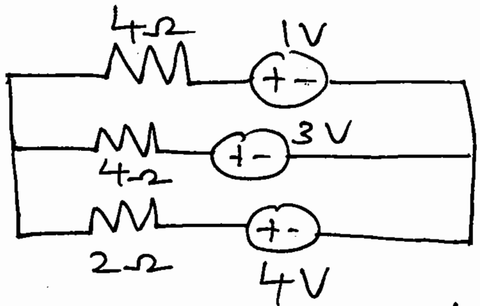
$$\boxed{V_a = 9.23\,V}$$

Reduce the network into a single source network

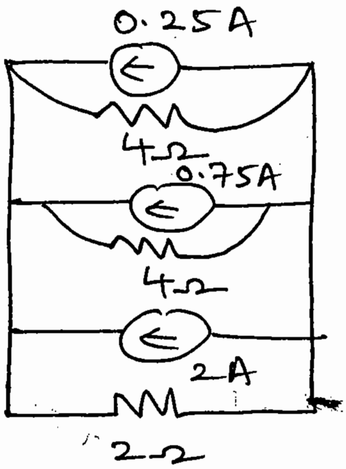








converting all voltage sources into its equivalent current sources

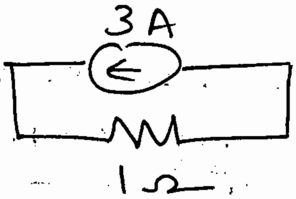


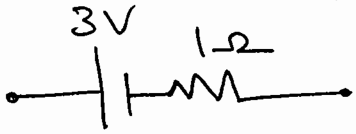
$\Rightarrow$

$$4 \parallel 4 = \frac{16}{8} = 2\Omega$$

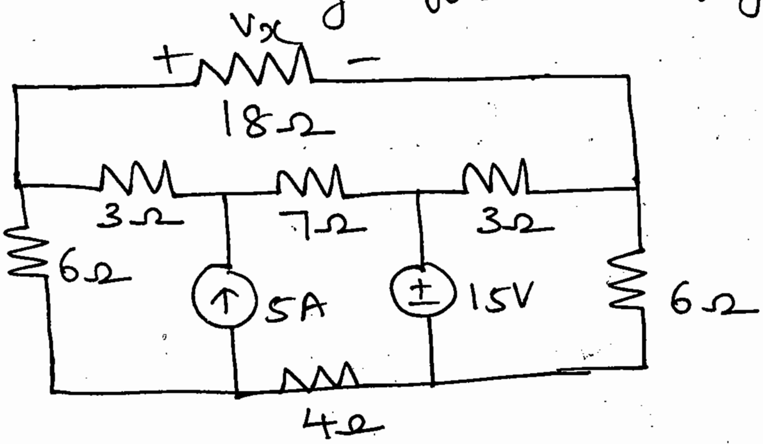
$$2 \parallel 2 = \frac{2\times 2}{4} = 1\Omega$$

$$0.25 + 0.75 + 2 = 3A$$

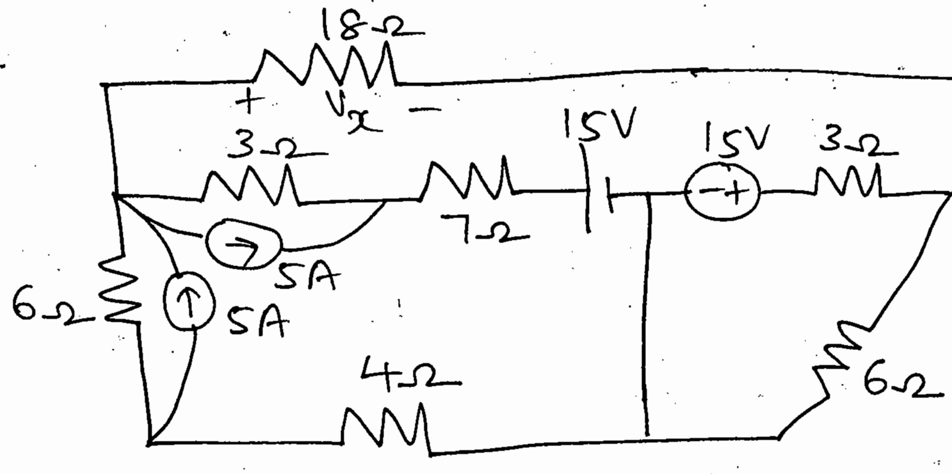


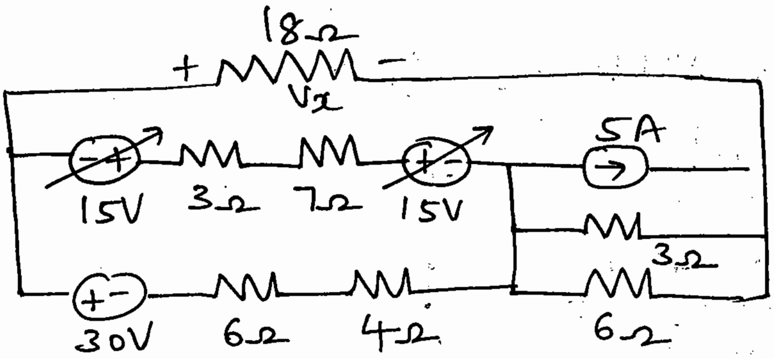


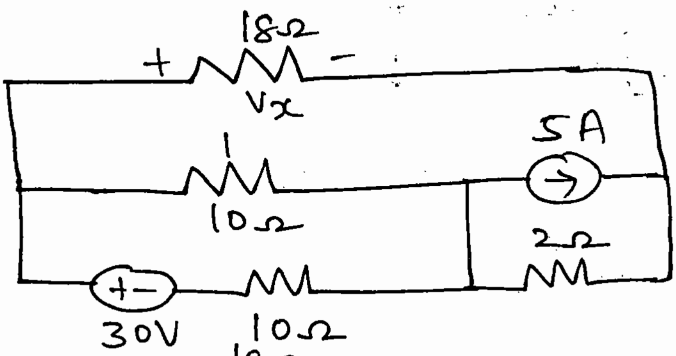
Find the value of V_x using the source mobility for the figure shown

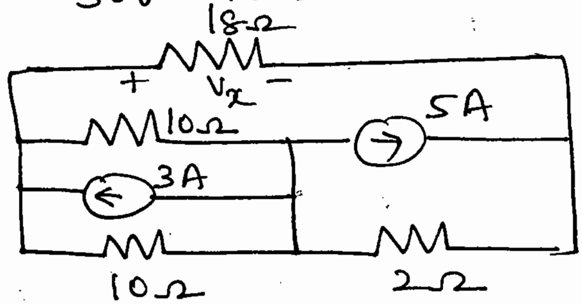


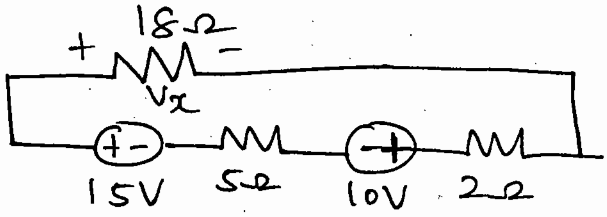
Soln

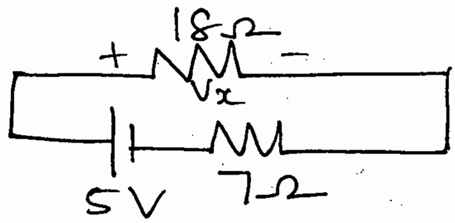










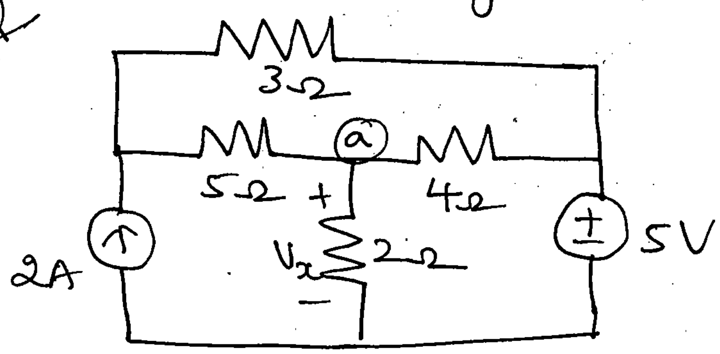


using voltage divider rule

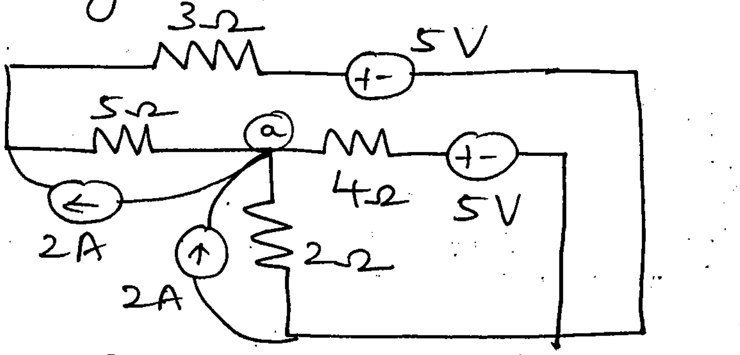
$$\boxed{V_x = \frac{5 \times 18}{18+7} = \frac{90}{25} \Rightarrow 3.6V}$$

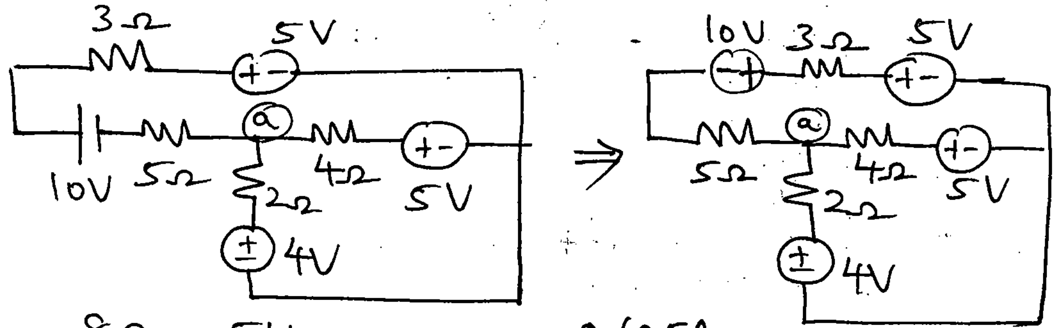
Imp

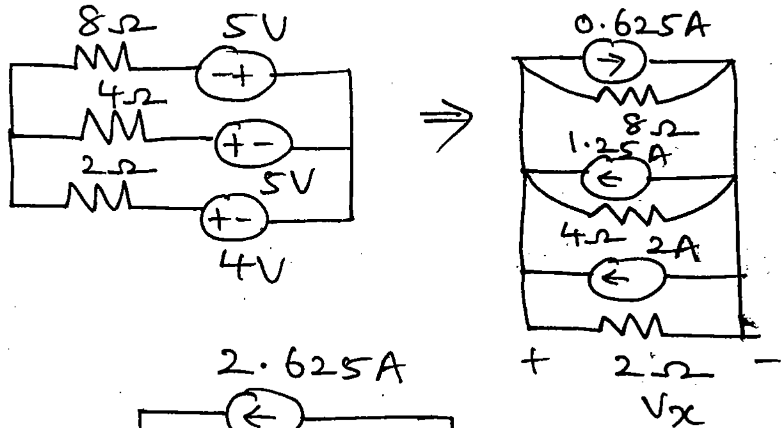
Find the voltage V_x

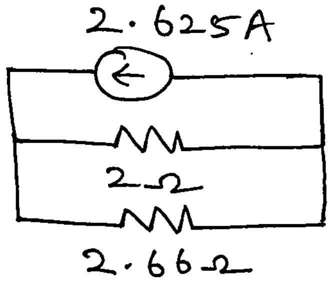


using E-shift & I-shift









using current divider rule

$$I_{2\Omega} = \frac{2.625 \times 2.66}{4.66}$$

$$= 1.5A$$

$\therefore V_x = 2 \cdot I$

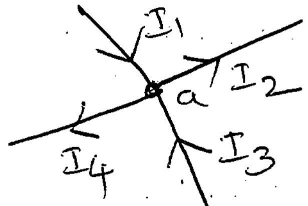
$\boxed{V_x = 3V}$

Kirchoff's Laws ⇒

The entire circuit analysis is based on Kirchoff's laws only

Kirchoff's current Law [KCL (or) point Law]

The algebraic sum of currents meeting at a junction (or) node in an electric circuit is zero. i.e, $\Sigma I = 0$



consider few conductors carrying currents I_1, I_2, I_3 & I_4 meeting at a point a

assuming incoming currents to be positive and outgoing currents negative

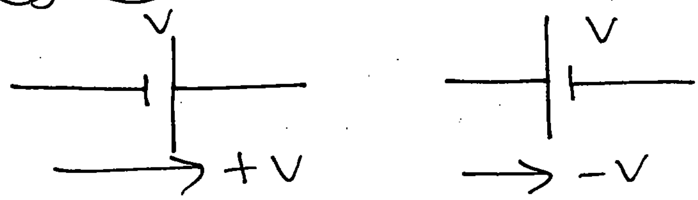
$$I_1 - I_2 + I_3 - I_4 = 0$$

$$I_1 + I_3 = I_2 + I_4$$

Kirchoff's Voltage Law

The algebraic sum of all the voltages in any closed circuit or mesh or Loop is zero i.e, $\Sigma \text{Emf} = \Sigma IR$

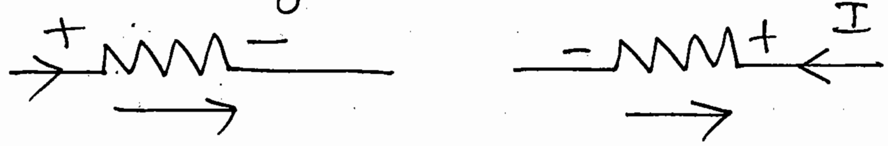
Sign convention



A rise in potential can be assumed to be positive while a fall in potential can be considered negative

If we go from positive terminal of a battery to the negative terminal there is a fall in potential & so emf is negative

When the current flows through a resistor, there is a voltage drop across it. If we go through the resistance in the same direction as the current, there is a fall in the potential & so the sign of this voltage drop is negative. If we go opposite to the direction of the current flow, there is a rise in potential and hence this voltage drop should be given a positive sign.



(a) Fall in Voltage (b) Rise in Voltage

MESH analysis

A mesh is defined as a loop which does not contain any other loops within it. mesh analysis is applicable for a planar network [Elements of network not crossing with Each other]

While applying mesh analysis circuit should contain voltage sources & Kirchoff's Voltage Law is used

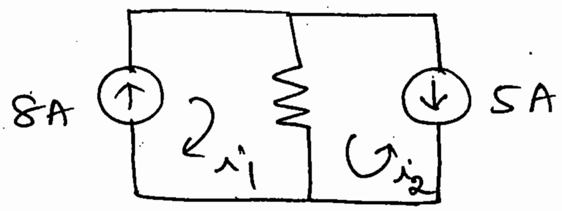
Steps to be followed in mesh analysis

1) Choose a conventional current flow.
2) Identify and number the loops for convenience
3) Apply Kirchoff's voltage law for the identified Loops
4) use ohm's law to express branch voltages in terms of unknown mesh currents & the resistance
5) Formulate the circuit equations
6) Solve the simultaneous equations

When the current source is in the Perimeter | Edge | corner | boundary of the circuit

We can read directly the value of the current because if their exist a current source in any branch of a network, then a Loop cannot be ~~identified~~ defined through current source as the drop across the current source is unknown

Ex ⇒



$\boxed{i_1 = 8A}$, $\boxed{i_2 = -5A}$

Super Loop | super mesh

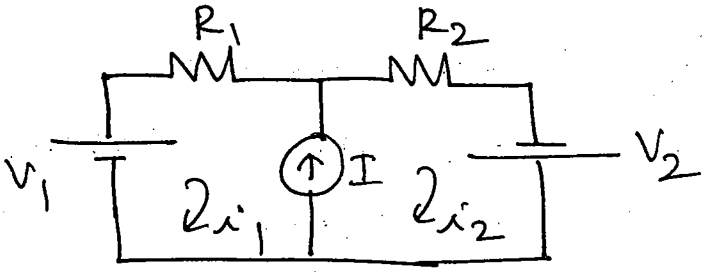
meshes that share a current source with other meshes, none of which contains a current source in the outer loop, form a super mesh. A path around a supermesh doesn't pass through a current source. A path around each mesh contained with in a supermesh passes through a current source

The total number of equations required for a supermesh equal to the number of meshes contained in the supermesh. A supermesh requires one mesh current equation, that is a KVL equation

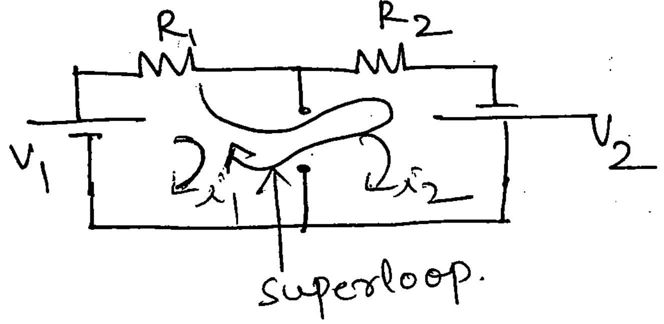
when the current source lies in the mutual branch position called super loop

Procedure ⇒

1) Get the equation for the source current in terms of loop currents
2) For the time being neglecting the current source consider the larger loop common to two loops and this loop is refered as super Loop
3) write the KVL equation for super loop by considering the respective loop currents
4) Determine the different Loop currents by applying KVL



constraint equation $I = i_2 - i_1 \rightarrow (1)$



apply KVL $V_1 - i_1 R_1 - i_2 R_2 + V_2 = 0 \rightarrow (2)$

2 unknowns 2 equations
solve the simultaneous equations

If there is no dependent source and no current source in the given circuit, make use of general equations given by for 3 Loops is as follows

Loop (1) $\Rightarrow I_1 R_{11} - I_2 R_{12} - I_3 R_{13} = E_1$

Loop (2) $\Rightarrow -I_1 R_{21} + I_2 R_{22} - I_3 R_{23} = E_2$

Loop (3) $\Rightarrow -I_1 R_{31} - I_2 R_{32} + I_3 R_{33} = E_3$

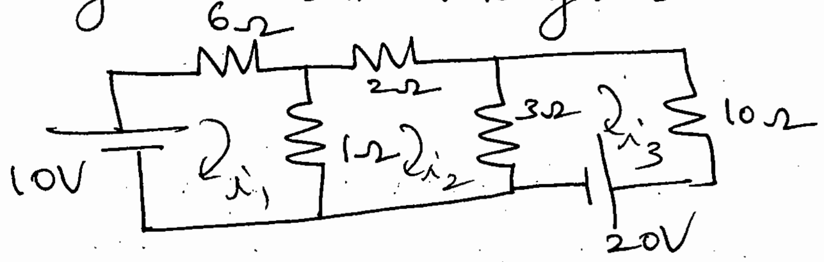
In general

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$R_{11}, R_{22}, R_{33} \Rightarrow$ are called self resistances i.e, the resistances connected to corresponding loops which has positive sign

$R_{13}, R_{23}, R_{32}, R_{12}, R_{21} \Rightarrow$ mutual resistances i.e, resistances common to the 2 loops which has negative sign

Find the current through 2Ω resistor using mesh analysis



mesh 1 $\Rightarrow$

$$10 - 6i_1 - 1(i_1 - i_2) = 0$$

$$-7i_1 + i_2 = -10$$

$$\boxed{7i_1 - i_2 = 10} \Rightarrow (1)$$

mesh 2 $\Rightarrow$

$$-1(i_2 - i_1) - 2i_2 - 3(i_2 - i_3) = 0$$

$$-i_2 + i_1 - 2i_2 - 3i_2 + 3i_3 = 0$$

$$\boxed{i_1 - 6i_2 + 3i_3 = 0} \Rightarrow (2)$$

mesh 3

$-3(i_3 - i_2) - 10 i_3 - 20 = 0$

$-3i_3 + 3i_2 - 10 i_3 = 20$

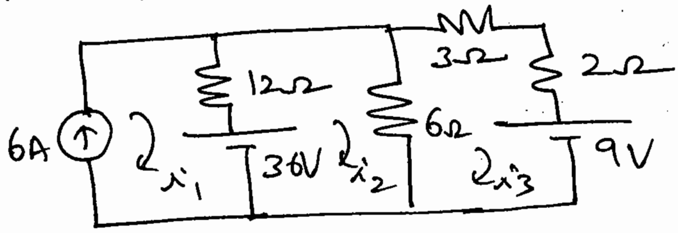
$\boxed{3i_2 - 13 i_3 = 20} \Rightarrow (3)$

solving 3 equations

$i_1 = 1.34\,A$ $i_2 = -0.61\,A$, $i_3 = -1.68A$

& the current through 2Ω is $-0.16A$

Find the current through 2Ω resistor



Loop 1 contains a current source, hence KVL for mesh 1 cannot be written. since the direction of current source & mesh current I_1 are same

$\boxed{i_1 = 6A}$

loop 2

$-12i_1 + 18 i_2 - 6 i_3 = 36$

$-12(6) + 18i_2 - 6i_3 = 36$

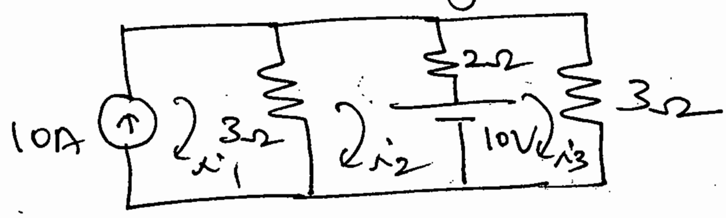
$\boxed{18i_2 - 6i_3 = 108} \Rightarrow (1)$

loop (3)

$\boxed{-6i_2 + 11 i_3 = -9} \Rightarrow (2)$

$\boxed{i_2 = 7A}$, $\boxed{i_3 = 3A = I_{2\Omega}}$

For the network shown write the mesh equations, determine the currents using Loop analysis



$i_1 = 10 A$

KVL to loop 2

$-2i_3 - 3i_1 + 5i_2 = -10$

$-2i_3 - 3(10) + 5i_2 = -10$

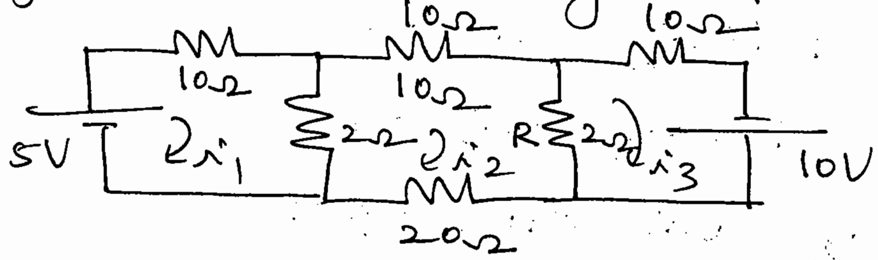
$\therefore$ $\boxed{+5i_2 - 2i_3 = 20}$

KVL to loop 3

$\boxed{-2i_2 + 5i_3 = 10}$

$\boxed{i_1 = 10A}$, $\boxed{i_2 = 5.71 A}$, $\boxed{i_3 = 4.28 A}$

July 2013 Find the voltage across R for the figure shown using Loop analysis.



voltage across 2Ω = $V_{2\Omega} = 2(i_2 - i_3)$

applying KVL to the loops

loop ① $\Rightarrow 12 i_1 - 2 i_2 = 5 \rightarrow$ ①

Loop ② $\Rightarrow -2i_1 + 34 i_2 - 2 i_3 = 0 \rightarrow$ ②

loop ③ $\Rightarrow -2 i_2 + 12 i_3 = 10 \rightarrow$ ③

Solving $i_1 = 0.429\,A$, $i_2 = 0.075\,A$

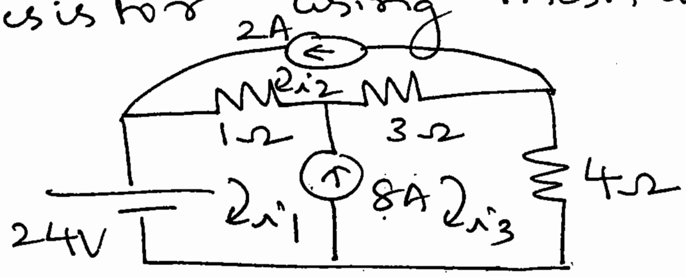
$i_3 = 0.8458\,A$

$V_R = 2(i_2 - i_3) = 2(0.075 - 0.8458)$

$$\boxed{V_R = -1.5416 \text{ volts}}$$

emp

Find the power delivered to the 4Ω resistor using mesh analysis



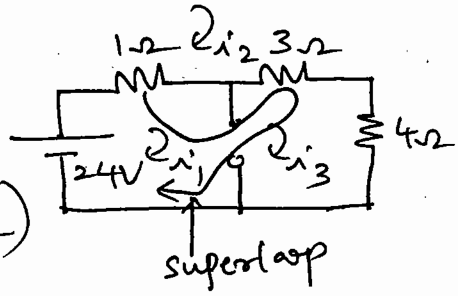
DIVAKAR. B.C

$i_2 = -2\,A$ [current source is @ the perimeter]

loops ① & ③ forms a super mesh

$\boxed{i_3 - i_1 = 8} \Rightarrow$ ①

KVL to supermesh



$24 - 1(i_1 - i_2) - 3(i_3 - i_2)$
$- 4 i_3 = 0$

$-i_1 + 4 i_2 - 7 i_3 = -24$

$\boxed{i_1 + 7i_3 = 16}$ $\quad -i_1 + 4(-2) - 7i_3 = -24$

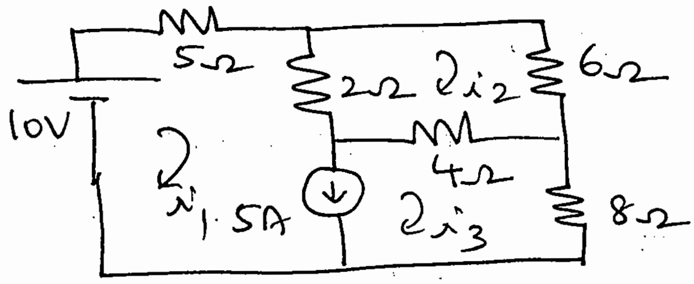
$-i_1 - 7i_3 = -16$

$i_1 = -5\,A$

$i_3 = 3\,A$

$$\boxed{P_{4\Omega} = i_3^2 \times 4 = 9(4) = 36\,\text{Watts}}$$

Determine the loop currents for the circuit shown



1 & 3 forms a super mesh

$\therefore \boxed{i_1 - i_3 = 5} \rightarrow 1$

KVL to loop 2

$$\boxed{-2i_1 + 12i_2 - 4i_3 = 0} \Rightarrow 2$$

apply KVL to super mesh

$$10 - 5i_1 - 2(i_1 - i_2) - 4(i_3 - i_2) - 8i_3 = 0$$

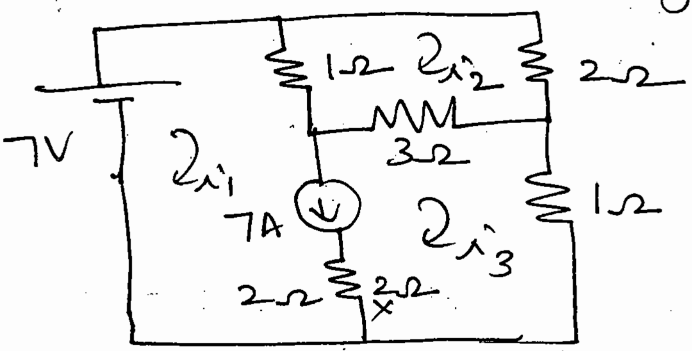
$$10 - 5i_1 - 2i_1 + 2i_2 - 4i_3 + 4i_2 - 8i_3 = 0$$

$$\boxed{7i_1 - 6i_2 + 12i_3 = 10} \Rightarrow 3$$

solving $i_1 = 3.75\,A$, $i_2 = 0.208\,A$

$i_3 = -1.25\,A$

Find the current through 3Ω resistor



2Ω is a redundant element

Loops 1 & 3 forms a supermesh

$$\therefore \boxed{i_1 - i_3 = 7} \Rightarrow (1)$$

KVL to loop 2

$$\boxed{-i_1 + 6i_2 - 3i_3 = 0} \Rightarrow (2)$$

KVL to supermesh

$$7 - 1(i_1 - i_2) - 3(i_3 - i_2) - i_3 = 0$$

$$-i_1 + 4i_2 - 4i_3 + 7 = 0$$

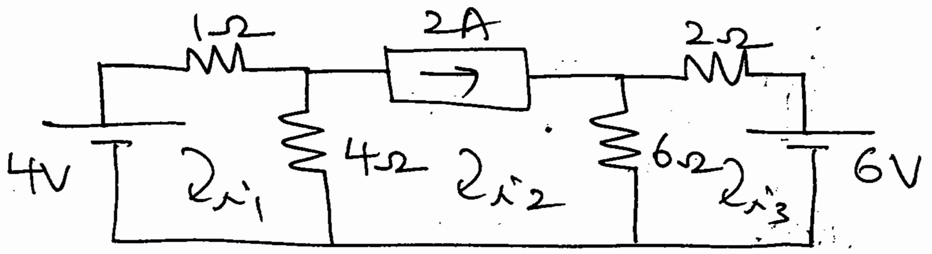
$$\boxed{i_1 - 4i_2 + 4i_3 = 7} \Rightarrow (3)$$

$$\boxed{i_1 = 9\text{ A}, \quad i_2 = 2.5\text{ A}, \quad i_3 = 2\text{ A}}$$

$\therefore$ current through 3Ω is

$$\boxed{I_{3\Omega} = i_2 - i_3 = 0.5\text{ A}}$$

For the circuit shown find loop currents



For the given circuit an ideal current source 2 A is present in the second loop since it is not possible to convert it into a voltage source.

For an ideal current source, the current through it cannot change whatever may be the effect of source outside it

$\therefore \boxed{i_2 = 2A}$

KVL to loop 1

$$5i_1 - 4i_2 = 4 \Rightarrow 1$$

$\boxed{\text{KVL to loop } 3 \Rightarrow -6i_2 + 8i_3 = -6}$

$$-6(2) + 8i_3 = -6$$

$$\therefore 8i_3 = 12 - 6$$

$$\therefore 8i_3 = 6$$

$$\boxed{i_3 = 0.75\,A}$$

$$5i_1 = 4 + 4i_2$$

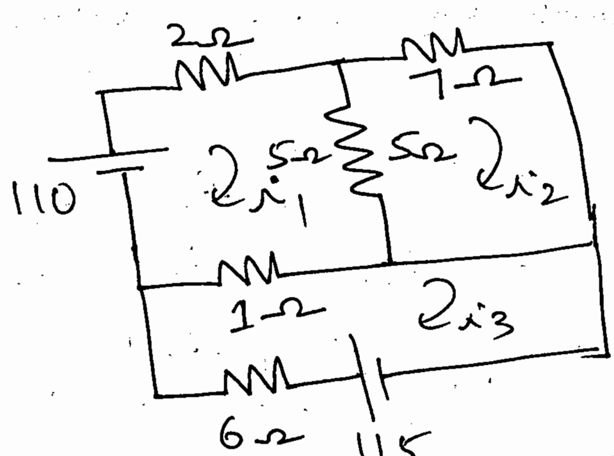
$$\therefore \boxed{i_1 = \frac{4 + 4(2)}{5} = 2.4\,A}$$

Loop equations are given derive the network

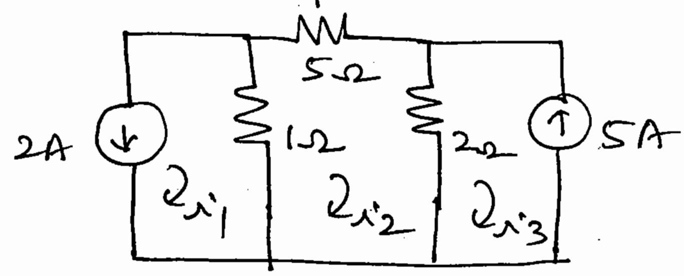
$$8i_1 - 5i_2 - i_3 = 110$$
$$-5i_1 + 12i_2 = 0$$
$$-i_1 + 7i_3 = 115$$

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} 8 & -5 & -1 \\ -5 & 12 & 0 \\ -1 & 0 & 7 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 110 \\ 0 \\ 115 \end{bmatrix}$$



Determine loop currents



$$i_1 = -2A, \quad i_3 = -5A$$

KVL to loop 2

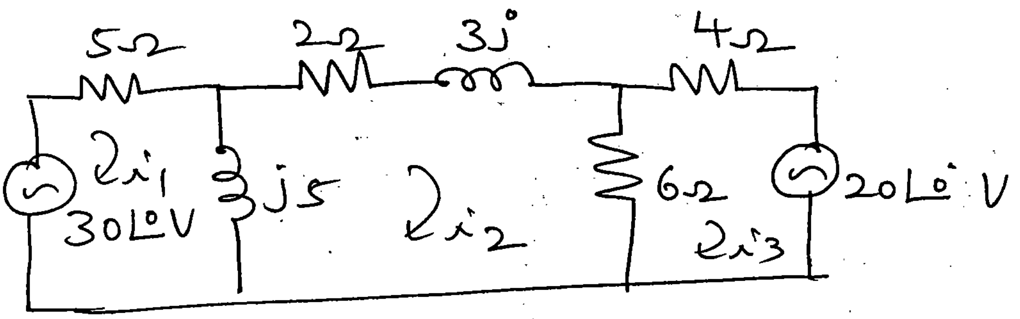
$$+i_2 8 - 2i_3 - i_1 = 0$$

$$8i_2 + 2 + 10 = 0 \qquad 8i_2 = -12$$

$$\boxed{i_2 = -1.5A}$$

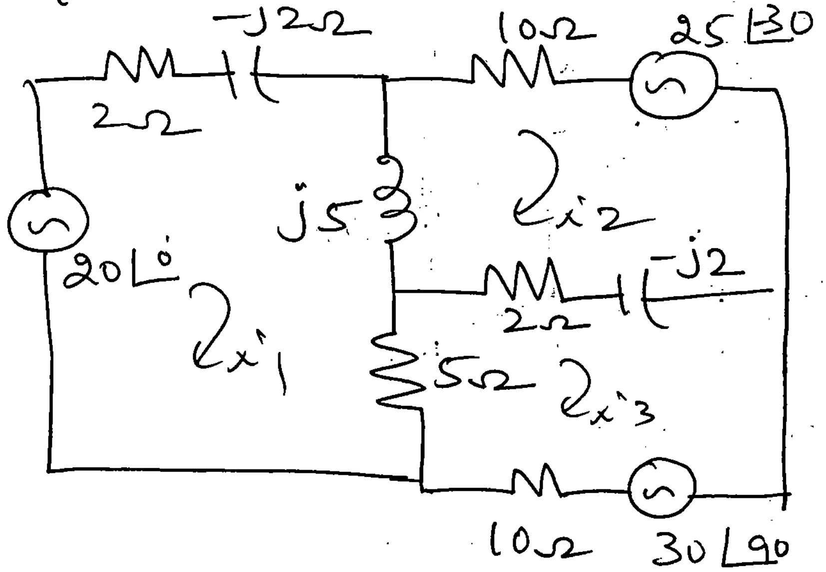
The loop equations are as given below write the corresponding circuit

$$\begin{bmatrix} 5+5j & -5j & 0 \\ -j5 & 8+8j & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 30\angle 0 \\ 0 \\ -20\angle 0 \end{bmatrix}$$

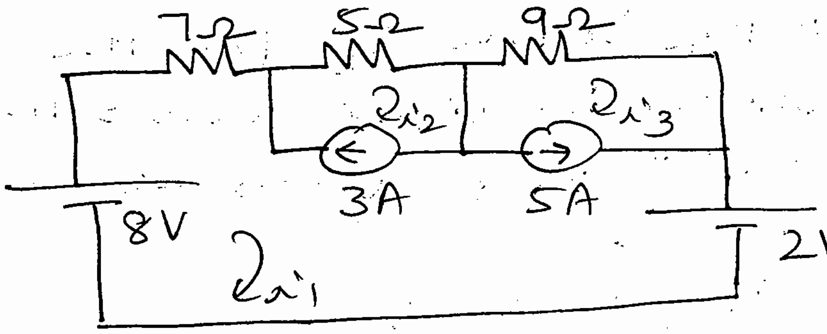


For the given loop equations draw the network

$$\begin{bmatrix} 7+3j & -5j & -5 \\ -5j & 12+3j & -(2-j2) \\ -5 & -(2-2j) & 17-2j \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 20\angle 0 \\ 25\angle -30 \\ 30\angle 90 \end{bmatrix}$$



Determine the loop currents



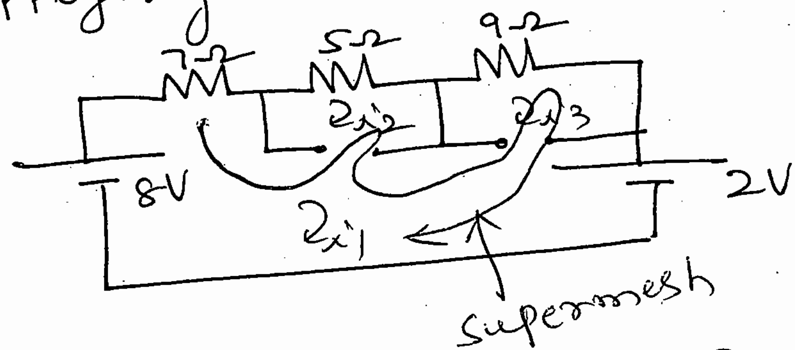
Loops ① & ② forms a superloop

$\therefore \quad i_2 - i_1 = 3 \Rightarrow$ ①

Loops ① & ③ forms a superloop

$i_1 - i_3 = 5 \Rightarrow$ ②

applying KVL to supermesh



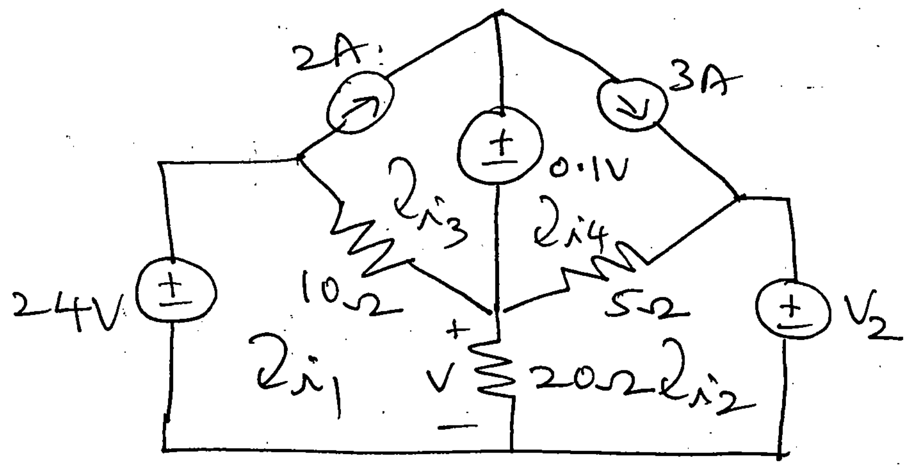
$8 - 7i_1 - 5i_2 - 9i_3 - 2 = 0$

$-7i_1 - 5i_2 - 9i_3 = -6$

$\boxed{7i_1 + 5i_2 + 9i_3 = 6} \Rightarrow$ ③

$\boxed{i_1 = 1.71\text{ A}}$ $\boxed{i_2 = 4.71\text{ A}}$ $\boxed{i_3 = -3.28\text{ A}}$

Use mesh analysis to find, what value of voltage source V_2 for the network shown in figure causes the voltage $V =$ "0" volts across 20Ω resistor



$i_3 = 2A$ $\therefore$ current source lies at the perimeter
$i_4 = 3A$

Given $V = 0$ across 20Ω resistor

$$\therefore \quad V_{20} = 20(i_1 - i_2)$$

$$20 i_1 - 20 i_2 = 0$$

which $\Rightarrow$ $i_1 = i_2$

KVL to loop 1

$$30 i_1 - 20 i_2 - 10 i_3 = 24$$

$$30 i_1 - 20 i_2 - 10(2) = 24$$

$$\boxed{30 i_1 - 20 i_2 = 44}$$

$$30 i_1 - 20 i_1 = 44$$

$$\boxed{i_1 = i_2 = 4.4A}$$

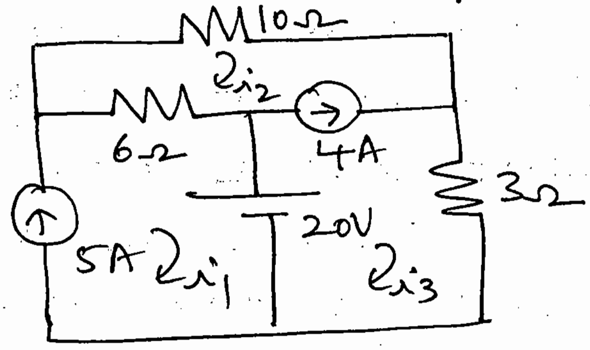
KVL to loop 2

$$-20 i_1 + 25 i_2 - 5 i_4 = -V_2$$

$$-20(4.4) + 25(4.4) - 5(3) = -V_2$$

$$\therefore \boxed{V_2 = -7V}$$

Find the loop currents



loops 2 & 3 forms a super loop

$$\boxed{i_3 - i_2 = 4}$$

$\boxed{i_1 = 5A}$ [current source lies @ the perimeter]

KVL to super loop

$$-6(i_2 - i_1) - 10 i_2 - 3 i_3 + 20 = 0$$

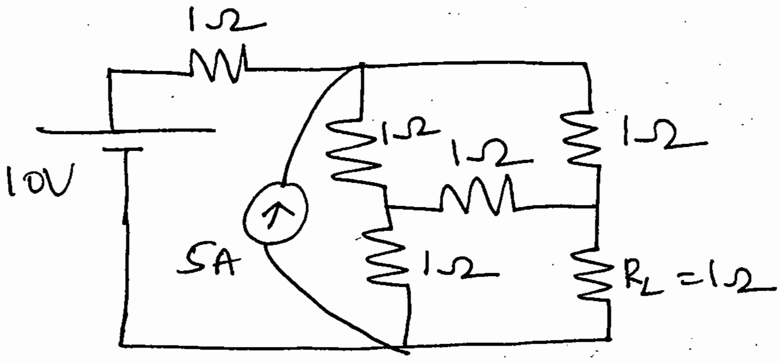
$$-6 i_2 + 6 i_1 - 10 i_2 - 3 i_3 = -20$$

$$6 i_1 - 16 i_2 - 3 i_3 = -20$$

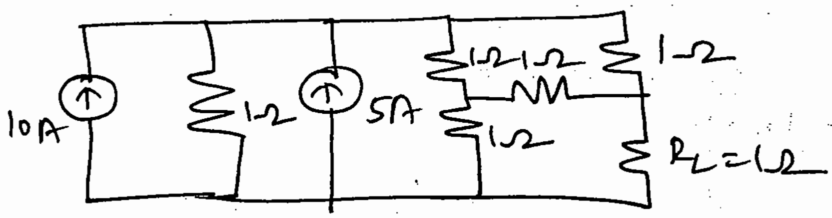
$$-16 i_2 - 3 i_2 = -50$$

$$\boxed{i_2 = 2A} \; ; \; \boxed{i_3 = 6A}$$

Find the current across the load resistance R_L for the circuit shown

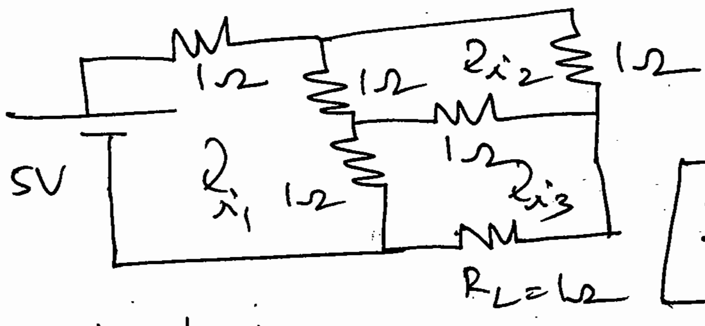


while solving the network using mesh analysis, the circuit should contain only voltage sources ∴ first convert 10V in series with 1Ω to current source, add these 2 current sources & reduce the network as shown below



10+5 = 15A in parallel with 1Ω

∴ convert this in to voltage source



$$\boxed{I_{R_L} = 3.75A = i_3}$$

apply KVL

loop 1 ⇒ $3i_1 - i_2 - i_3 = 15 \rightarrow (1)$

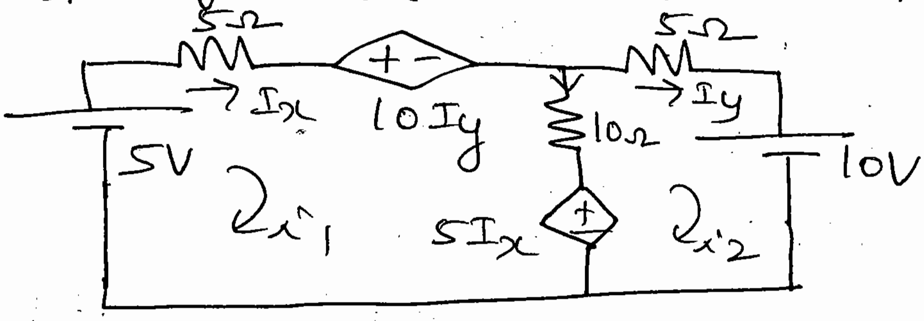
loop 2 ⇒ $-i_1 + 3i_2 - i_3 = 0 \rightarrow (2)$

Loop 3 ⇒ $-i_1 - i_2 + 3i_3 = 0 \rightarrow (3)$

$$\boxed{i_1 = 7.5A \quad i_2 = 3.75A = i_3}$$

Dependent sources

Ex: Determine the current across 10Ω resistor for the circuit shown



from the figure $I_x = i_1$ & $I_y = i_2$

$I_{10\Omega} = i_1 - i_2$

apply KVL to loop 1

$$5 - 5i_1 - 10\,I_y - 10(i_1 - i_2) - 5\,I_x = 0$$

$$5 - 5i_1 - 10i_2 - 10(i_1 - i_2) - 5i_1 = 0$$

$$\boxed{-20i_1 = -5} \qquad i_1 = \frac{1}{4} = 0.25A$$

KVL to loop 2

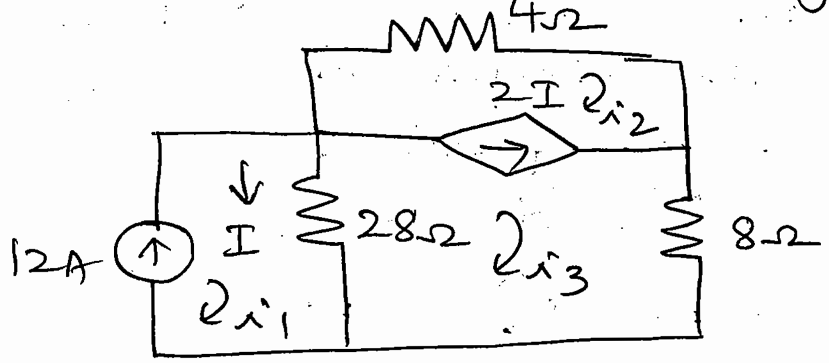
$$-5i_2 - 10 + 5i_1 - 10(i_2 - i_1) = 0$$

$$\boxed{15i_1 - 15i_2 = 10}$$

$$i_2 = -0.416A$$

$$\therefore \boxed{I_{10\Omega} = i_1 - i_2 = 0.666A}$$

Eg: Find the current through 28Ω resistor



Since 12 A current source is at the perimeter $\boxed{i_1 = 12A}$

loops (2) & (3) forms a superloop

$$i_3 - i_2 = 2I$$

From the figure $\boxed{I = i_1 - i_3}$

$$\therefore \quad i_3 - i_2 = 2(i_1 - i_3)$$

$$i_3 - i_2 = 2i_1 - 2i_3$$

$$i_3 - i_2 = 2(12) - 2i_3$$

$$24 - 3i_3 = -i_2$$

$$\boxed{-i_2 + 3i_3 = 24 \Rightarrow (1)}$$

KVL to super mesh

$$-28(i_3 - i_1) - 4i_2 - 8i_3 = 0$$

$$-28i_3 - 28i_1 - 4i_2 - 8i_3 = 0$$

$$4i_2 + 36i_3 = 28i_1$$

$$4(i_2 + 9i_3) = 28 \times 12$$

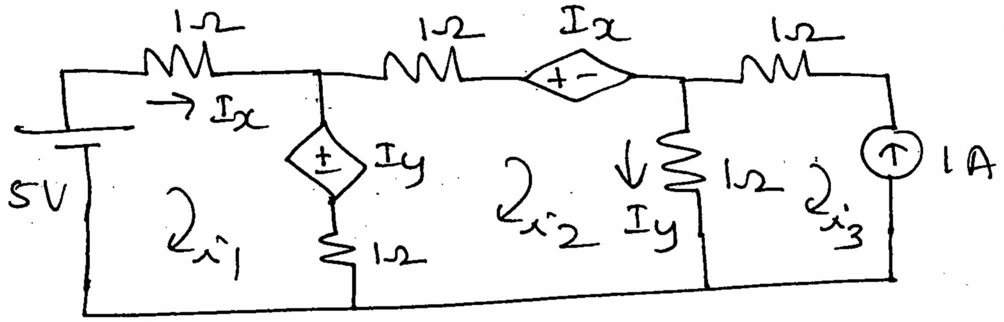
$$\therefore \quad \boxed{i_2 + 9i_3 = 84} \Rightarrow (2)$$

$i_2 = 3A$

$i_3 = 9A$

$\therefore \boxed{I_{2\,8\Omega} = i_1 - i_3 = 12 - 9 = 3A}$

Determine the currents in 3 meshes



from the fig $I_x = i_1$; $I_y = i_2 - i_3$

$\boxed{i_3 = -1A}$ $\qquad \therefore I_y = i_2 + 1$

KVL to loop 1

$5 - i_1 - I_y - (i_1 - i_2) = 0$

$5 - i_1 - (i_2 + 1) - i_1 + i_2 = 0$

$5 - i_1 - (i_2 + 1) - i_1 + i_2 = 0$

$5 - i_1 - \cancel{i_2} - 1 - i_1 + \cancel{i_2} = 0$

$-2i_1 + 4 = 0 \qquad \boxed{i_1 = 2A}$

KVL to loop 2

$-i_2 - I_x - (i_2 - i_3) - (i_2 - i_1) + I_y = 0$

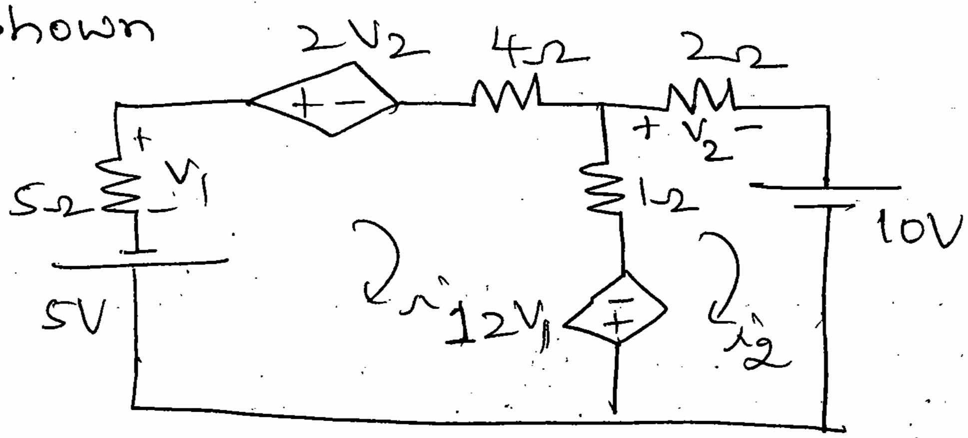
$-\cancel{i_2} - \cancel{i_1} - i_2 + i_3 - i_2 + \cancel{i_1} + \cancel{i_2} + 1 = 0$

$-2i_2 + i_3 = -1$

$-2i_2 = -1 - i_3 = -1 + 1 = 0$

$\therefore \boxed{i_2 = 0}$

Find the mesh currents for the network shown



from the fig $V_2 = 2i_2$, $V_1 = -5i_1$

KVL to loop (1)

$$-5 + V_1 - 2V_2 - 4i_1 - (i_1 - i_2) + 2V_1 = 0$$

$$-5i_1 - 2(2i_2) - 4i_1 - i_1 + i_2 + 2(-5i_1) = 5$$

$$-5i_1 - 4i_2 - 4i_1 - i_1 + i_2 - 10i_1 = 5$$

$$\boxed{20i_1 + 3i_2 = -5 \Rightarrow (1)}$$

Loop (2)

$$-(i_2 - i_1) - V_2 - 10 - 2V_1 = 0$$

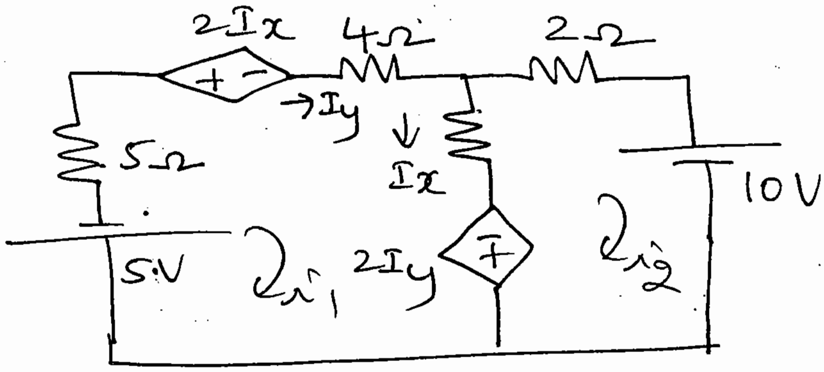
$$-i_2 + i_1 - 2i_2 - 2(-5i_1) = 10$$

$$-i_2 + i_1 - 2i_2 + 10i_1 = 10$$

$$\boxed{11i_1 - 3i_2 = 10 \Rightarrow (2)}$$

$$\boxed{i_1 = 0.161\text{ A}} \qquad \boxed{i_2 = -2.741\text{ A}}$$

Find the currents I_x & I_y



from the figure $I_y = i_1$; $I_x = i_1 - i_2$

KVL to loop (1)

$-5 - 5i_1 - 2i_x - 4i_1 - (i_1 - i_2) + 2I_y = 0$

$-5i_1 - 2(i_1 - i_2) - 4i_1 - i_1 + i_2 + 2i_1 = 5$

$-5i_1 - \cancel{2i_1} + 2i_2 - 4i_1 - i_1 + i_2 + \cancel{2i_1} = 5$

$\boxed{-10i_1 + 3i_2 = 5 \Rightarrow (1)}$

KVL to loop 2

$-(i_2 - i_1) - 2i_2 - 10 - 2I_y = 0$

$-i_2 + i_1 - 2i_2 - 10 - 2i_1 = 0$

$\boxed{-i_1 - 3i_2 = 10 \Rightarrow (2)}$

$i_1 = -1.364\,A$

$i_2 = -2.878\,A$

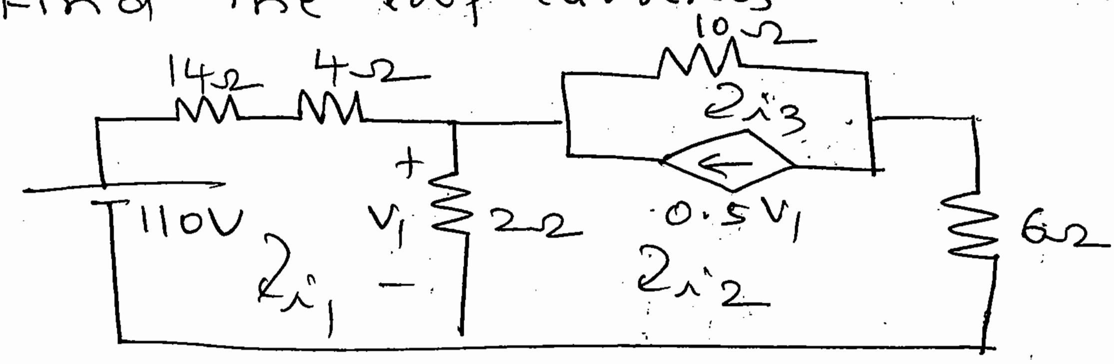
$I_y = i_1 = -1.364\,A$

$I_x = i_1 - i_2$

$= -1.364 - (-2.878)$

$\boxed{I_x = 1.514\,A}$

Find the loop currents



$$V_1 = 2(i_1 - i_3)$$

Loops ② & ③ forms a supermesh

$\therefore\ i_3 - i_2 = 0.5\,V_1$

$\therefore\ i_3 - i_2 = 0.5 \times 2(i_1 - i_3)$

$i_3 - \cancel{i_2} = i_1 - \cancel{i_2}$

$\boxed{i_3 = i_1} \Rightarrow \boxed{i_1 - i_3 = 0} \Rightarrow ①$

KVL to super mesh

$-10i_3 - 6i_2 - 2(i_2 - i_1) = 0$

$-10i_3 - 6i_2 - 2i_2 + 2i_1 = 0$

$\boxed{2i_1 - 8i_2 - 10i_3 = 0} \Rightarrow ②$

KVL to loop ①

$110 - 18i_1 - 2(i_1 - i_2) = 0$

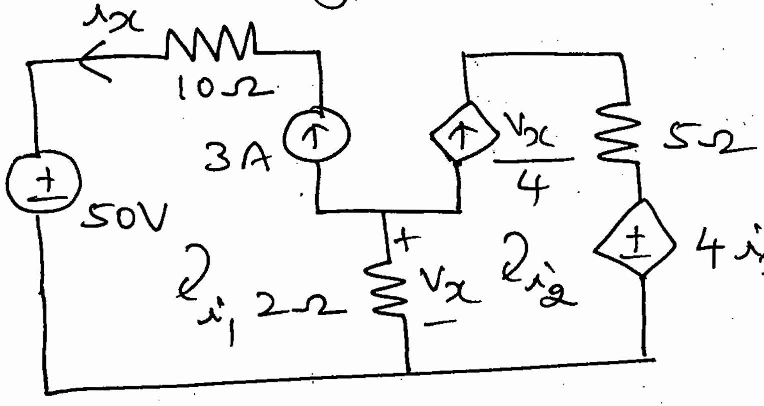
$-18i_1 - 2i_1 + 2i_2 = -110$

$\boxed{-20i_1 + 2i_2 = -110} \Rightarrow ③$

Solving equations ①, 2 & ③

$\boxed{i_1 = 5A}$ $\boxed{i_2 = -5A}$ $\boxed{i_3 = 5A}$

Jan 2014 Find I_x & V_x for the circuit shown using mesh analysis 28



from the figure $i_x = -i_1$ & $i_1 = -3A$

& $i_2 = \frac{V_x}{4}$ since the current source is at the periphery

$$i_2 = \frac{V_x}{4} \quad \& \quad V_x = 2(i_1 - i_2)$$

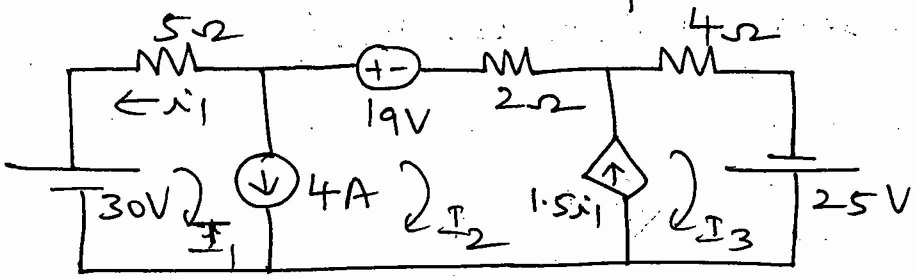
$$V_x = 2\left(-3 - \frac{V_x}{4}\right)$$

$$\frac{V_x}{2} = -3 - \frac{V_x}{4} \quad \therefore \quad \frac{V_x}{2} + \frac{V_x}{4} = -3$$

$$-\cancel{3} = \frac{\cancel{3} V_x}{4} \quad \therefore \quad \boxed{V_x = -4V}$$

$$\boxed{i_2 = \frac{-4}{4} = -1\,A}$$

Find the current i_1



$$i_1 = -I_1$$

loops ① & ② forms a supermesh

$$I_1 - I_2 = 4 \rightarrow ①$$

Loops ② & ③ forms a supermesh

$$I_3 - I_2 = 1.5 i_1$$

$$I_3 - I_2 = -1.5 I_1$$

$$\boxed{1.5 I_1 - I_2 + I_3 = 0} \Rightarrow ②$$

KVL to super loop

$$30 - 5I_1 - 19 - 2I_2 - 4I_3 + 25 = 0$$

$$-5I_1 - 2I_2 - 4I_3 = -36 \Rightarrow ③$$

$$\boxed{I_1 - I_2 = 4} \Rightarrow ①$$

solving $I_1 = 12\,A$ $\quad\therefore$ $\boxed{i_1 = -I_1 = -12\,A}$

Nodal method analysis

It is a technique to solve network problems based on kirchoff's current Law.

Steps involved in application of Nodal voltage method

1. Identify & name the nodes for the convenience
2. choose one node as a reference node [preferably this should be the one which has more number of branches connected from it]
3. Identify the node at which Node voltage is known
4. write down the KCL for the remaining nodes in terms of unknown nodal voltages
5. Formulate the circuit equations & solve the simultaneous equations

Super Node ⇒ when there is no resistance connected in series for the given voltage source, the node is termed as supernode i.e, a Ideal voltage source present between the two nodes

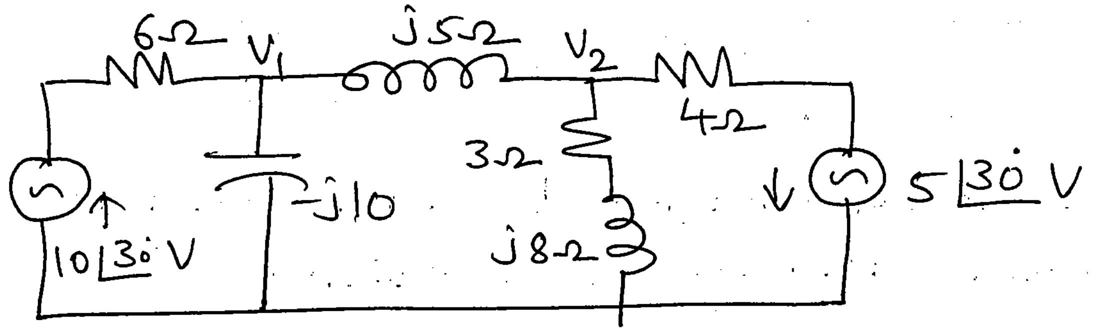
The nodal equations are given derive the network

$$\left(\frac{1}{6}+\frac{1}{j5}+\frac{1}{-j10}\right)V_1 - \frac{1}{j5}V_2 = \frac{10\angle 30}{5}$$

$$\frac{-1}{j5}V_1 + \left(\frac{1}{j5}+\frac{1}{3+j8}+\frac{1}{4}\right)V_2 = \frac{-5\angle 30}{4}$$

comparing with $V_1 G_{11} - V_2 G_{12} = I_1$

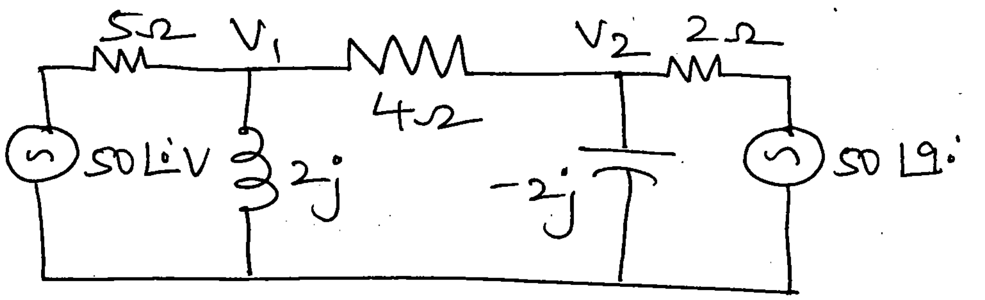
$-V_1 G_{21} + V_2 G_{22} = I_2$



Qmp For the given nodal equations derive the network

$$\left(\frac{1}{5}+\frac{1}{2j}+\frac{1}{4}\right)V_1 - \frac{1}{4}V_2 = \frac{50\angle 0}{5}$$

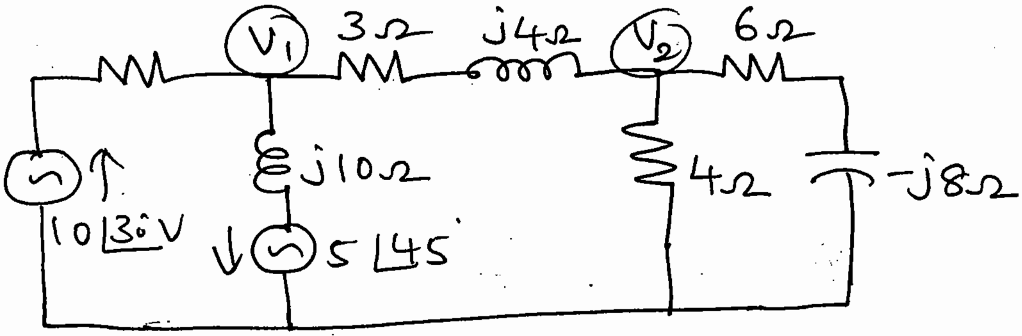
$$-\frac{1}{4}V_1 + \left(\frac{1}{4}+\frac{1}{-2j}+\frac{1}{2}\right)V_2 = \frac{50\angle 90°}{2}$$



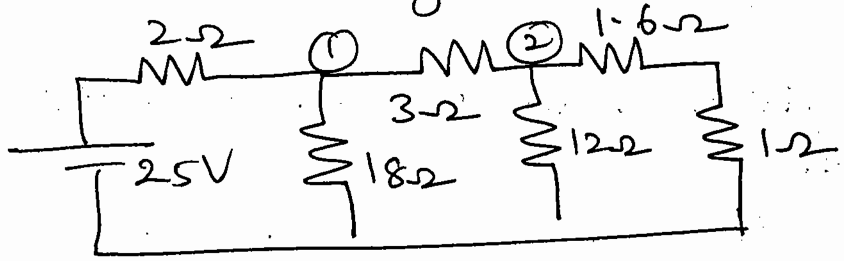
For the given nodal equations derive the network

$$\left(\frac{1}{5}+\frac{1}{j10}+\frac{1}{3+j4}\right)V_1-\left(\frac{1}{3+j4}\right)V_2=\frac{10\angle 30^\circ}{5}-\frac{5\angle 45^\circ}{j10}$$

$$\frac{-1}{3+j4}V_1+\left(\frac{1}{3+j4}+\frac{1}{6-j8}+\frac{1}{4}\right)V_2=0$$



use nodal method to determine the current through 3Ω resistor



$$I_{3\Omega}=\frac{V_1-V_2}{3}$$

KCL @ node 1

$$\frac{V_1-25}{2}+\frac{V_1}{18}+\frac{V_1-V_2}{3}=0$$

$$\frac{V_1}{2}+\frac{V_1}{18}+\frac{V_1}{3}-\frac{V_2}{3}=12.5$$

$$V_1\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{18}\right)-\frac{V_2}{3}=12.5$$

$$\boxed{0.8889\,V_1-0.334\,V_2=12.5\rightarrow (1)}$$

KCL @ node 2

$$\frac{V_2 - V_1}{3} + \frac{V_2}{12} + \frac{V_2}{2.6} = 0$$

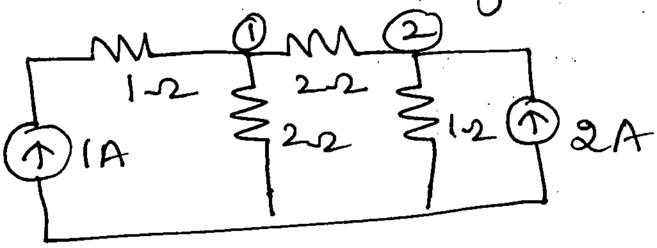
$$-\frac{V_1}{3} + V_2\left(\frac{1}{3} + \frac{1}{12} + \frac{1}{2.6}\right) = 0$$

$$\boxed{-0.334V_1 + 0.80128V_2 = 0 \rightarrow 2}$$

Solving $V_1 = 16.66\,V$, $V_2 = 6.93V$

$$\therefore \boxed{I_{3\Omega} = \frac{16.66 - 6.93}{3} = 3.24\,A}$$

Find the voltages at nodes 1 & 2



1Ω ⇒ Redundant element

KCL @ node 1

$$\frac{V_1}{2} + \frac{V_1 - V_2}{2} = 1$$

$$V_1 + V_1 - V_2 = 2$$

$$\boxed{2V_1 - V_2 = 2} \Rightarrow 1$$

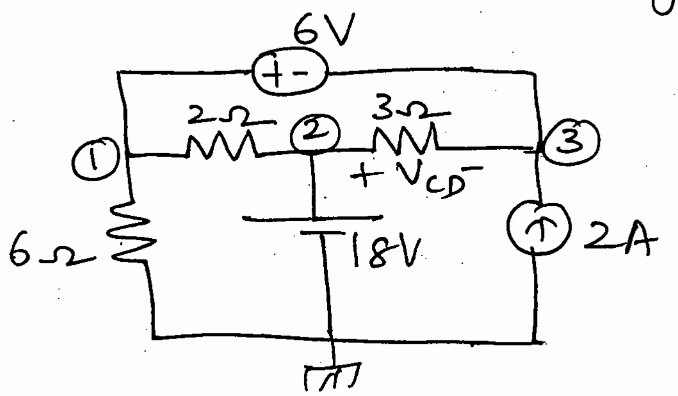
KCL @ node 2 $\quad \frac{V_2}{1} + \frac{V_2 - V_1}{2} = 2$

$$2V_2 + V_2 - V_1 = 4$$

$$\boxed{3V_2 - V_1 = 4} \Rightarrow 2$$

$$\boxed{V_1 = V_2 = 2V}$$

Find the voltage V_{CD} using nodal analysis



$V_{CD} = V_2 - V_3$

$V_2 = 18\ V$ ∴ KCL @ node 1 & 3

Nodes 1 & 3 forms a supernode

∴ $\boxed{V_1 - V_3 = 6}$ ⇒ 1

KCL @ super node

$$\frac{V_1 - V_2}{2} + \frac{V_1}{6} + \frac{V_3 - V_2}{3} = 2$$

$$\frac{V_1 - 18}{2} + \frac{V_1}{6} + \frac{V_3 - 18}{3} = 2$$

$$V_1\left(\frac{1}{2} + \frac{1}{6}\right) + V_3\left(\frac{1}{3}\right) = 6 + 2 + 9$$

$\boxed{0.666 V_1 + 0.33 V_3 = 17}$ ⇒ 2

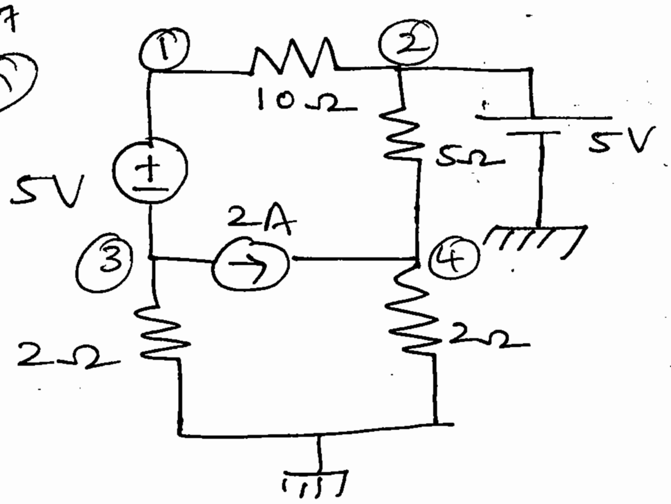
solving 1 & 2 $V_1 = 19V$, $V_3 = 13V$

∴ $V_{CD} = V_2 - V_3 = 18 - 13$

$\boxed{V_{CD} = 5\ V}$

Imp
June 2017
(8m)

Determine the nodal voltages



$V_2 = 5$ V [as it is directly connected to the Datum node]

Nodes 1 & 3 forms a supernode

$$\boxed{V_1 - V_3 = 5} \rightarrow (1)$$

KCL @ supernode

$$\frac{V_1 - V_2}{10} + \frac{V_3}{2} = -2$$

$$\boxed{0.1V_1 + 0.5V_3 = -1.5} \rightarrow (2)$$

Apply KCL @ node 4

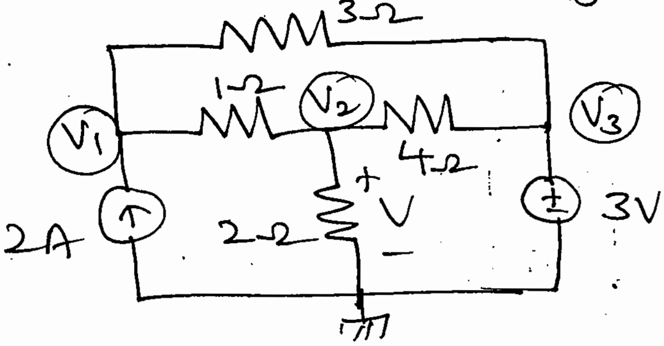
$$\frac{V_4 - V_2}{5} + \frac{V_4}{2} = 2$$

$$\boxed{\frac{V_4}{5} + \frac{V_4}{2} = 3}$$

$$\therefore \boxed{V_4 = 4.2857\,V}$$

$\boxed{V_1 = 1.667\,V}$ $\boxed{V_3 = -3.33\,V}$

Determine the voltage V using nodal method



from the figure $V = V_2$ & $\boxed{V_3 = 3V}$

KCL @ node 1

$$\frac{V_1 - V_3}{3} + \frac{V_1 - V_2}{1} = 2$$

$$\frac{V_1 - 3}{3} + \frac{V_1 - V_2}{1} = 2$$

$$\boxed{1.33 V_1 - V_2 = 3} \Rightarrow (1)$$

KCL @ node 2

$$\frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{4} + \frac{V_2}{2} = 0$$

$$V_2(1 + 0.5 + 0.25) - V_1 = \frac{3}{4}$$

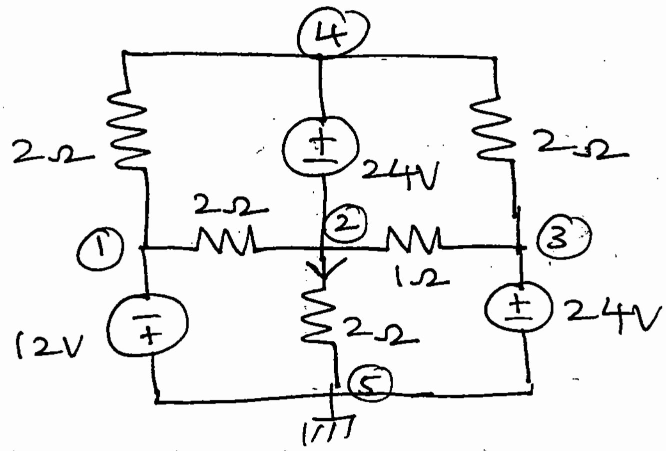
$$\boxed{1.75 V_2 - V_1 = 0.75} \Rightarrow (2)$$

solving $\boxed{V_1 = 4.5V}$

$\boxed{V_2 = 3V}$

$\therefore \boxed{V = V_2 = 3V}$

Jan 2014 Solve for the current I_o using nodal analysis



Ans $V_1 = -12V, \quad V_3 = 24V$

Nodes 2 & 4 forms a supernode

$$\boxed{V_4 - V_2 = 24} \Rightarrow (1)$$

from the figure $I_o = \dfrac{V_2}{2}$

KCL @ super node

$$\frac{V_2}{2} + \frac{V_4 - V_1}{2} + \frac{V_4 - V_3}{2} + \frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{1} = 0$$

$$\frac{V_2}{2} + \frac{V_4 + 12}{2} + \frac{V_4 - 24}{2} + \frac{V_2 + 12}{2} + V_2 - 24 = 0$$

$$V_2\left(\frac{1}{2} + 1 + \frac{1}{2}\right) + V_4\left(\frac{1}{2} + \frac{1}{2}\right) = 24 - 6 + 12 - 6$$

$$\boxed{2V_2 + V_4 = 24} \Rightarrow (2)$$

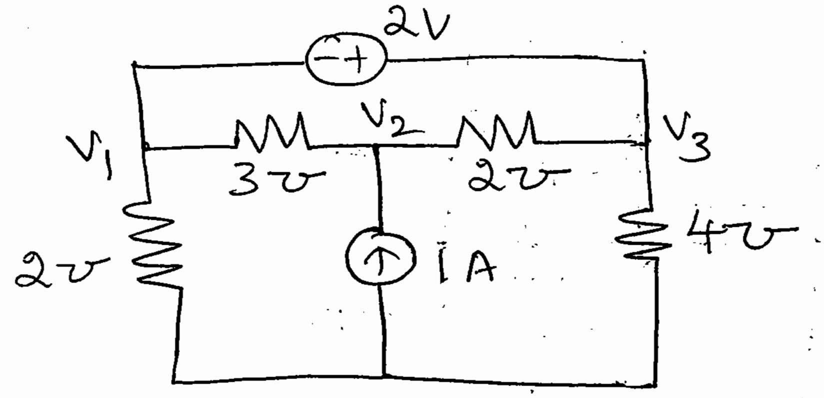
$V_2 = 0V$

$V_4 = 24V$

$$\boxed{I = 0 \text{ amperes}}$$

Imp

Find V_1, V_2 & V_3 using nodal method



Nodes 1 & 3 forms a supernode

$\therefore \boxed{V_3 - V_1 = 2} \Rightarrow (1)$

KCL @ supernode

$2V_1 + 3(V_1 - V_2) + 4V_3 + 2(V_3 - V_2) = 0$

$\boxed{5V_1 - 5V_2 + 6V_3 = 0} \Rightarrow (2)$

KCL @ node 2

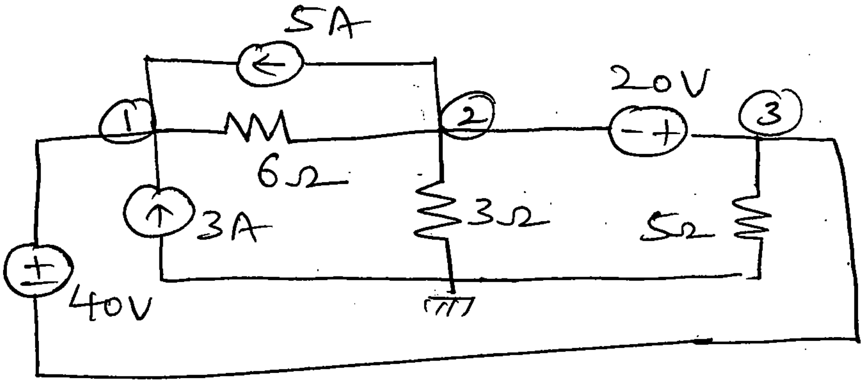
$3(V_2 - V_1) + 2(V_2 - V_3) = 1$

$\boxed{-3V_1 + 5V_2 - 2V_3 = 1} \Rightarrow (3)$

solving equations

$\boxed{V_1 = -1.16V}$; $\boxed{V_2 = -0.166V}$, $\boxed{V_3 = 0.833V}$

Calculate the power dissipated by 5Ω resistor



$$\boxed{V_3 - V_2 = 20} \rightarrow 1$$

$$\boxed{V_1 - V_3 = 40} \rightarrow 2$$

Nodes 2 & 3 & 1 & 3 forms a super node

KCL @ supernode

$$\frac{V_1-\cancel{V_2}}{\cancel{6}} + \frac{V_2-\cancel{V_1}}{\cancel{6}} + \frac{V_2}{3} + \frac{V_3}{5} = 3+\cancel{5}-\cancel{5}$$

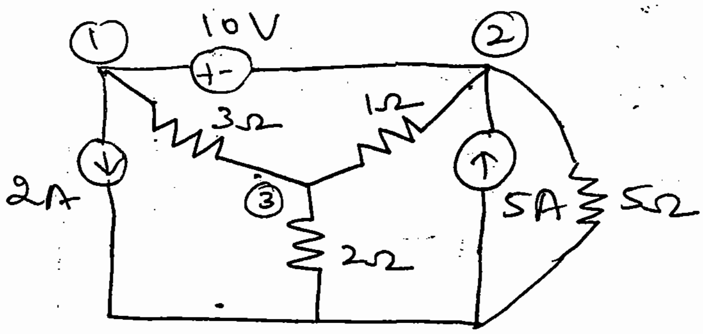
$$\boxed{0.3V_2 + 0.2V_3 = 3} \Rightarrow 3$$

$V_1 = 58\,V$, $V_2 = -2\,V$, $V_3 = 18\,V$

$\therefore \quad P_{5\Omega} = VI = \frac{V^2}{R}$

$$\boxed{P = 64.8 \text{ watts}}$$

Find the power delivered by 5A current source using nodal method



Nodes 1 & 2 forms a supernode

$$\boxed{V_1 - V_2 = 10 \rightarrow (1)}$$

KCL to supernode

$$\frac{V_1 - V_3}{3} + \frac{V_2 - V_3}{1} + \frac{V_2}{5} = 5 - 2$$

$$\boxed{0.33V_1 + 1.2V_2 - 1.33V_3 = 3 \rightarrow (2)}$$

KCL to Node 3

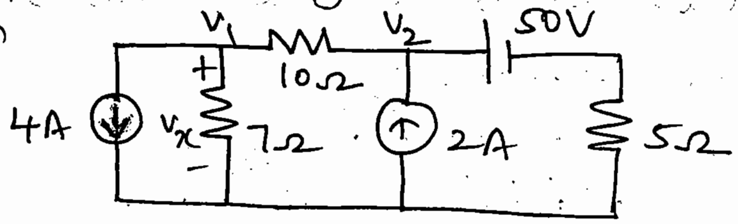
$$\boxed{-0.33V_1 - V_2 + 1.833V_3 = 0}$$

$$\frac{V_3 - V_1}{3} + \frac{V_3}{3} + \frac{V_3 - V_2}{1} = 0$$

$V_1 = 13.71V$, $V_2 = 3.71V$, $V_3 = 4.5V$

$$P_{5A} = 5 \times V_2 = 5 \times 3.71$$
$$= 18.55 \text{ watts}$$

Find the voltage V_x for the circuit shown (35)



from the figure. $V_x = V_1$

KCL @ node 1

$$\frac{V_1 - V_2}{10} + \frac{V_1}{7} = -4$$

$$V_1\left(\frac{1}{7} + \frac{1}{10}\right) - \frac{V_2}{10} = -4$$

$$V_1(0.1428 + 0.1) - 0.1V_2 = -4$$

$$\boxed{0.2428\, V_1 - 0.1\, V_2 = -4 \Rightarrow (1)}$$

KCL @ node 2

$$\frac{V_2 - 50}{5} + \frac{V_2 - V_1}{10} = 2$$

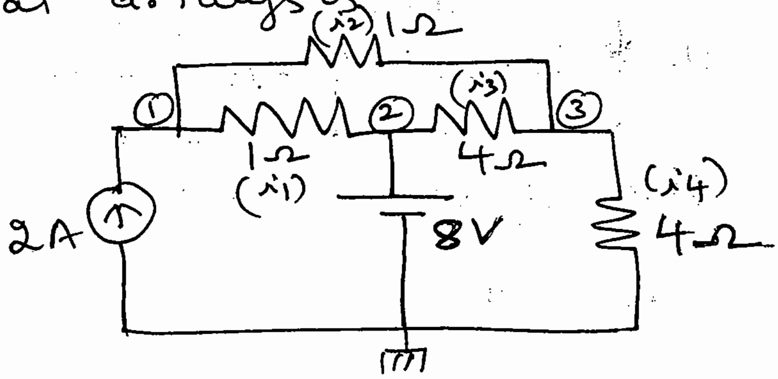
$$-\frac{V_1}{10} + V_2\left(\frac{1}{5} + \frac{1}{10}\right) = 12$$

$$\boxed{-0.1V_1 + V_2(0.3) = 12} \Rightarrow (2)$$

$V_1 = 0V$, $V_2 = 40V$, $\boxed{V_1 = V_x = 0\,V}$

DEC 2011

Find currents in all resistors by nodal analysis



$V_2 = 8V$

apply KCL @ node 1

$$\frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{1} = 2$$

$$V_1 - 8 + V_1 - V_3 = 2$$

$$\boxed{2V_1 - V_3 = 10} \Rightarrow (1)$$

KCL @ Node 3

$$\frac{V_3}{4} + \frac{V_3 - V_2}{4} + \frac{V_3 - V_1}{1} = 0$$

$$\frac{V_3}{4} + \frac{V_3}{4} - \frac{V_2}{4} + V_3 - V_1 \quad \therefore \boxed{V_2 = 8V}$$

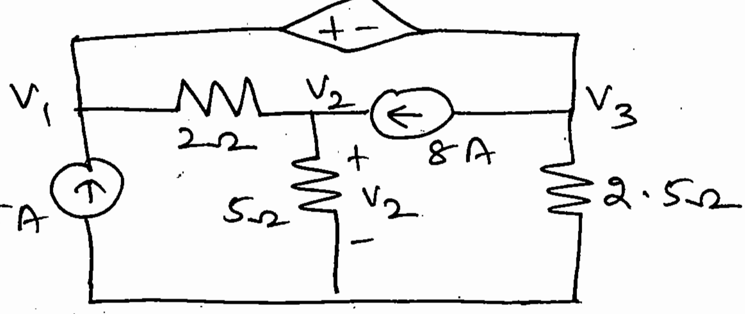
$$\boxed{-V_1 + 1.5 V_3 = 2} \Rightarrow (2)$$

$$i_1 = \frac{V_1 - V_2}{1} = 0.5A \; ; \; i_2 = \frac{V_2 - V_3}{4} = 0.25A$$

$$i_3 = \frac{V_1 - V_3}{1} = 1.5A$$

$$i_{4\Omega} = \frac{V_3}{4} = 1.75A$$

Determine the nodal voltages for the circuit shown (36)



Nodes ① & ③ forms a supernode

$$V_1 - V_3 = 0.8\,V_2$$

$$\boxed{V_1 - 0.8\,V_2 - V_3 = 0} \rightarrow ①$$

KCL @ super node

$$\frac{V_1 - V_2}{2} + \frac{V_3}{2.5} = +5 - 8$$

DIVAKAR. B. C

$$\frac{V_1}{2} - \frac{V_2}{2} + \frac{V_3}{2.5} = -3$$

$$\boxed{0.5\,V_1 - 0.5\,V_2 + 0.4\,V_3 = -3} \rightarrow ②$$

KCL @ node 2

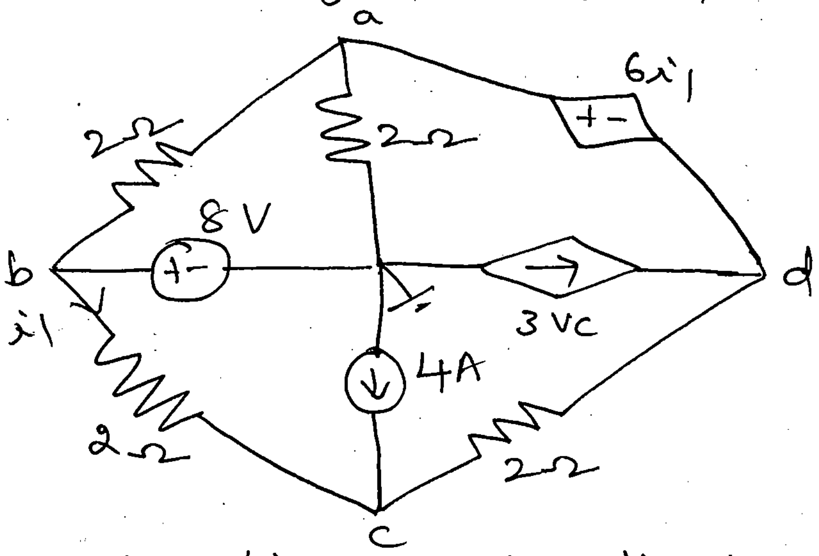
$$\frac{V_2 - V_1}{2} + \frac{V_2}{5} = 8$$

$$-\frac{V_1}{2} + \frac{V_2}{2} + \frac{V_2}{5} = 8$$

$$\boxed{-0.5\,V_1 + 0.7\,V_2 = 8} \Rightarrow ③$$

$$V_1 = 20.27\,V,\ V_2 = 25.9\,V,\ V_3 = -0.45\,V$$

3) For the network shown find the node voltages V_c & V_d



from the figure $i_1 = \frac{V_b - V_c}{2}$

$\boxed{V_b = 8\,V}$

Nodes (a) & (d) forms a super node

$\therefore V_a - V_d = 6i_1 \Rightarrow \boxed{V_a = 6i_1 + V_d}$

KCL @ super node

$$\frac{V_a}{2} + \frac{V_a - V_b}{2} + \frac{V_d - V_c}{2} = 3V_c$$

$$0.5V_a + 0.5V_a - 0.5V_b + 0.5V_d - 0.5V_c = 3V_c$$

$$V_a + 0.5V_d = 3V_c + 4 + 0.5V_c$$

$$6i_1 + V_d + 0.5V_d = 3V_c + 4 + 0.5V_c$$

$$6i_1 + 1.5V_d = 3V_c + 4 + 0.5V_c$$

$$6i_1 + 1.5V_d - 3.5V_c = 4$$

$$6\left[\frac{V_b - V_c}{2}\right] + 1.5V_d - 3.5V_c = 4$$

$$\boxed{-6.5V_c + 1.5V_d = -20}$$

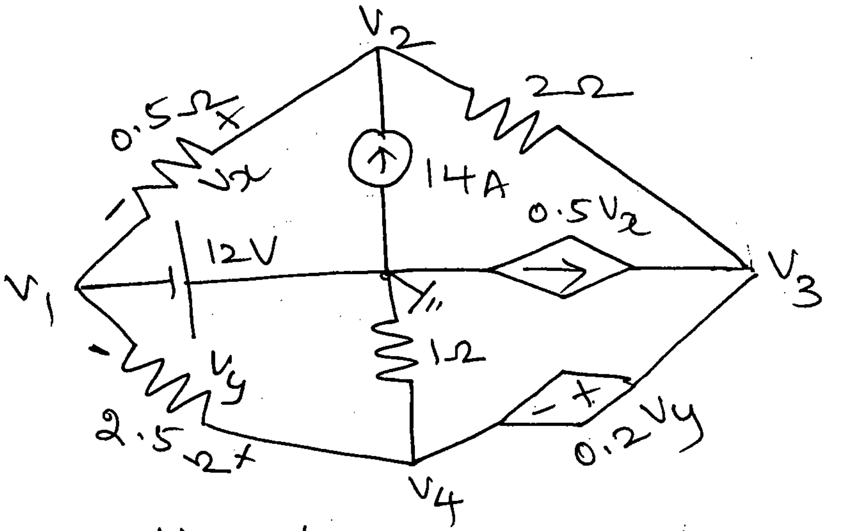
KCL @ node c

$$\frac{V_c - V_d}{2} + \frac{V_c - V_b}{2} = 4$$

$$\boxed{V_c - 0.5V_d = 8}$$

$\boxed{V_c = -1.14 \text{ volts}}$, $\boxed{V_d = -18.3 \text{ volts}}$

3) For the network shown find V_x & V_y



from the figure $V_x = V_2 - V_1$

$$V_y = V_4 - V_1$$

& $V_1 = -12V$

Nodes (3) & (4) forms a supernode

$$V_3 - V_4 = 0.2V_y$$

$$V_3 - V_4 = 0.2(V_4 - V_1)$$

$$V_3 - V_4 = 0.2V_4 - 0.2V_1$$

$$0.2V_1 - 1.2V_4 + V_3 = 0$$

$$\boxed{V_3 - 1.2V_4 = 2.4} \Rightarrow (1)$$

KCL @ Super node

$$V_4 + \frac{V_4 - V_1}{2.5} + \frac{V_3 - V_2}{2} = 0.5\, V_x$$

$$\left(1+\frac{1}{2.5}\right)V_4 + 4.8 + \frac{V_3}{2} - \frac{V_2}{2} = 0.5(V_2+12)$$

$$\boxed{-V_2 + \frac{V_3}{2} + 1.4V_4 = 1.2} \Rightarrow (2)$$

KCL @ node 2

$$\frac{V_2 - V_1}{0.5} + \frac{V_2 - V_3}{2} = 14 \qquad \because \boxed{V_1 = -12V}$$

$$V_2\left(\frac{1}{0.5}+\frac{1}{2}\right) - \frac{V_3}{2} = -10$$

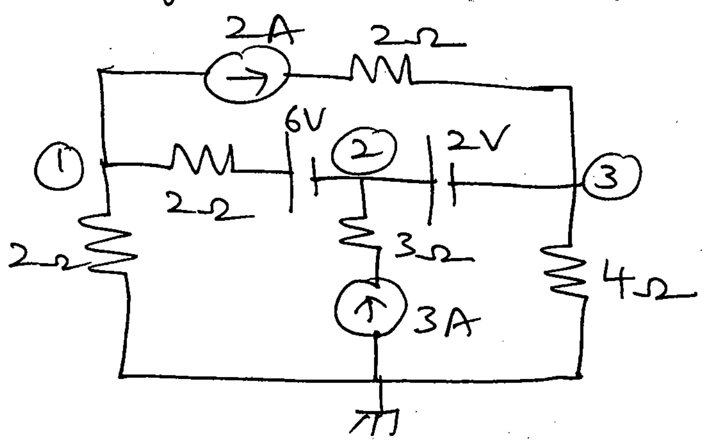
$$\boxed{2.5\, V_2 - 0.5\, V_3 = -10} \Rightarrow (3)$$

$V_2 = -4V$, $V_3 = 0\,V$, $V_4 = -2V$

$\boxed{V_x = V_2 - V_1}$ $\qquad$ $\boxed{V_y = V_4 - V_1}$

$V_x = 8V$ $\qquad$ $V_y = 10V$

Find the current through 4Ω resistor for the circuit shown



2Ω in series with 2A &
3Ω in series with 3A are redundant

Nodes 2 & 3 forms a super node

$$V_2 - V_3 = 2 \rightarrow (1)$$

+12)

KCL @ supernode

$$\frac{V_2 + 6 - V_1}{2} + \frac{V_3}{4} = 2 + 3$$

$$-\frac{V_1}{2} + \frac{V_2}{2} + \frac{V_3}{4} = 5 - 3$$

$$\boxed{-0.5V_1 + 0.5V_2 + 0.25V_3 = 2} \Rightarrow (2)$$

KCL @ node 1

$$\frac{V_1}{2} + \frac{V_1 - 6 - V_2}{2} = -2$$

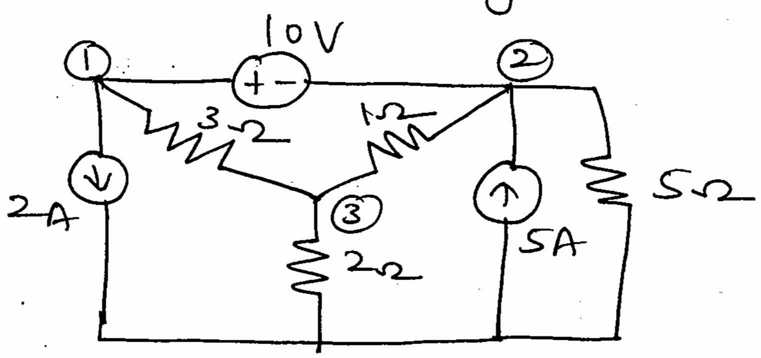
$$V_1 - \frac{V_2}{2} = -2 + 3$$

$$\boxed{V_1 - 0.5\,V_2 = 1} \Rightarrow (3)$$

$$\boxed{V_1 = 4V}, \boxed{V_2 = 6V}, \boxed{V_3 = 4V}$$

$$\boxed{I_{4\Omega} = \frac{V_3}{4} = \frac{4}{4} = 1\,A}$$

Find the power delivered by 5A current source using Nodal method



Nodes ① & ② forms a super node

$$V_1 - V_2 = 10$$

KCL @ super node

$$\frac{V_1 - V_3}{3} + \frac{V_2 - V_3}{1} + \frac{V_2}{5} = 5 - 2$$

$$\boxed{0.33V_1 + 1.2V_2 - 1.33V_3 = 3}$$

KCL @ node 3

$$-0.33V_1 - V_2 + 1.833V_3 = 0$$

$V_1 = 13.71V$, $V_2 = 3.71V$, $V_3 = 4.5V$

$$P_{5A} = 5 \times 3.71 = 18.55 \text{ Watts}$$

Star - delta transformation

When a circuit cannot be simplified by normal series parallel reduction techniques, the star-delta transformation can be used. Fig (a) shows 3 resistances R_A, R_B & R_C connected in delta

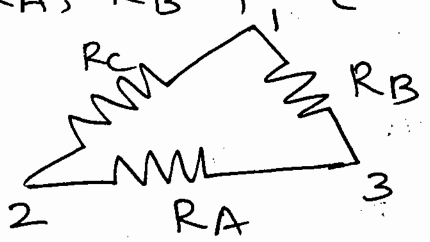


Fig (a)
Delta connection

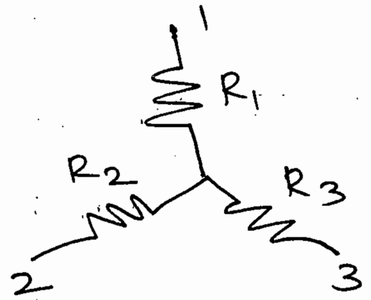


fig (b) star connection

fig (b) shows 3 resistances R_1, R_2 & R_3 connected in star

These 2 networks will be electrically equivalent if the resistances as measured between any pair of terminals is the same in both arrangements

Delta to star transformation

Referring to the delta network as shown in fig (a)

The resistance between terminals 1 & 2

$$= R_C \parallel (R_A + R_B) = \frac{R_C \times (R_A + R_B)}{R_A + R_B + R_C}$$

Referring to star network the resistance between terminals 1 & 2 is $R_1 + R_2$

since the network are electrically equivalent

$$R_1 + R_2 = \frac{R_C(R_A+R_B)}{R_A+R_B+R_C} \Rightarrow (1)$$

$|||^{ly}$

$$R_2 + R_3 = \frac{R_A(R_B+R_C)}{R_A+R_B+R_C} \Rightarrow (2)$$

$$R_3 + R_1 = \frac{R_B(R_C+R_A)}{R_A+R_B+R_C} \Rightarrow (3)$$

(1) − (2)

$$R_1 - R_3 = \frac{R_C(R_A+R_B) - R_A(R_B+R_C)}{R_A+R_B+R_C}$$

$$R_1 - R_3 = \frac{\cancel{R_C R_A} + R_B R_C - R_A R_B - \cancel{R_A R_C}}{R_A+R_B+R_C}$$

$$R_1 - R_3 = \frac{R_B R_C - R_A R_B}{R_A+R_B+R_C} \Rightarrow (4)$$

(3) + (4)

$$2R_1 = \frac{\cancel{R_A R_B} + R_B R_C + R_B R_C - \cancel{R_A R_B}}{R_A+R_B+R_C}$$

$$2R_1 = \frac{2 R_B R_C}{R_A+R_B+R_C}$$

$$\Rightarrow \boxed{R_1 = \frac{R_B R_C}{R_A+R_B+R_C}} \Rightarrow (5); \quad \boxed{R_2 = \frac{R_A R_C}{R_A+R_B+R_C}} \Rightarrow (6)$$

$$\boxed{R_3 = \frac{R_A R_B}{R_A+R_B+R_C}} \Rightarrow (7)$$

Star to Delta transformation

multiplying the above equations

$$R_1 R_2 = \frac{R_A R_B R_C^2}{(R_A+R_B+R_C)^2} \Rightarrow (8)$$

$$R_2 R_3 = \frac{R_A^2 R_B R_C}{(R_A+R_B+R_C)^2} \Rightarrow (9)$$

$$R_3 R_1 = \frac{R_A R_B^2 R_C}{(R_A+R_B+R_C)^2} \Rightarrow (10)$$

$(8) + (9) + (10) \Rightarrow$

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_A R_B R_C [R_C + R_A + R_B]}{(R_A+R_B+R_C)^2}$$

$$= \frac{R_A R_B R_C}{R_A+R_B+R_C} \quad \text{using } (5)$$

$$= R_A R_1 = R_B R_2 + R_C R_3$$

$$\Rightarrow R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

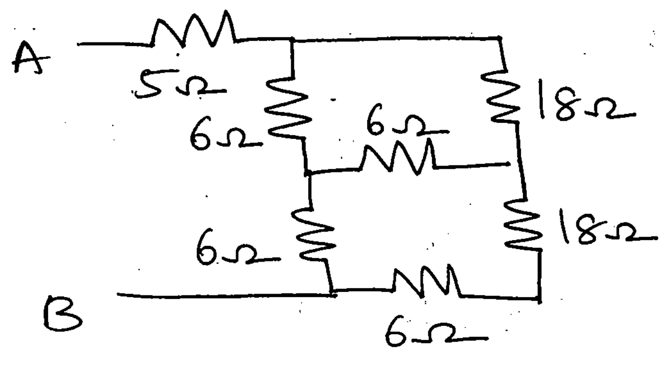
R_C

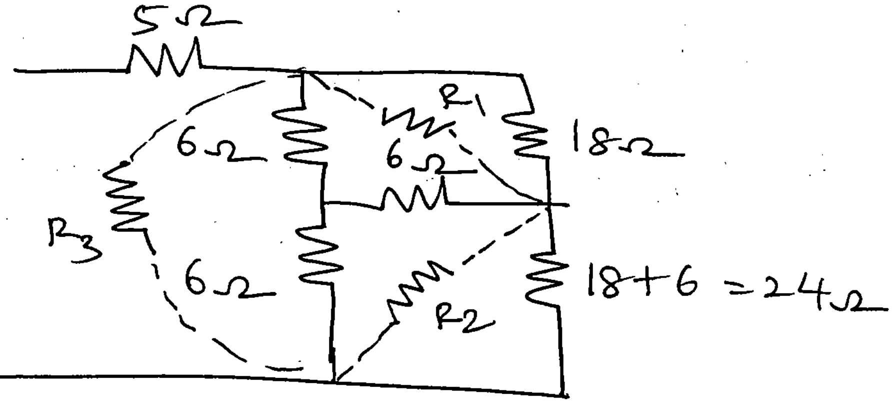
R_A	$=$	$R_2 + R_3 + \dfrac{R_2 R_3}{R_1}$	
R_B	$=$	$R_1 + R_3 + \dfrac{R_3 R_1}{R_2}$	
R_C	$=$	$R_1 + R_2 + \dfrac{R_1 R_2}{R_3}$	

Thus delta resistance between the 2 terminals is the sum of 2 star resistances connected to the same terminals plus product of the 2 resistances divided by the remaining third star resistance

July 2017 ~~sup~~

(08m) Find the equivalent resistance R_{AB} using star & delta transformation for the network shown in figure

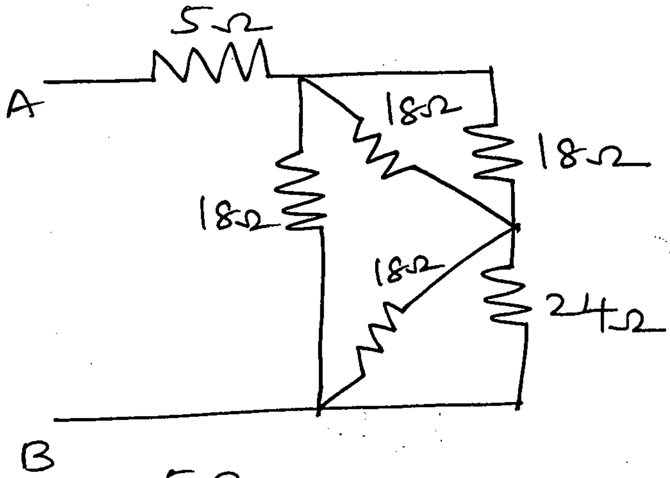




$$R_1 = \frac{(6\times6)+(6\times6)+(6\times6)}{6} = 18\Omega$$

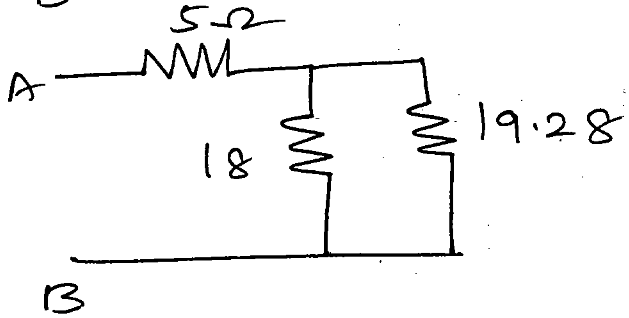
$$R_2 = \frac{(6\times6)+(6\times6)+(6\times6)}{6} = 18\Omega$$

$$R_3 = \frac{(6\times6)+(6\times6)+(6\times6)}{6} = 18\Omega$$



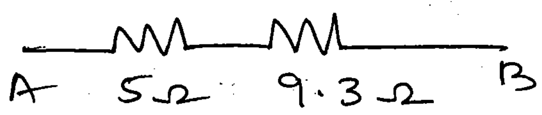
$18 \parallel 18 = \frac{324}{36}$
$= 9\Omega$

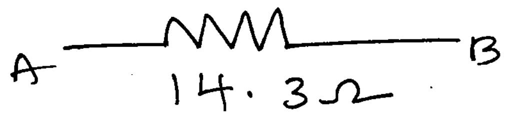
$18 \parallel 24 = 10.28$



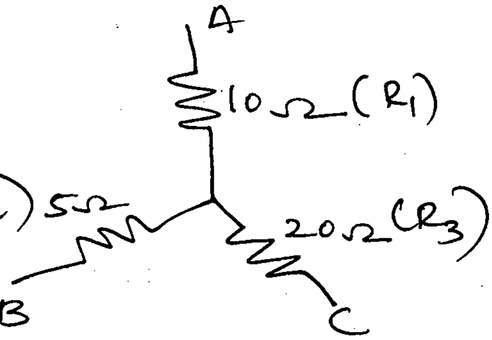
$10.28 + 9$
$= 19.28$

$$= \frac{19.28 \times 18}{19.28 + 18} = 9.30\Omega$$

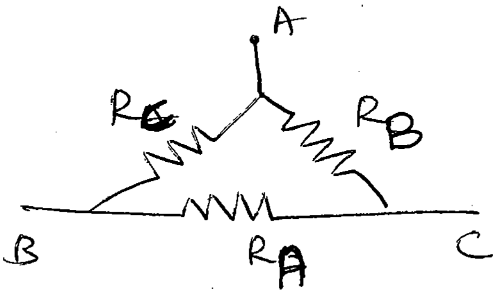




obtain the delta connected equivalent for the star connected circuit shown



Delta connected equivalent of the given star connected circuit



$$R_A = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

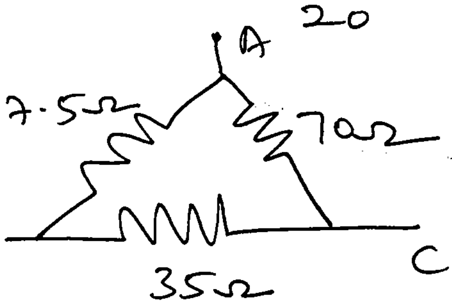
$$= 5 + 20 + \frac{(5)(20)}{10} = 35\Omega$$

$$R_B = R_1 + R_3 + \frac{R_3 R_1}{R_2}$$

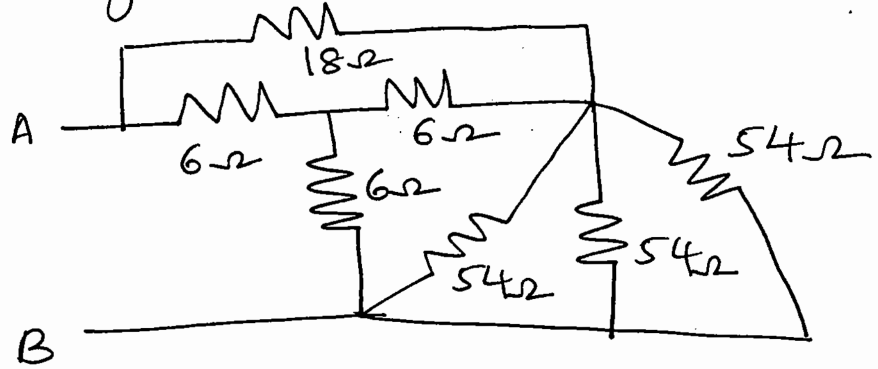
$$= 10 \times 20 + \frac{(10)(20)}{5} = 70\Omega$$

$$R_C = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$10 + 5 + \frac{(5)(10)}{20} = 17.5\Omega$$

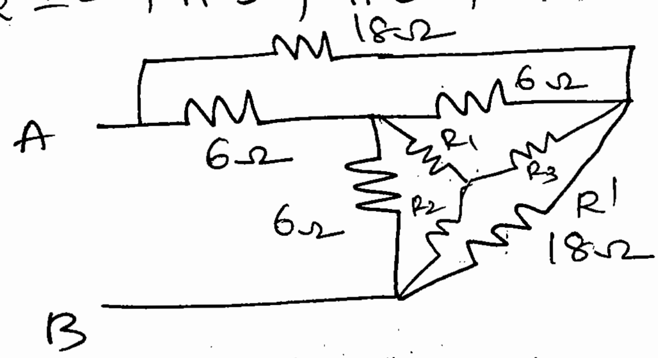


Compute the resistance across the terminals A & B of the n/w shown using Y – Δ transformation



Three 54Ω resistances are in parallel

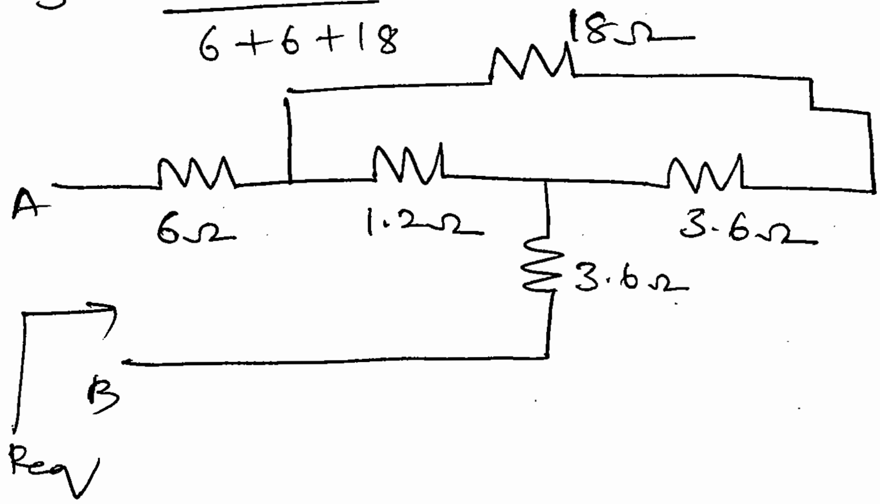
$R' = 54 \parallel 54 \parallel 54 = 18\Omega$

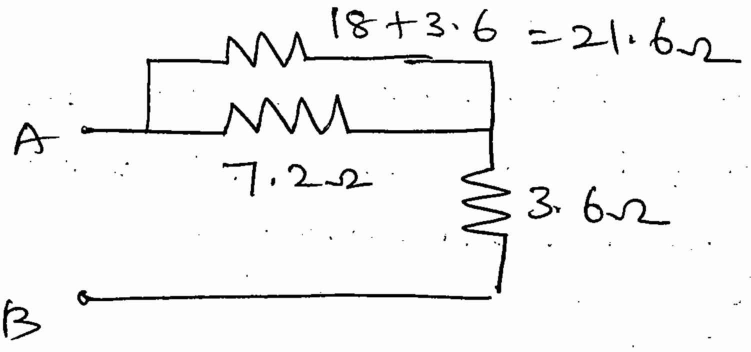


transforming Δ formed by 6Ω, 6Ω & 18Ω

$R_1 = \frac{(6)(6)}{6+6+18} = 1.2\Omega$; $R_2 = \frac{(6).(18)}{6+6+18} = 3.6\Omega$

$R_3 = \frac{(6)(18)}{6+6+18} = 3.6\Omega$



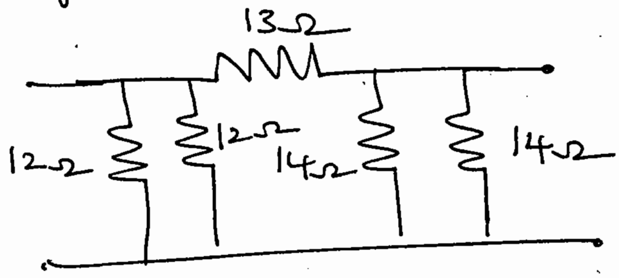


$$R_{eq} = (21.6 \parallel 7.2) + 3.6$$

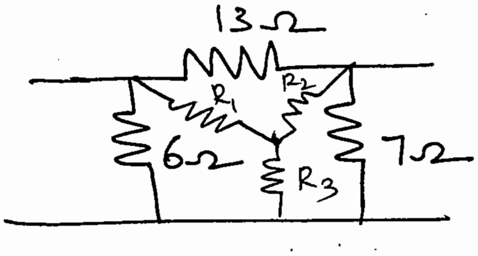
$$= 5.4 + 3.6$$

$$\boxed{R_{eq} = 9\,\Omega}$$

Three impedances are connected in star. find the star equivalent



$12 \parallel 12 = 6\,\Omega$ $\qquad$ $14 \parallel 14 = 7\,\Omega$

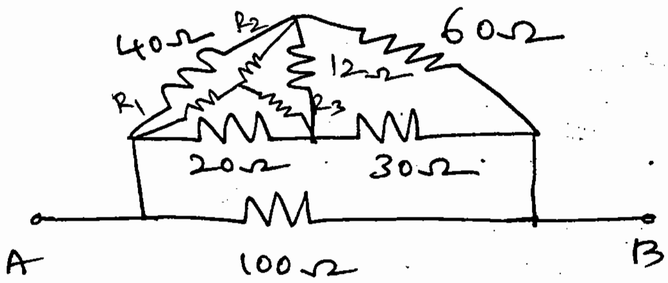


$$R_2 = \frac{13 \times 7}{13+6+7} = 3.5\,\Omega$$

$$R_1 = \frac{13 \times 6}{13+6+7} = 3\,\Omega$$

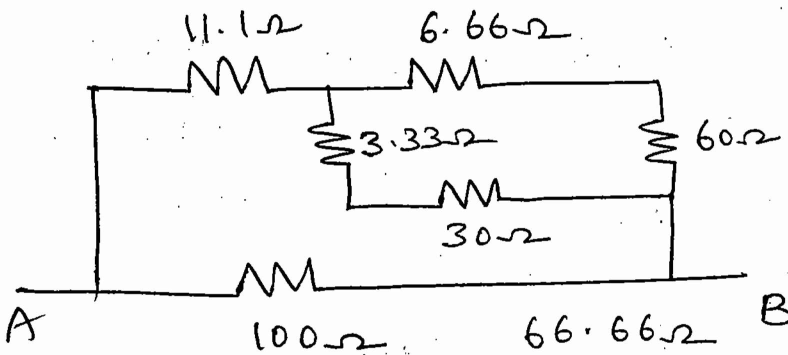
$$R_3 = \frac{6 \times 7}{13+6+7} = 1.615\,\Omega$$

Find the equivalent resistance between A & B 43



$$R_1 = \frac{40 \times 20}{40+20+12} = 11.1\,\Omega$$

$$R_2 = \frac{40 \times 12}{40+20+12} = 6.66\,\Omega \; ; \; R_3 = \frac{20 \times 12}{40+12+20} = 3.33\,\Omega$$



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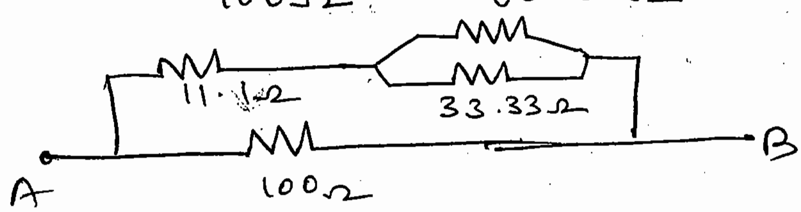
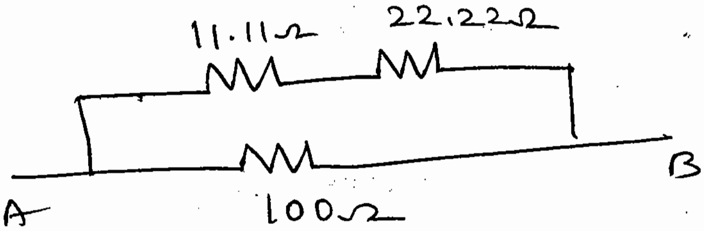
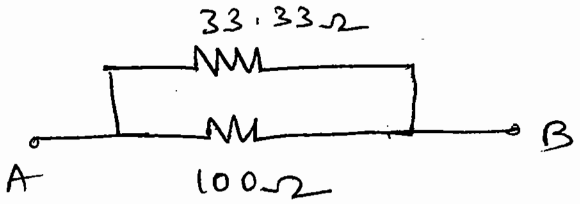


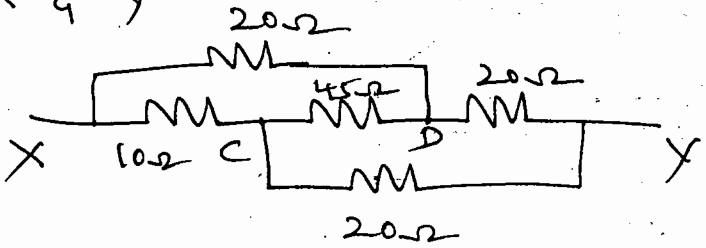
$66.66 \parallel^{\text{lel}} 33.33 = 22.22\,\Omega$

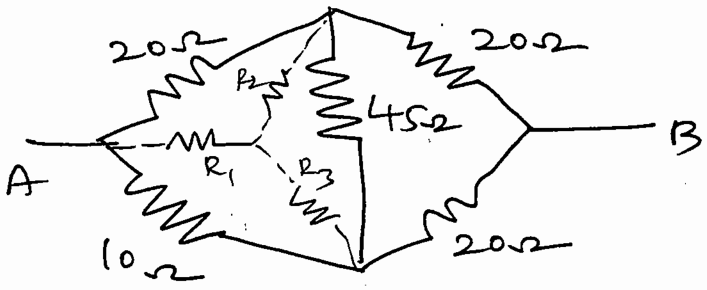




$$= \frac{33.33 \times 100}{33.33 + 100} = 24.99\,\Omega = 25\,\Omega$$

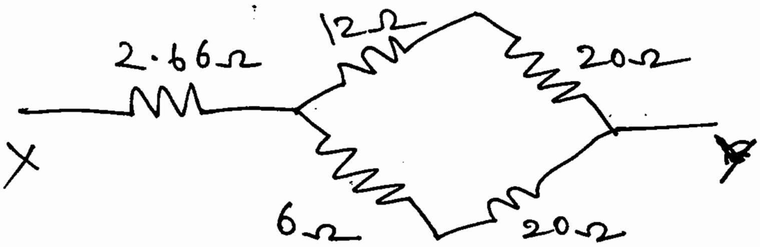
Find the equivalent resistance b/w X & Y

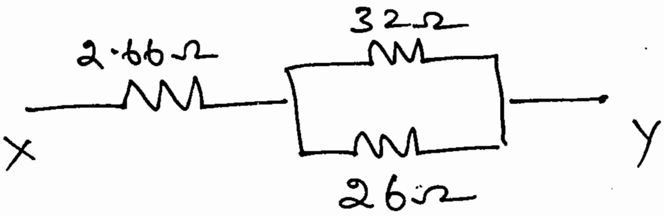




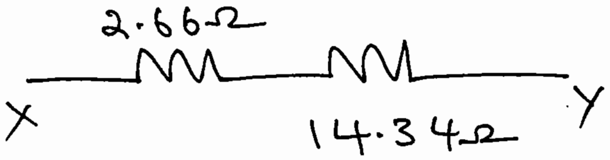
$$R_1 = \frac{20 \times 10}{20+10+45} = 2.66\Omega \quad ; \quad R_3 = \frac{10 \times 45}{20+10+45} = 6\Omega$$

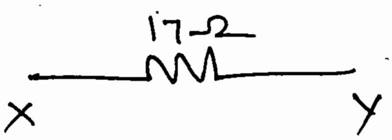
$$R_2 = \frac{20 \times 45}{20+10+45} = 12\Omega$$



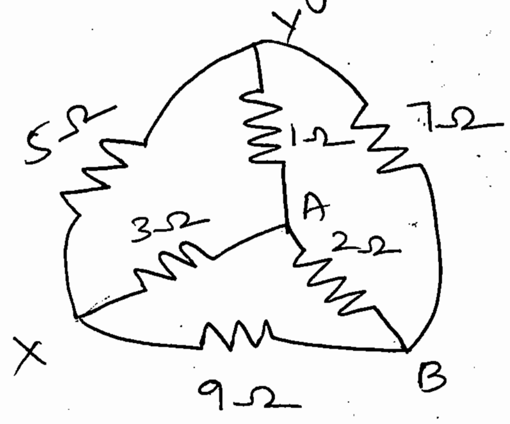


$$32 \parallel^{lel} 26 = \frac{32 \times 26}{32+26} = 14.34\Omega$$





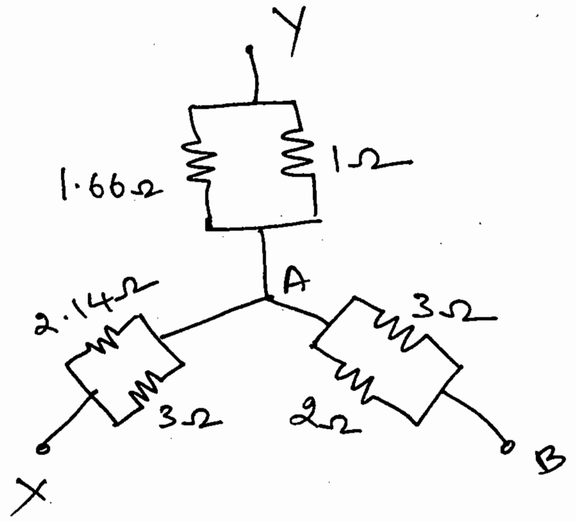
Find R_{AB} by solving the outer delta (X – B – Y) only



$$R_1 = \frac{5 \times 9}{5+9+7} = 2.14\Omega$$

$$R_2 = \frac{5 \times 7}{5+9+7} = 1.66\Omega$$

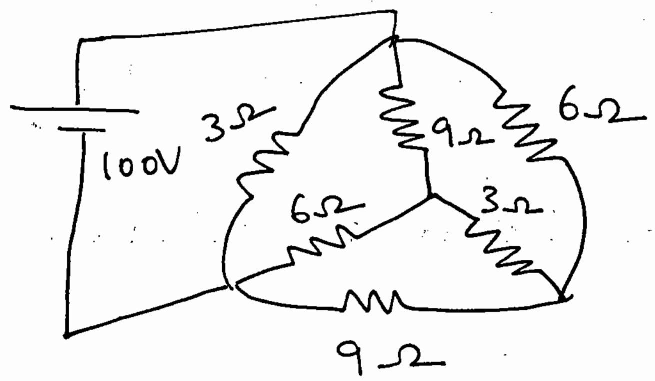
$$R_3 = \frac{7 \times 9}{5+9+7} = 3\Omega$$



$$3 \parallel 2 = \frac{6}{5} = 1.2\Omega$$

$$R_{AB} = 1.2\Omega$$

Determine the power supplied to the network

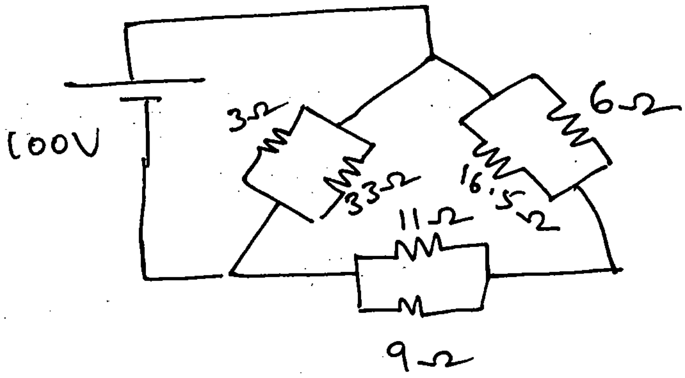


converting inner star to Δ

$$R_A = 6+9 + \frac{6 \times 9}{3} = 33\Omega$$

$$R_B = 9+3 + \frac{9 \times 3}{6} = 16.5\Omega$$

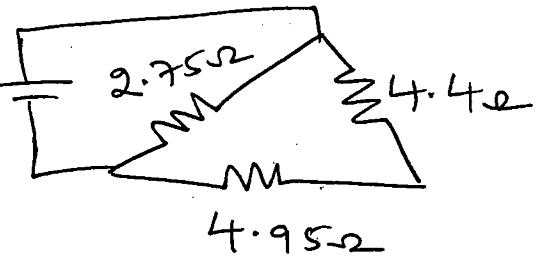
$$R_C = 6+3 + \frac{6 \times 3}{9} = 11\Omega$$

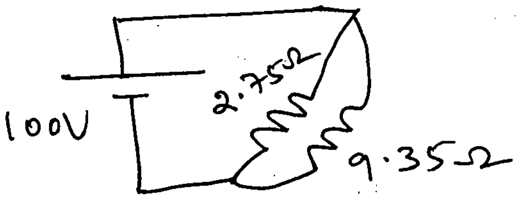


$3 \parallel 33 = 2.75\Omega$

$6 \parallel 16.5 = 4.4\Omega$

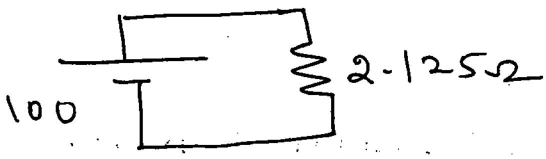
$11 \parallel 9 = 4.95\Omega$





$9.35 \parallel 2.75$

$= 2.125\Omega$

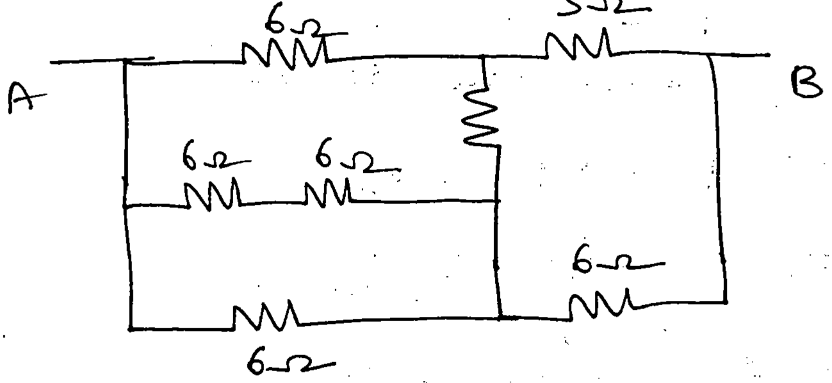


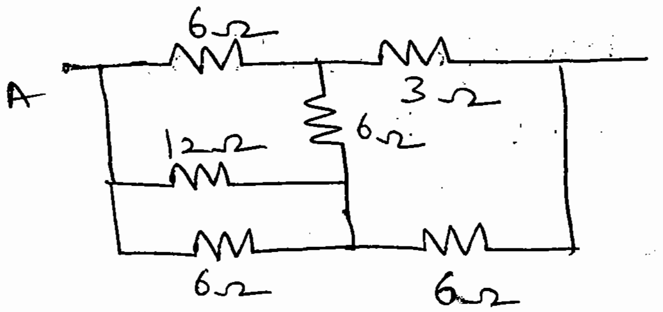
$$I = \frac{100}{2.125} = 47.05A$$

power $= VI = 100 \times 47.05$

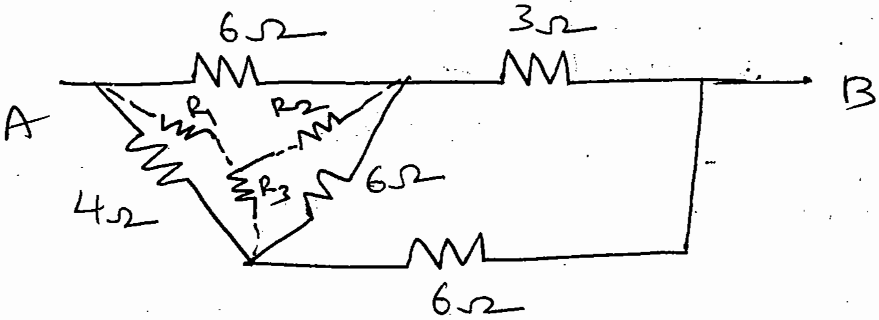
$\boxed{P = 4705.8 \text{ watts}}$

Find the equivalent resistance between A & B





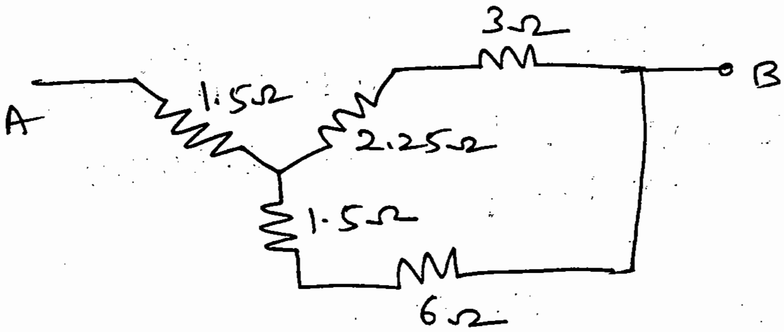
$6 \parallel 12 = 4\Omega$

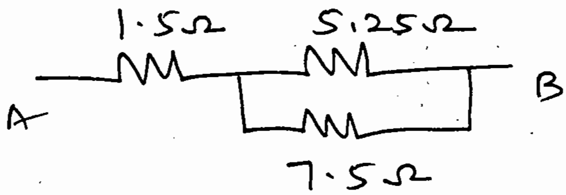


Δ to star

$$R_3 = R_1 = \frac{6 \times 4}{6+4+6} = 1.5\Omega$$

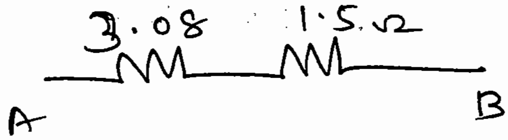
$$R_2 = \frac{6 \times 6}{6+4+6} = 2.25\Omega$$

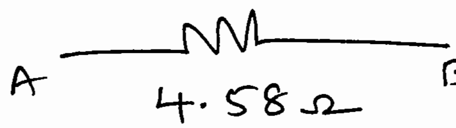




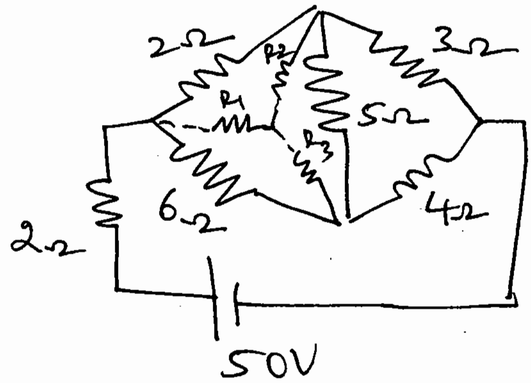
$7.5 \parallel 5.25$

$= 3.08\Omega$





using Y – Δ transformation find the current I

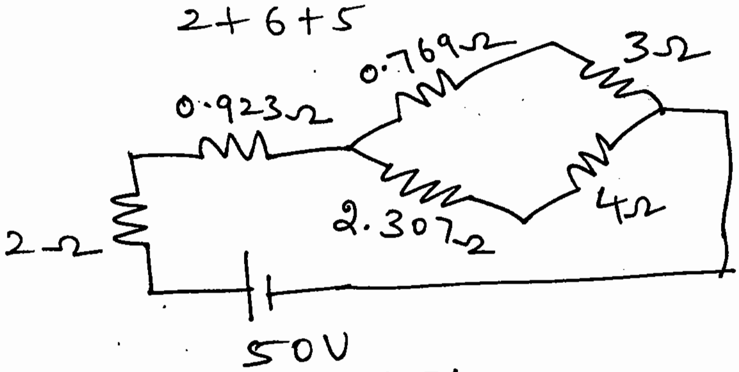


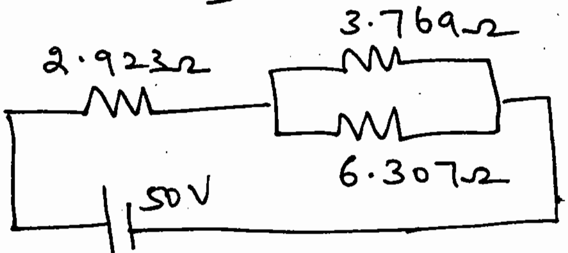
Converting Δ to star

$$R_1 = \frac{2 \times 6}{2+6+5} = 0.923\,\Omega$$

$$R_2 = \frac{2 \times 5}{2+6+5} = 0.769\,\Omega$$

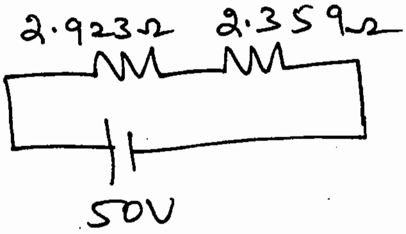
$$R_3 = \frac{6 \times 5}{2+6+5} = 2.307\,\Omega$$

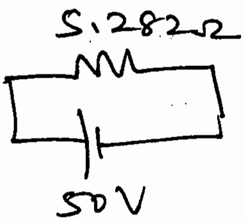




$3.769 \parallel^{lel} 6.307$

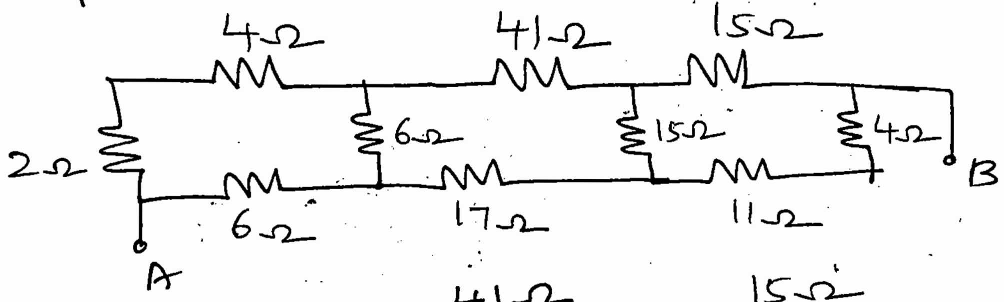
$= 2.359\,\Omega$

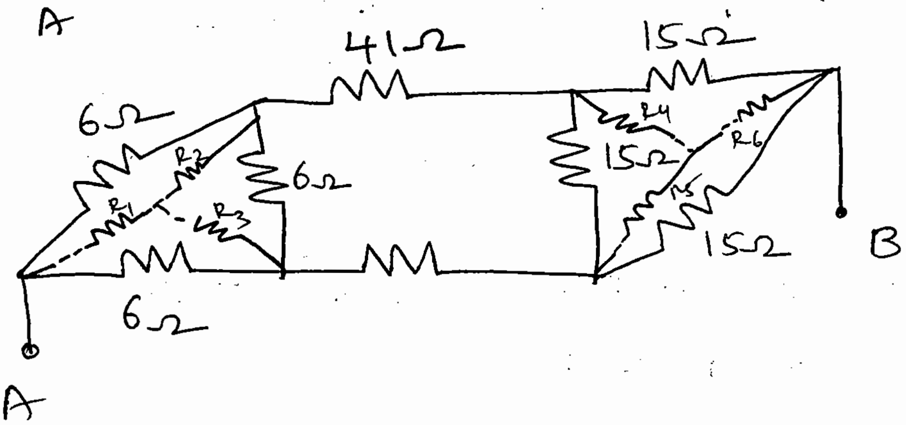




$$\boxed{I = \frac{50}{5.282} = 9.465\,A}$$

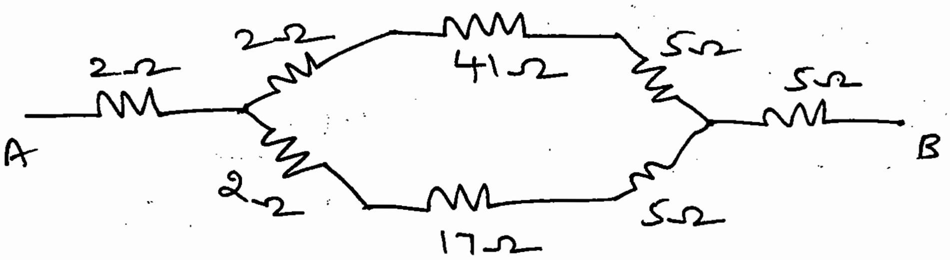
Find an equivalent resistance b/n A & B

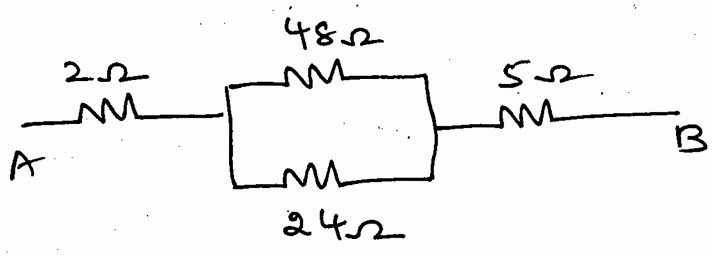




$$R_1 = R_2 = R_3 = \frac{6 \times 6}{6+6+6} = \frac{36}{18} = 2\Omega$$

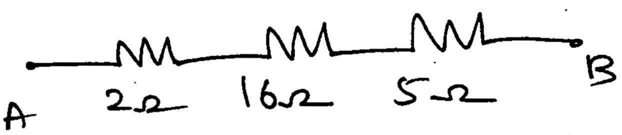
$$R_4 = R_5 = R_6 = \frac{15 \times 15}{15+15+15} = 5\Omega$$

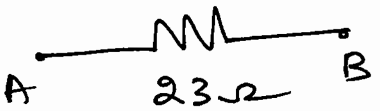




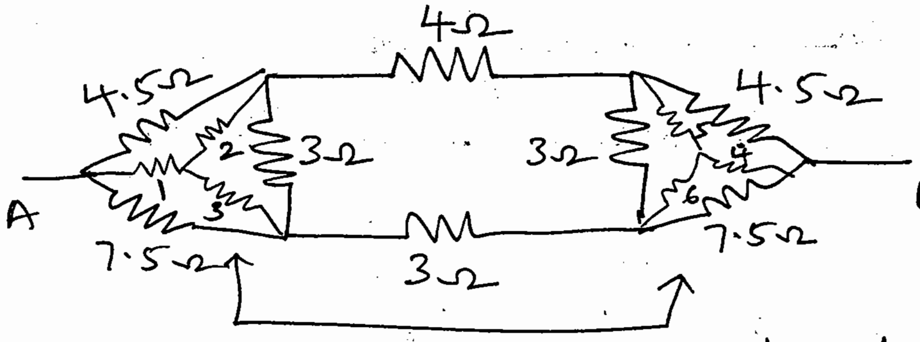
$48 \parallel^{lel} 24$

$= 16\Omega$





Find the equivalent resistance between A & B

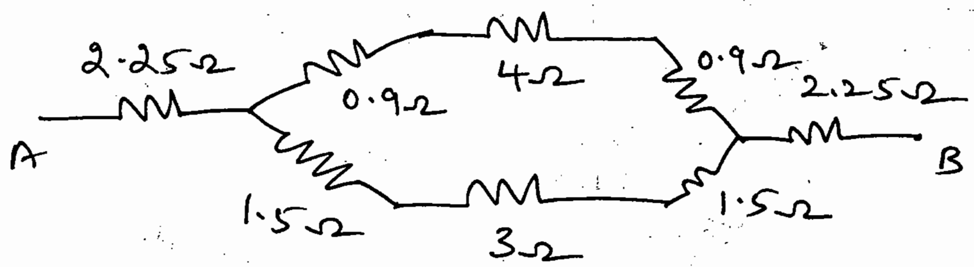


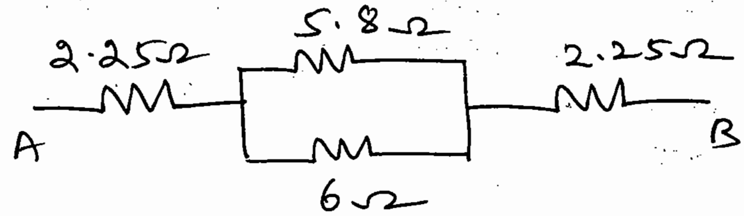
converting Δ to star

$$R_1 = \frac{4.5 \times 7.5}{4.5 + 7.5 + 3} = 2.25 = R_4$$

$$R_2 = \frac{4.5 \times 3}{4.5 + 7.5 + 3} = 0.9\Omega = R_5$$

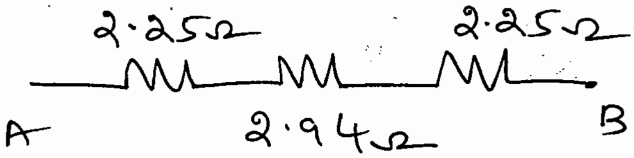
$$R_3 = \frac{7.5 \times 3}{4.5 + 7.5 + 3} = 1.5\Omega = R_6$$

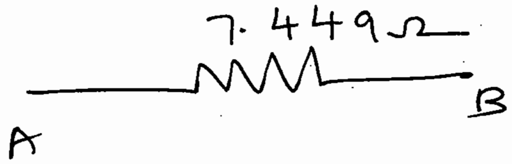




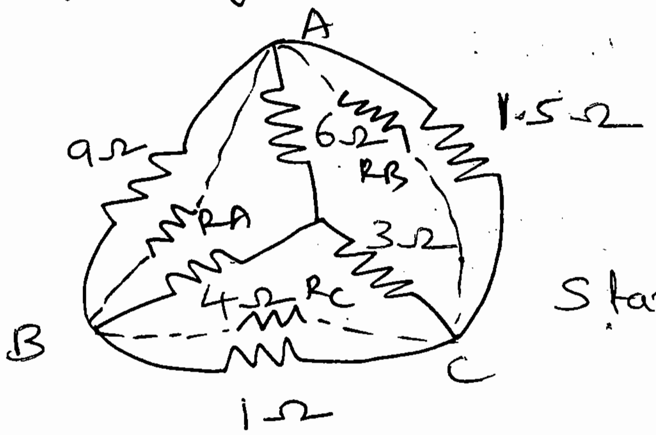
5.8 ||^{el} 6

= 2.94Ω





Find the equivalent resistance between A & B 47

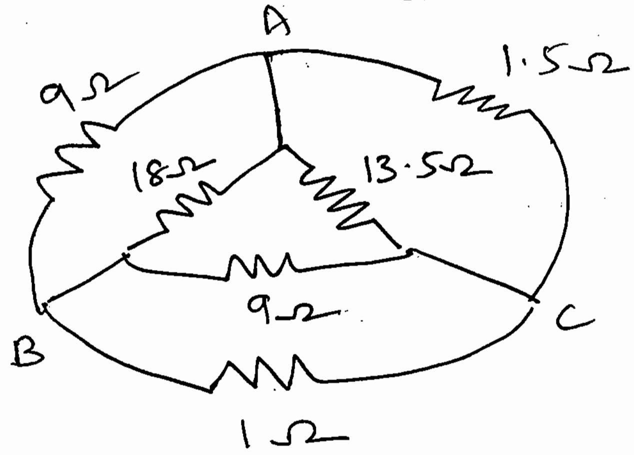


Star to Δ

$$R_A = 6+4+\frac{6\times4}{3} = 10+\frac{24}{3} = 18\Omega$$

$$R_B = 6+3+\frac{6\times3}{4} = 9+\frac{18}{4} = 13.5\Omega$$

$$R_C = 4+3+\frac{4\times3}{6} = 7+\frac{12}{6} = 9\Omega$$



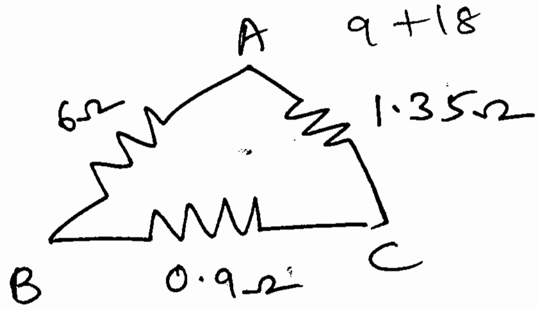
$1.5 \parallel^{lel} 13.5$

$= 1.35\Omega$

$1 \parallel^{lel} 9 = \frac{9}{10}$

$= 0.9\Omega$

$$9 \parallel^{lel} 18 = \frac{9\times18}{9+18} = 6\Omega$$

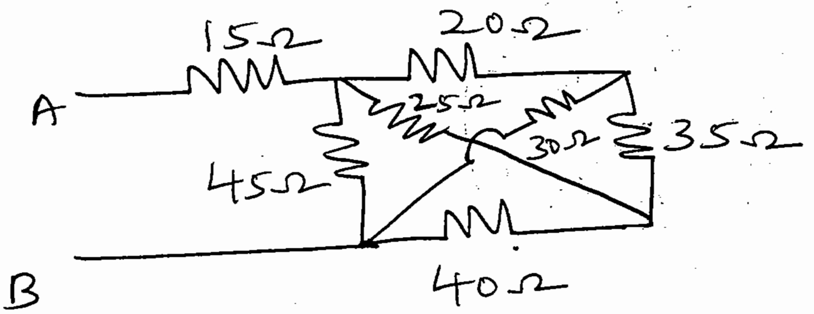


Resistance between A & B is

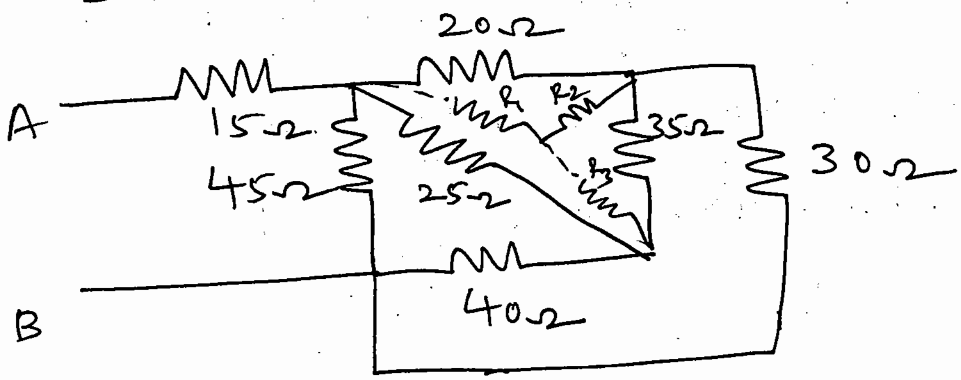
$$6 \parallel^{lel} (1.35+0.9)$$

$$= \frac{6\times2.25}{6+2.25} = 1.63\Omega$$

Find an equivalent resistance between A & B



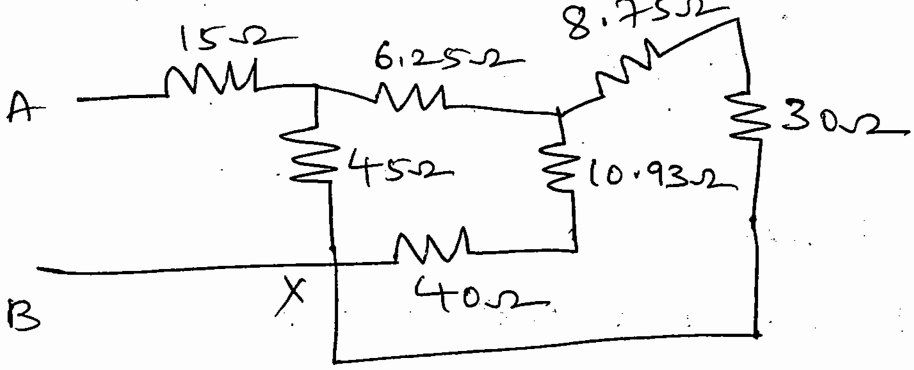
Δ to star

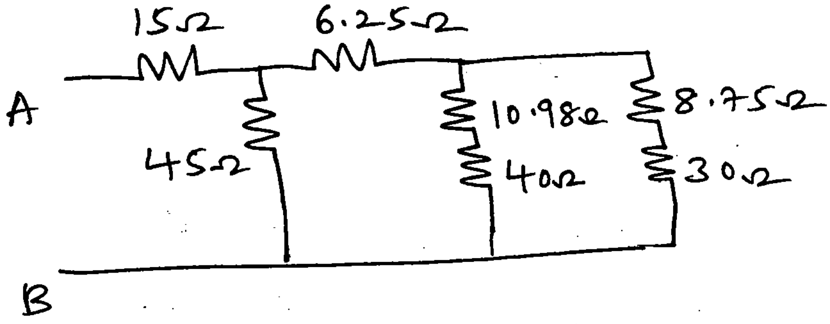


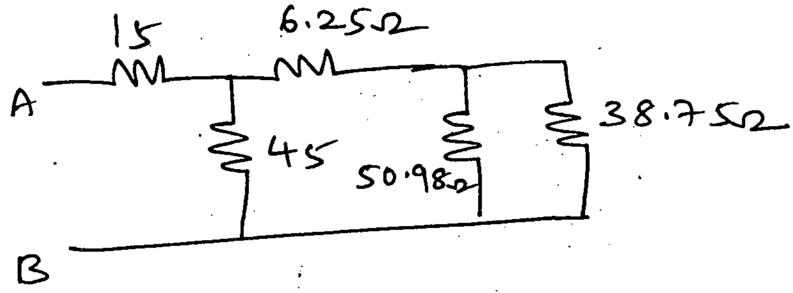
$$R_1 = \frac{25 \times 20}{25+20+35} = 6.25\Omega$$

$$R_2 = \frac{20 \times 35}{25+20+35} = 8.75\Omega$$

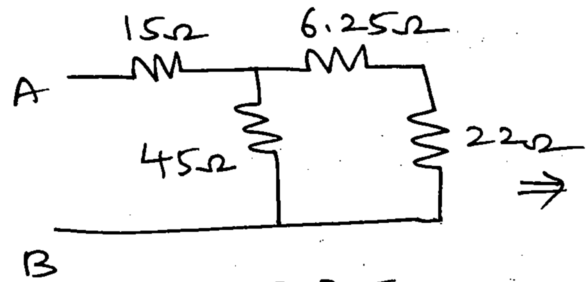
$$R_3 = \frac{25 \times 35}{25+35+20} = 10.93\Omega$$



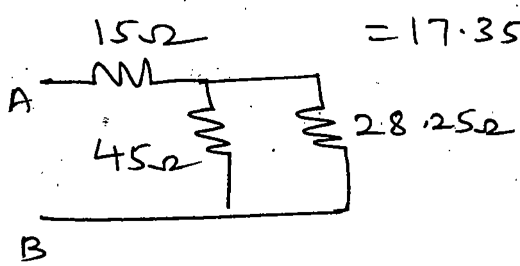


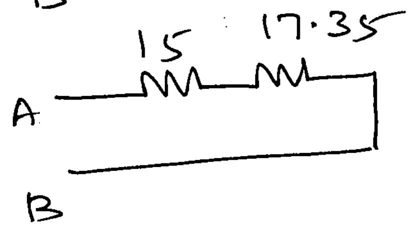


50.98 || 38.75 = 22Ω   45 ||'el 28.25 = 17.35Ω

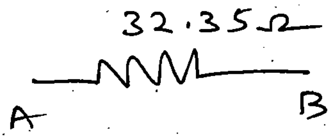


⇒

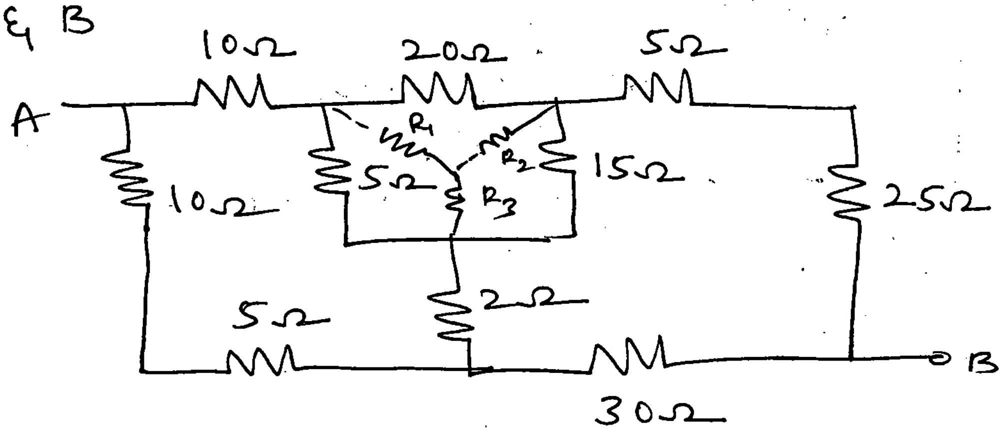




⇒

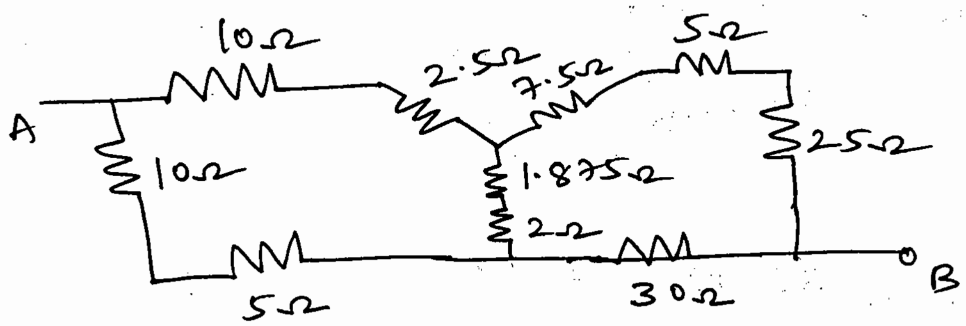


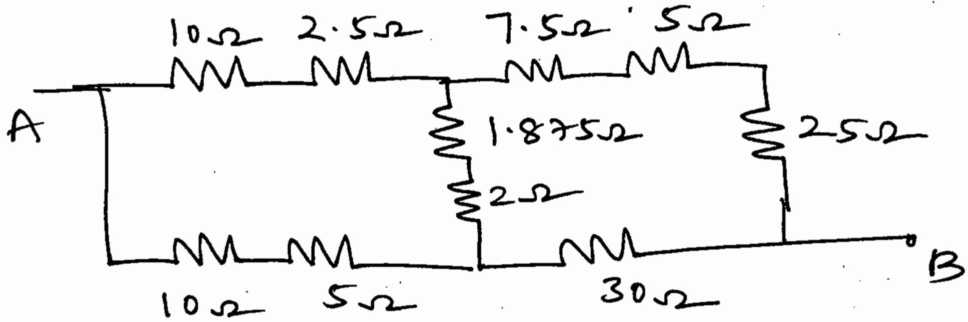
Imp Find the equivalent resistance between A & B

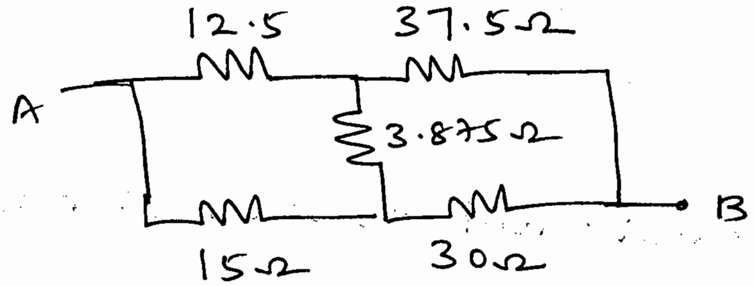


$$R_1 = \frac{20 \times 5}{20 + 5 + 15} = 2.5\,\Omega$$

$$R_2 = \frac{20 \times 15}{20 + 5 + 15} = 7.5\,\Omega \; ; \; R_3 = \frac{5 \times 15}{20 + 5 + 15} = 1.875\,\Omega$$



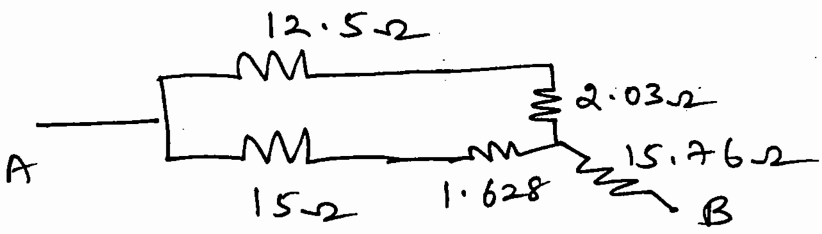


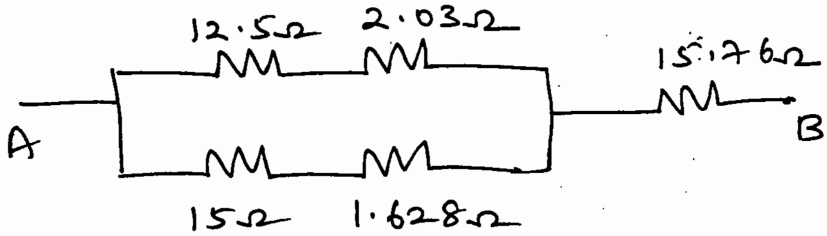


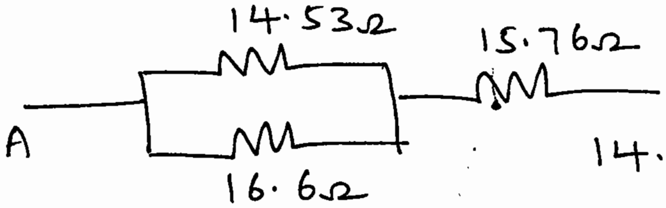
$$R_1 = \frac{3.875 \times 37.5}{3.875 + 37.5 + 30} = 2.03\,\Omega$$

$$R_2 = \frac{37.5 \times 30}{3.875 + 30 + 37.5} = 15.76\,\Omega$$

$$R_3 = \frac{3.875 \times 10}{3.875 + 37.5 + 30} = 1.628\,\Omega$$

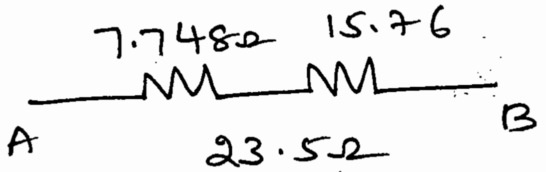


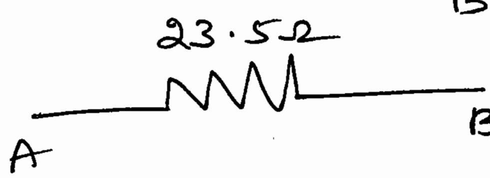




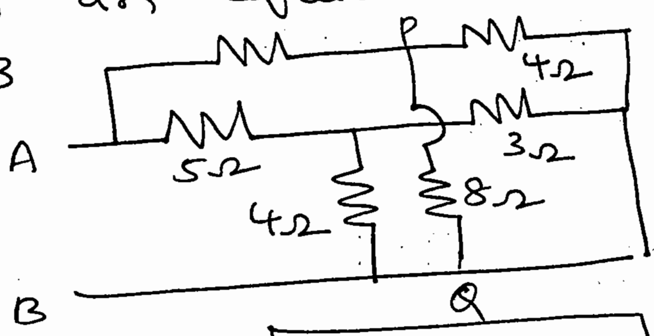
$14.53 \parallel^{lel} 16.6$

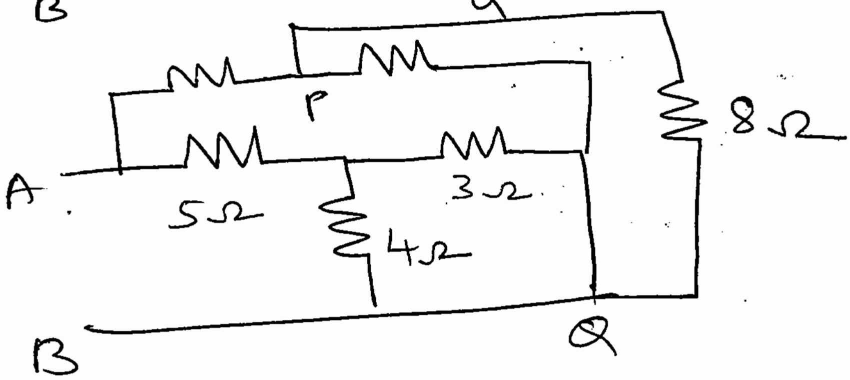
$= 7.748\,\Omega$

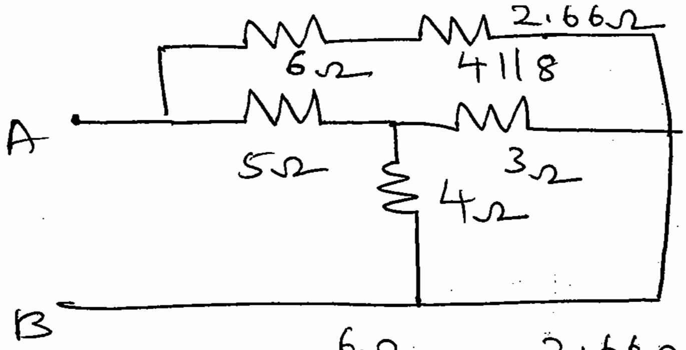


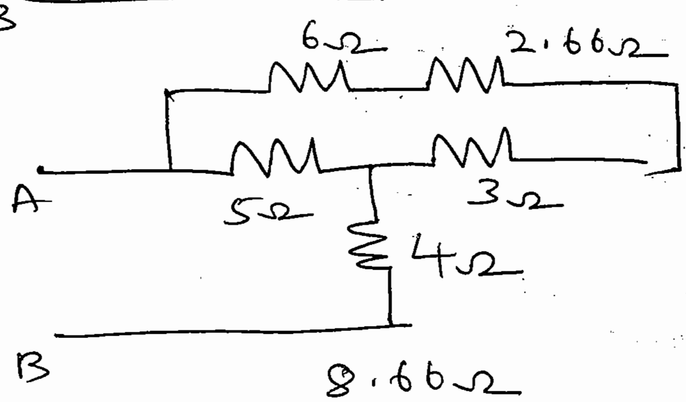


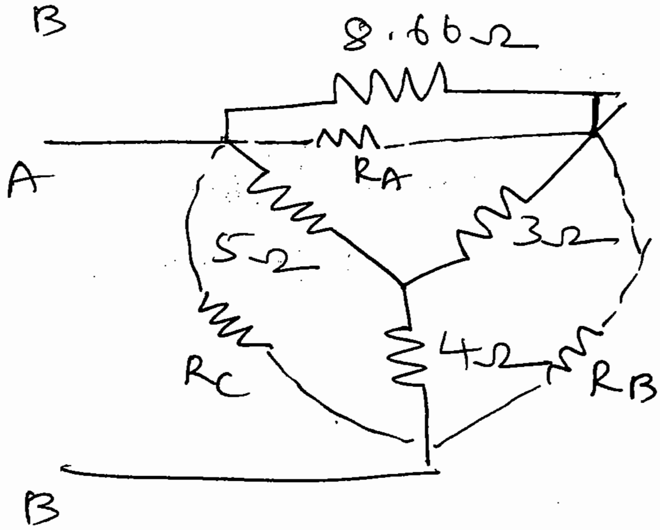
Find an equivalent resistance between A & B









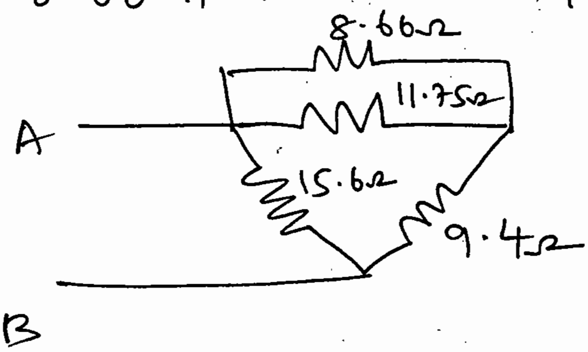


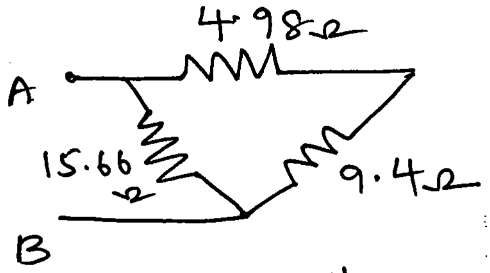
$$R_A = 5+3+\frac{5\times3}{4} = 11.75\Omega$$

$$R_B = 4+3+\frac{4\times3}{5} = 9.4\Omega$$

$$R_C = 5+4+\frac{5\times4}{3} = 15.66\Omega.$$

$$8.66 \parallel 11.75 = 4.98\Omega$$

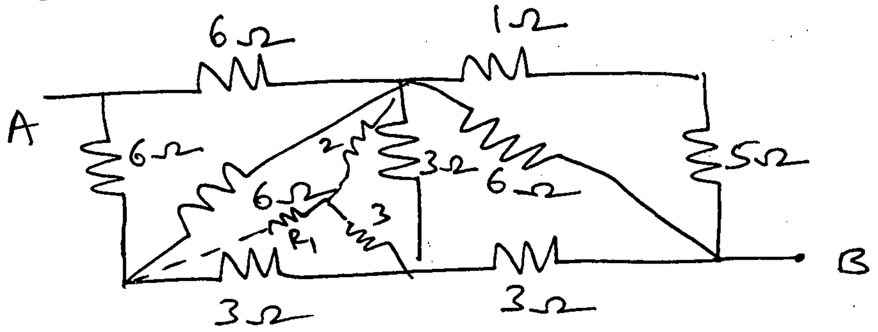




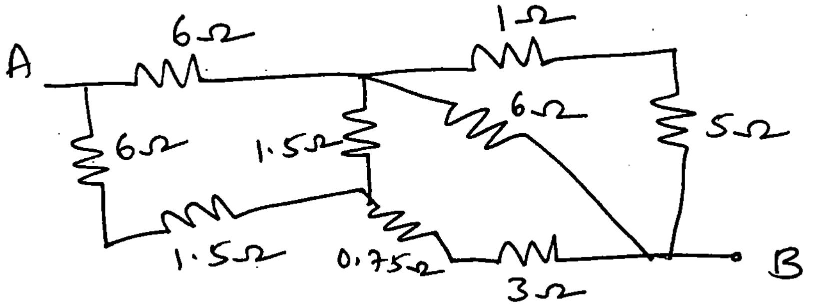
$$15.66 \parallel^{lel} (4.98 + 9.4)$$

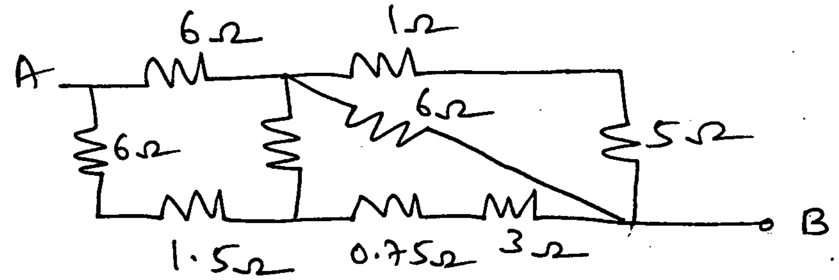
$$= \frac{15.66 \times 14.38}{15.66 + 14.38} = 7.49\Omega$$

Find an equivalent resistance between A & B

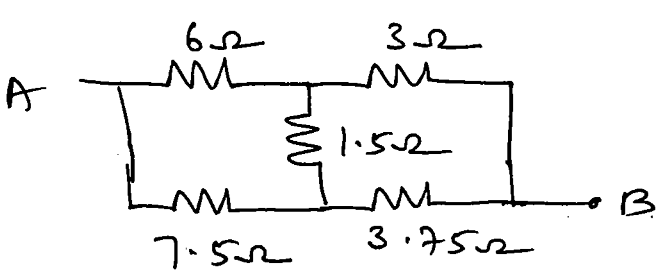


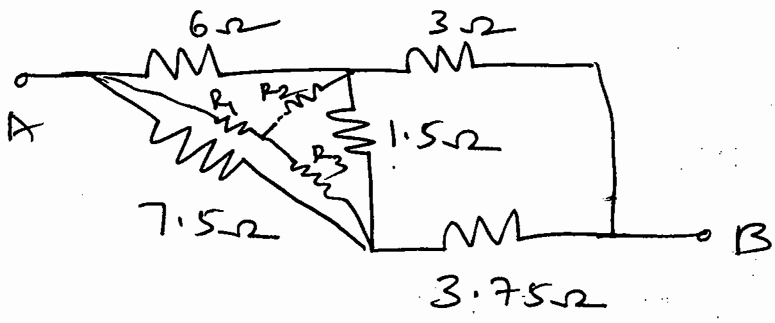
$$R_1 = R_2 = \frac{3 \times 6}{3+3+6} = 1.5\Omega \; ; \quad R_3 = \frac{3 \times 3}{3+3+6} = 0.75\Omega$$





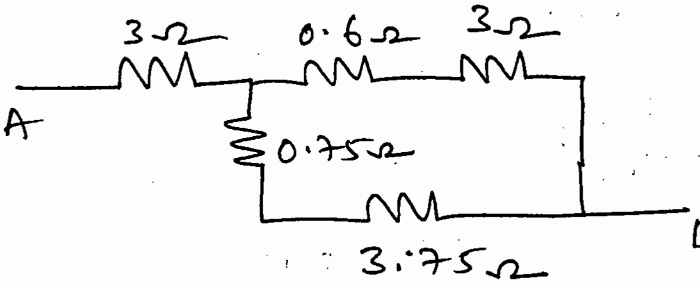
$$6 \parallel^{lel} 6 = 3\Omega$$

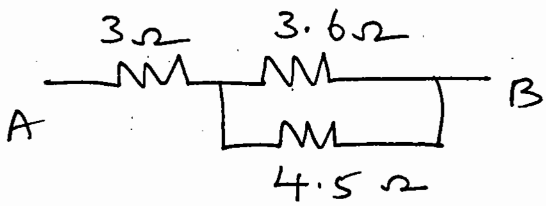




$$R_1 = \frac{6 \times 7.5}{6 + 7.5 + 1.5} = 3\Omega \qquad R_2 = \frac{6 \times 1.5}{6 + 1.5 + 7.5} = 0.6\Omega$$

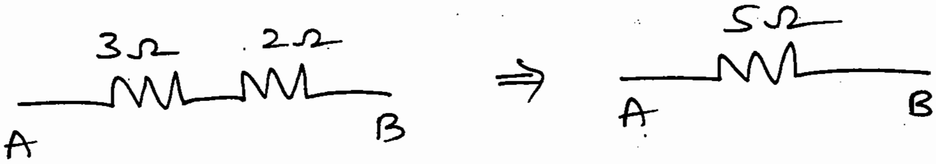
$$R_3 = \frac{7.5 \times 1.5}{6 + 1.5 + 7.5} = 0.75\Omega$$



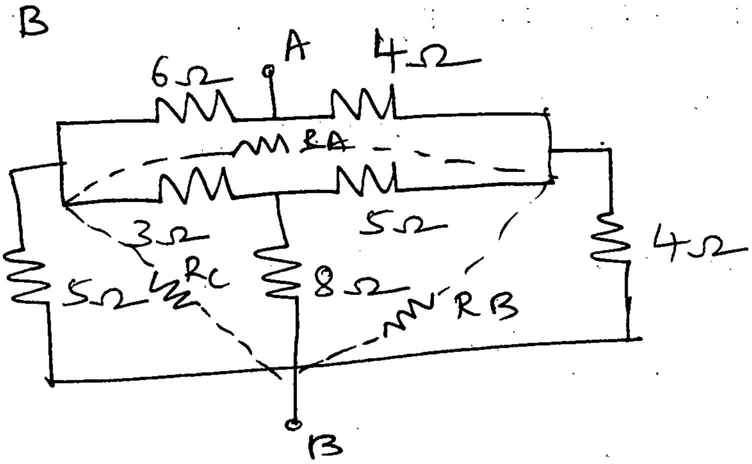


$4.5 \parallel^{lel} 3.6$

$= 2\Omega$



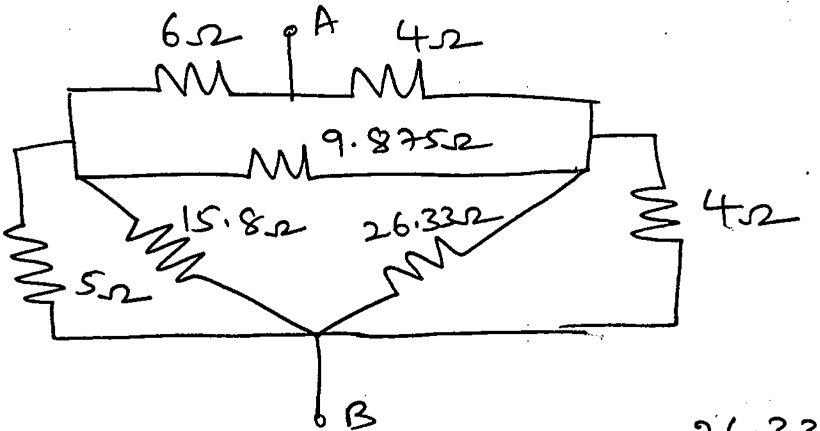
Find an equivalent resistance between A & B



$$R_A = 3+5+\frac{3\times5}{8} = 9.875\,\Omega$$

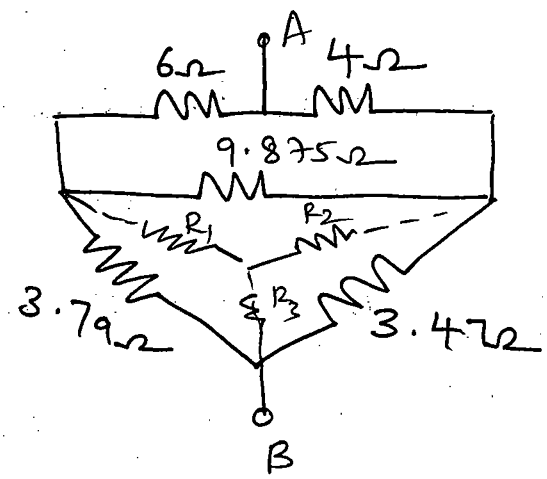
$$R_B = 8+5+\frac{8\times5}{3} = 26.33\,\Omega$$

$$R_C = 3+8+\frac{3\times8}{5} = 15.8\,\Omega$$



$15.8 \parallel^{el} 5 = 3.79\,\Omega$

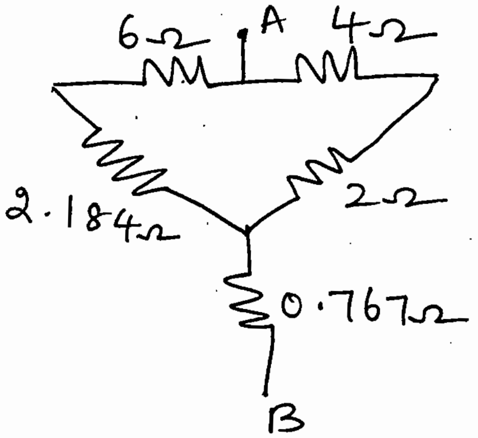
$26.33 \parallel^{el} 4 = 3.47\,\Omega$



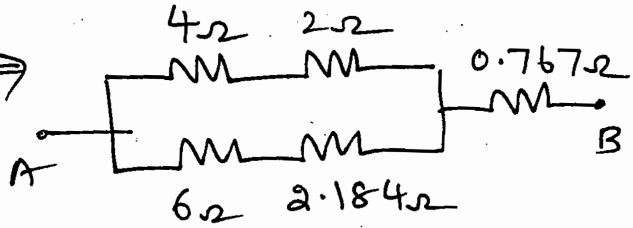
$$R_1 = \frac{9.875 \times 3.79}{9.875 + 3.79 + 3.47} = 2.184\Omega$$

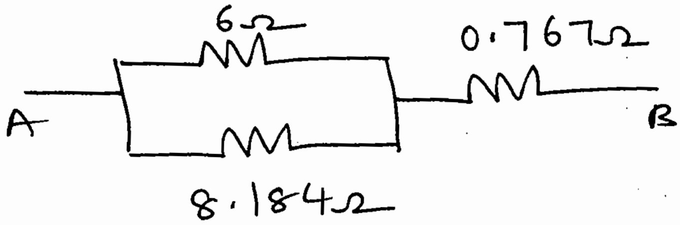
$$R_2 = \frac{9.875 \times 3.47}{17.135} = 1.999\Omega$$

$$R_3 = \frac{3.79 \times 3.47}{9.875 + 3.79 + 3.47} = 0.767\Omega$$

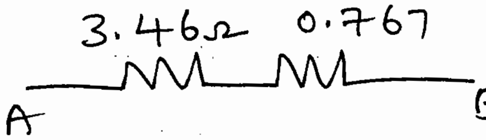


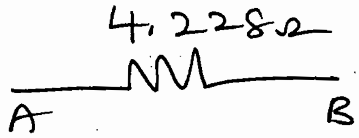
$\Rightarrow$



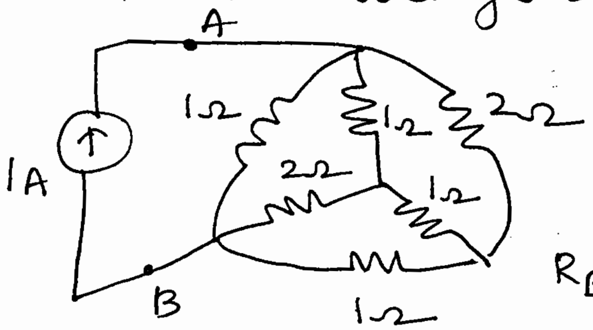


$6 \parallel^{lel} 8.184 = 3.46\Omega$





Find the voltage between terminals A & B

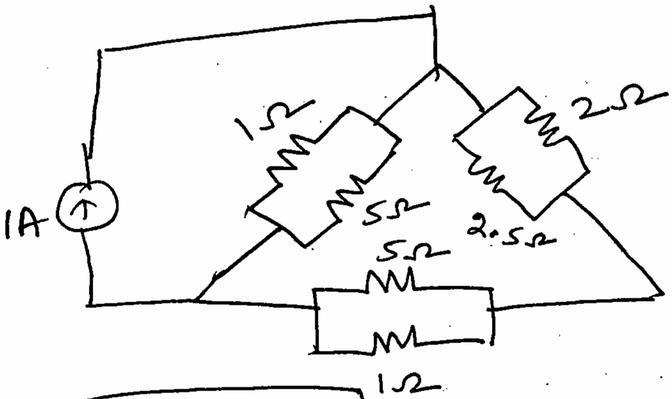


Inner star to Δ

$$R_A = 1+2 + \frac{2\times1}{1} = 5\Omega$$

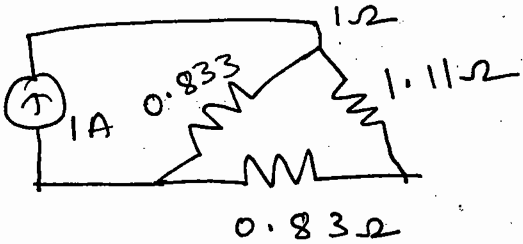
$$R_B = 1+1 + \frac{1\times1}{2} = 2.5\Omega$$

$$R_C = 2+1 + \frac{2\times1}{1} = 5\Omega$$



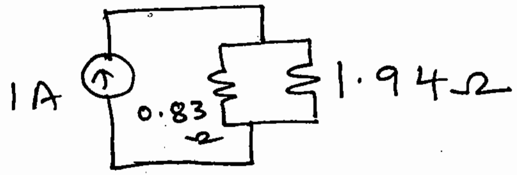
$1 \parallel 5 = \frac{5}{6} = 0.83\Omega$

$2 \parallel 2.5 = 1.11\Omega$

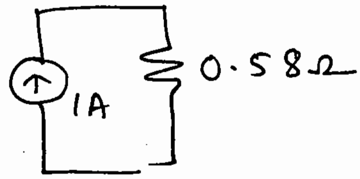


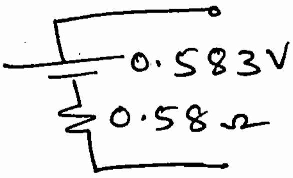
$0.833 \parallel 1.944$

$= 0.583$

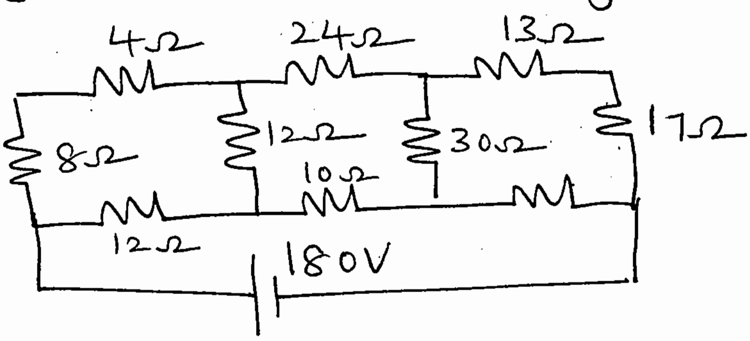


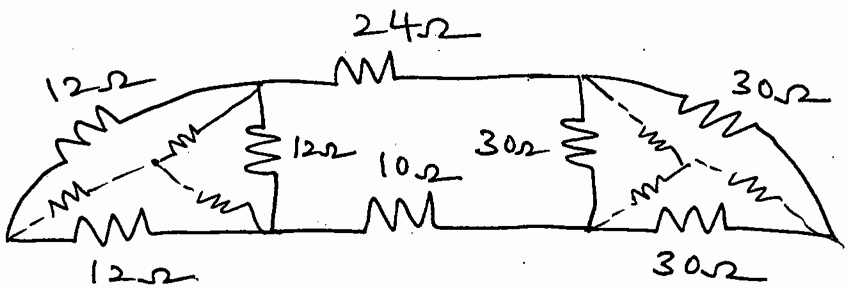
$\Rightarrow$





Determine the current through 10Ω resistor

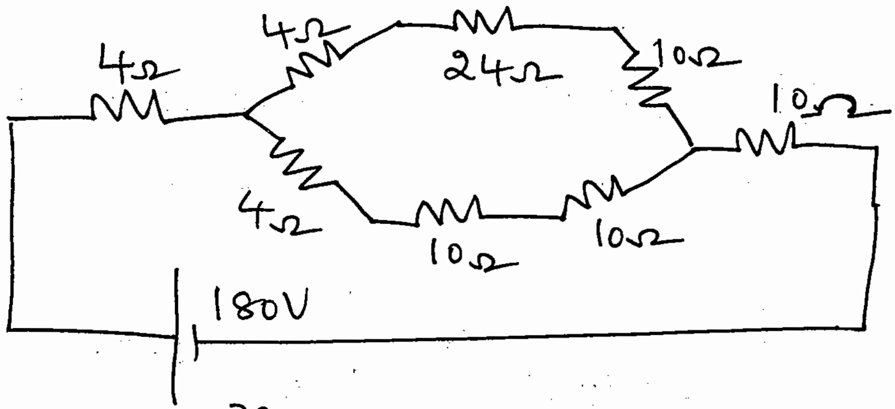


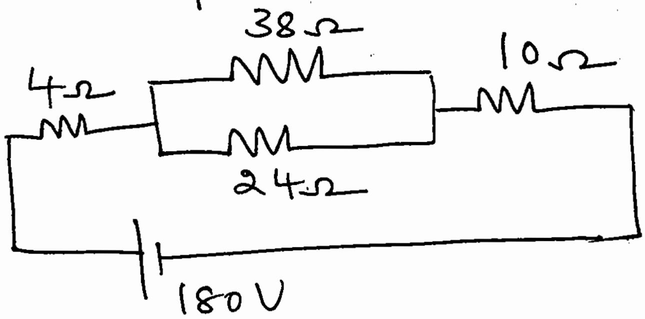


Converting Δ to star

$$R_1 = R_2 = R_3 = \frac{12 \times 12}{12+12+12} = 4\Omega$$

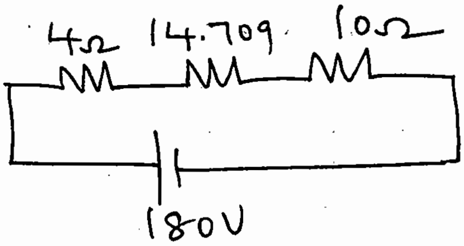
$$R_4 = R_5 = R_6 = \frac{30 \times 30}{30+30+30} = 10\Omega$$

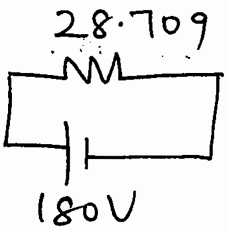




$38 \parallel^{ld} 24$

$= 14.709\Omega$





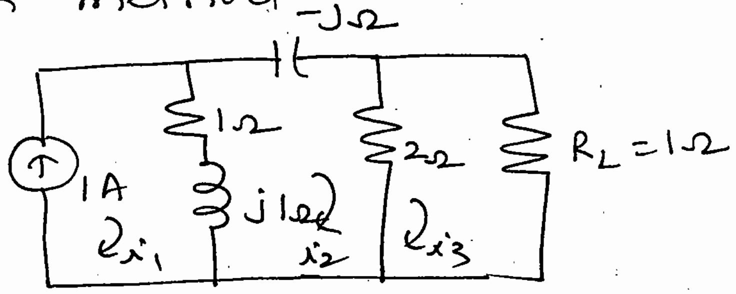
Total current $I = \frac{180}{28.709} = 6.269\,A$

using current divider rule

$$I_{10\Omega} = \frac{6.269 \times 38}{38+24} = 3.842A$$

problems on ac sources

Determine the current through load resistor R_L for the network using mesh method



$i_1 = 1$ A [current source lies @ the perimeter]

KVL to loop (2)

$$-i_1 R_{21} + i_2 R_{22} - i_3 R_{23} = E_2$$

$$-i_1(1+j1) + (3-j+j)i_2 - 2i_3 = 0$$

$$-3i_2 + 2i_3 = (-1+j) \Rightarrow (1)$$

KVL to loop (3)

$$-i_1 R_{31} \quad -i_2 R_{32} + i_3 R_{33} = E_3$$

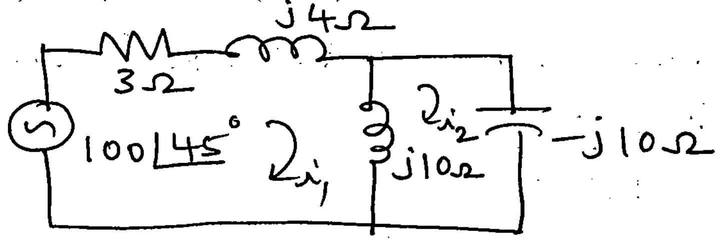
$$-2i_2 + 3i_3 = 0 \Rightarrow (2)$$

$$\Delta = \begin{bmatrix} -3 & 2 \\ -2 & 3 \end{bmatrix}$$

$$\Delta_3 = \begin{bmatrix} -3 & (-1+j) \\ 2 & 0 \end{bmatrix}$$

$$\therefore \quad i_3 = \frac{\Delta_3}{\Delta} = 0.5657 \angle 45^\circ \text{ A}$$

(2) Find the mesh currents for the circuit shown



KVL loop (1)

$$(3+j14)i_1 - 10j\,i_2 = 100\angle 45^\circ \rightarrow (1)$$

KVL to loop (2)

$$-j10\,i_1 + (j10 - j10)\,i_2 = 0$$

$$-j10\,i_1 = 0 \Rightarrow \boxed{i_1 = 0} \Rightarrow (2)$$

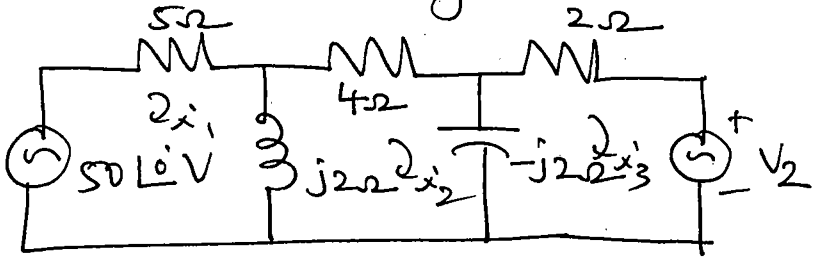
Put (2) in (1)

$$-j10\,i_2 = 100\angle 45^\circ$$

$$\therefore\ i_2 = \frac{100\angle 45}{-j10} = \frac{100\angle 45^\circ}{10\angle -90}$$

$$\boxed{i_2 = 10\angle 135^\circ\ A}$$

For the circuit shown find V_2 which results in zero current through 4Ω resistor using mesh analysis



given current through 4Ω is zero

$\therefore \quad i_2 = 0$

applying KVL to loop ①

$$(5+j2)\, i_1 - \cancel{j2\, i_2}^{\,0} = 50\angle 0^\circ$$

$$\therefore \quad (5+j2)\, i_1 = 50\angle 0^\circ$$

$$\therefore \quad i_1 = \frac{50\angle 0}{5+j2} = \frac{50\angle 0}{5.3851\angle 21.8}$$

$$\boxed{i_1 = 9.284\angle -21.8^\circ \text{ A}}$$

KVL to loop ③

$$\cancel{(-j2)\, i_2}^{\,0} + (2-j2)\, i_3 = -V_2$$

$$(2-j2)\, i_3 = -V_2$$

DIVAKARA BC

KVL to loop ②

$$-(j2)\, i_1 + \cancel{(4+j2)\, i_2}^{\,0} - (-j2)\, i_3 = 0$$

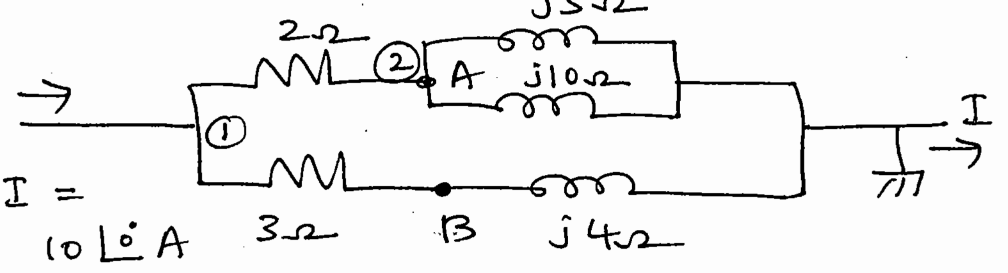
$$-j2\, i_1 = +j2\, i_3$$

$$\Rightarrow \boxed{i_1 = i_3 = 9.284\angle -21.8^\circ \text{ A}}$$

$$\therefore \quad = 2.8284\angle 135 \times 9.284\angle -21.8$$

$$\therefore \quad \boxed{V_2 = -26.261\angle 113.2^\circ \text{ V}}$$

Find the voltage V_{AB} for the network shown



KCL @ node ①

$$\frac{V_1 - V_2}{2} + \frac{V_1}{3+j4} = 10\angle 0^\circ$$

$$V_1\left(\frac{1}{2} + \frac{1}{3+j4}\right) + V_2\left(-\frac{1}{2}\right) = 10\angle 0^\circ$$

$$\boxed{(0.62 - j0.16)V_1 - 0.5V_2 = 10\angle 0^\circ \Rightarrow ①}$$

KCL @ node 2

$$\frac{V_2 - V_1}{2} + \frac{V_2}{j5} + \frac{V_2}{j10} = 0$$

$$V_1\left[-\frac{1}{2}\right] + V_2\left[\frac{1}{2} + \frac{1}{j5} + \frac{1}{j10}\right] = 0$$

$$\boxed{-0.5V_1 + V_2\left[0.5 - 0.3j\right] = 0 \rightarrow ②}$$

$$\Delta = \begin{bmatrix} (0.62 - 0.16j) & -0.5 \\ -0.5 & (0.5 - 0.3j) \end{bmatrix}$$

$$= \left[(0.64\angle -14.47)(0.58\angle -30.96) - 0.25\right]$$

$$= 0.3712\angle -45.43 - 0.25$$

$$= (0.2613 - 0.2651j) - 0.25$$

$$= 0.0113 - 0.2651j = 0.2653\angle -87.56$$

$$V_1 = \begin{bmatrix} 10\angle 0 & 0.5 \\ 0 & 0.5-0.3j \end{bmatrix}$$

$$= (10\angle 0)(0.58\angle -30.96)$$

$$= 5.8\angle -30.96 = 4.973 - 2.98j$$

$$\therefore \quad V_1 = \frac{5.8\angle -30.96}{0.265\angle -87.4} = \underline{21.7\angle 56.42^\circ\ V}$$

$$V_2 = \begin{bmatrix} (0.62-0.16j) & 10\angle 0 \\ -0.5 & 0 \end{bmatrix}$$

$$= (0.64\angle -14.47) \times 0 + 0.5\,(10\angle 0^\circ)$$

$$= 5\angle 0^\circ$$

$$V_2 = \frac{5\angle 0}{0.267\angle -87.42} = \underline{18.726\angle 87.42^\circ\ V}$$

$\boxed{V_2 = V_A}$

using voltage divider rule

$$V_B = \frac{V_1 \times (j4)}{3+j4}$$

$$V_B = \frac{21.7\angle 56.42 \times 4\angle 90}{5\angle 53.13}$$

$$= \frac{86.8\angle 146.42}{5\angle 53.13} = 17.36\angle 93.63^\circ\ V$$

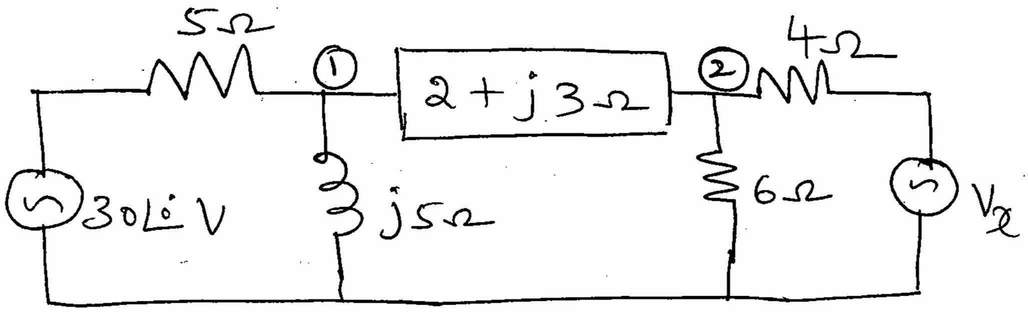
$$V_{AB} = V_A - V_B = [18.726\angle 87.42 - 17.36\angle 93.63]$$

$$= [0.843 + 18.7j - (-1.099 + 17.32j)]$$

$$= 1.942 + 1.38j$$

$$V_{AB} = 2.38\angle 35.39^\circ\ V$$

use nodal method to find the value of V_x for the circuit shown such that current through $2+j3\Omega$ is zero



Given current through $2+j3\Omega$ is zero

KCL @ node ①

$$\frac{V_1}{5} + \frac{V_1}{j5} + \cancel{\frac{V_1 - V_2}{2+j3}}^{\,0} = \frac{30\angle 0^\circ}{5}$$

$$V_1\left(\frac{1}{5} + \frac{1}{j5}\right) = 6\angle 0^\circ$$

$$V_1(0.2 - 0.2j) = 6\angle 0^\circ$$

$$V_1 = \frac{6\angle 0^\circ}{0.2 - j0.2} = \frac{6\angle 0^\circ}{0.2828\angle -45^\circ} = 21.21\angle 45^\circ \, V$$

KCL @ node 2

$$\cancel{\frac{V_2 - V_1}{2+j3}}^{\,0} + \frac{V_2}{4} + \frac{V_2}{6} = \frac{V_x}{4}$$

$$0.25\,V_2 + 0.166\,V_2 = 0.25\,V_x$$

$$0.416\,V_2 = 0.25\,V_x$$

$$V_x = \frac{0.416}{0.25} V_2$$

$\therefore$ $V_x = 1.664 V_2$

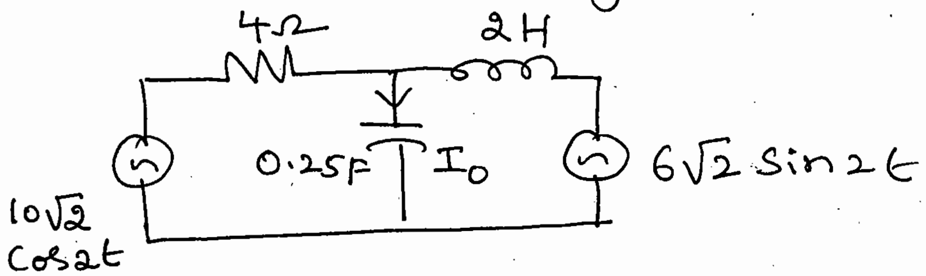
Since current flowing through $(2+j3)$ is zero, potential @ node 1 is equal to potential @ node 2

$\therefore$ $V_1 = V_2 = 21.21 \angle 45^\circ$ V

$\therefore$ $V_x = 1.664 \times 21.21 \angle 45^\circ$

$$\boxed{V_x = 35.35 \angle 45^\circ}$$

Find the value of I_0 for which the circuit shown using Loop analysis



$V = V_m \cos W_m t$

$V = 10\sqrt{2} \cos 2t \Rightarrow W = 2$

$X_L = WL = (2)(2) = j4\Omega$

Convert the given time domain to frequency domain

$$X_C = \frac{1}{WC} = \frac{1}{(2)(0.25)} \Rightarrow -j2\Omega$$

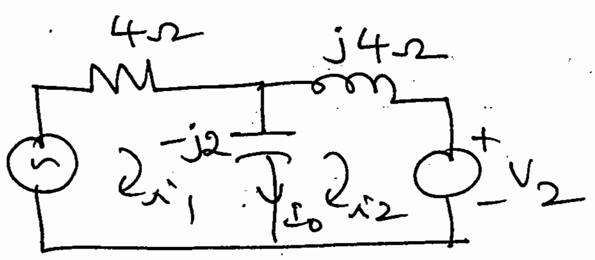
converting $10\sqrt{2}\cos 2t$ into sine

w.k.t $\quad \cos\omega t = \sin(\omega t + 90^\circ)$

$\sin\omega t = \cos(\omega t - 90^\circ)$

$V_{rms} = \dfrac{V_m}{\sqrt{2}}$ $\quad \therefore V_1 = \dfrac{10\sqrt{2}\sin(\omega t + 90^\circ)}{\sqrt{2}}$

$\boxed{V_1 = 10\angle 90^\circ V}$ $\quad V_2 = \dfrac{6\sqrt{2}\sin\omega t}{\sqrt{2}} = 6\angle 0^\circ$



$I_0 = i_1 - i_2$

KVL to loop 1

$(4-2j)i_1 + 2j\,i_2 = 10\angle 90^\circ \rightarrow 1$

$2j\,i_1 + 2j = -6\angle 0^\circ \rightarrow 2$

(−) $\quad$ (−) $\quad$ (+)

$(4-2j-2j)i_1 = 10\angle 90^\circ + 6\angle 0^\circ$

$(4-4j)i_1 = 10j + 6$

$\therefore i_1 = \dfrac{6+10j}{4-4j} = \dfrac{11.6619\angle 59.036}{5.656\angle -45^\circ}$

$\boxed{i_1 = 2.06\angle 104^\circ \text{ A}}$

$-(+2j\,i_1 + 2j\,i_2) = 6\angle 0^\circ$

$2\angle -90^\circ(2.06\angle 104) + (2\angle -90^\circ)i_2 = 6\angle 0^\circ$

$$2.06\angle 104 + i_2 = \frac{6\angle 0^\circ}{2\angle -90^\circ} = 3\angle 90^\circ$$

$$i_2 = 3\angle 90 - 2.06\angle 104$$

$$= 3j - (-0.5 + 2j)$$

$$= 0.5 + 1j$$

$$\boxed{i_2 = 1.118\angle 63.41^\circ \text{ A}}$$

$$\boxed{I_0 = i_1 - i_2}$$

$$= (2.068\angle 104^\circ) - (1.118\angle 63.41)$$

$$= (-0.5 + 2j) - (0.5 + 1j)$$

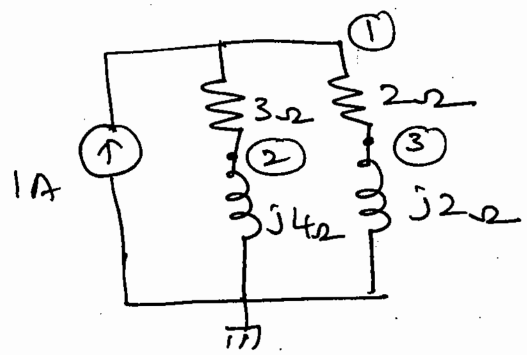
$$= -0.5 + 2j - 0.5 - 1j$$

$$I_0 = -1 + 1j$$

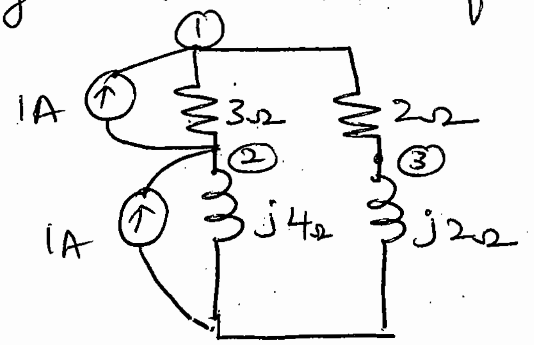
$$\boxed{I_0 = 1.414\angle 135^\circ \text{ A}}$$

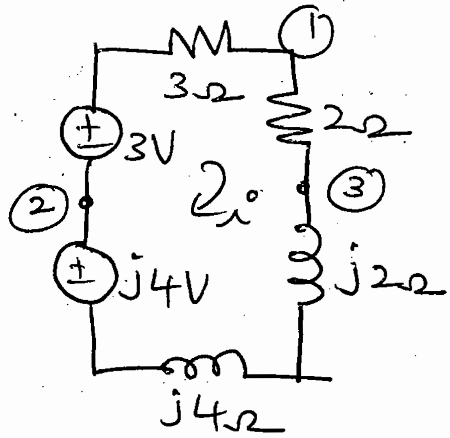
$$I_0 = 1.414\angle 135^\circ \text{ A}$$

Find V_{23} by mesh analysis



Since the current source is present by source transformation





apply KVL

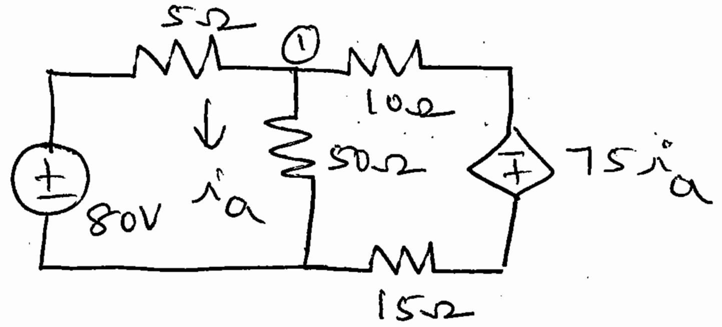
$$i(3+j4+2+j2) = 3+j4$$

$$i = \frac{3+j4}{5+j6} \Rightarrow$$

$$V_{23} = (3+2)\,i - 3$$

$$V_{23} = 0.256\angle 39.8^\circ \text{ V}$$

For the network shown find the power delivered by the dependent voltage source in the network (58)



KCL @ node 1

$$\frac{V_1 - 80}{5} + \frac{V_1}{50} + \frac{V_1 + 75 i_a}{25} = 0$$

$$i_a = \frac{V_1}{50}$$

$$\Rightarrow \frac{V_1 - 80}{5} + \frac{V_1}{50} + \frac{V_1 + 75\left(\frac{V_1}{50}\right)}{25} = 0$$

$$V_1 = 50V$$

$$i_a = \frac{V_1}{50} = \frac{50}{50} = 1\,A$$

$$i_1 = \frac{V_1 - (-75 i_a)}{25} = \frac{V_1 + 75 i_a}{25}$$

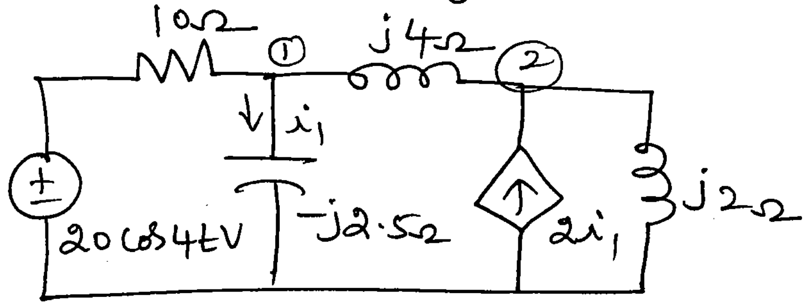
$$= \frac{50 + 75 \times 1}{25} = 5A$$

$$P_{75 i_a} = (75 i_a)\, i_1$$

$$= 75 \times 1 \times 5$$

$$= 375 \text{ watts}$$

Find i_1 using nodal analysis



KCL @ node ①

$$\frac{V_1 - 20\angle 0^\circ}{10} + \frac{V_1}{-j2.5} + \frac{V_1 - V_2}{j4} = 0$$

$$(1 + j1.5)V_1 + j2.5V_2 = 20$$

KCL @ node 2

$$\frac{V_2 - V_1}{j4} + \frac{V_2}{j2} = 2i_1$$

from the big $i_1 = \dfrac{V_1}{-j2.5}$

$$\frac{V_2 - V_1}{j4} + \frac{V_2}{j2} = \frac{2V_1}{-j2.5}$$

$$\Rightarrow -j0.55V_1 - 0.75jV_2 = 0$$

X^{ly} throughout by $j20$

$$11V_1 + 15V_2 = 0$$

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$V_1 = 18.97\angle 18.43^\circ \text{ V}$$

$$V_2 = 13.91\angle -161.56^\circ \text{ V}$$

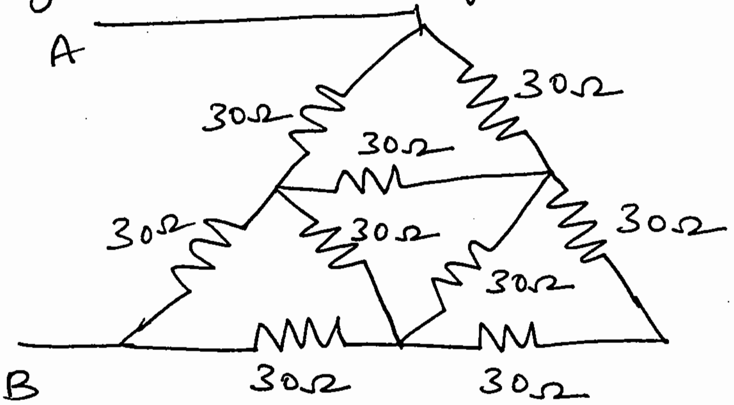
$$\therefore \quad \dot{I}_1 = \frac{V_1}{-j2.5} = 7.59\angle 108.4^\circ \text{ A}$$

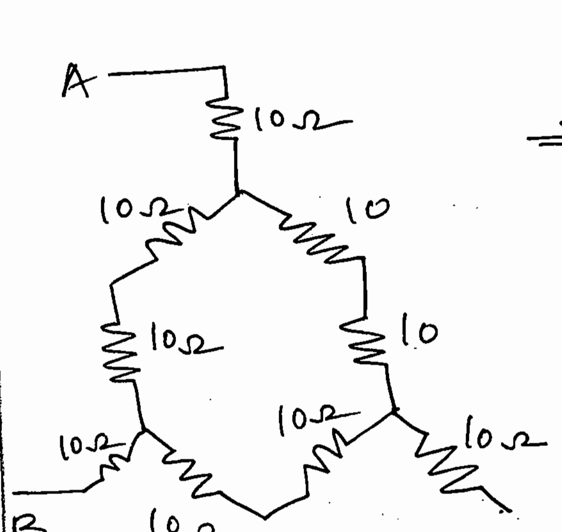
transforming this to time domain

$$i_1 = 7.59\cos(4t + 108.4^\circ) \text{ A}$$

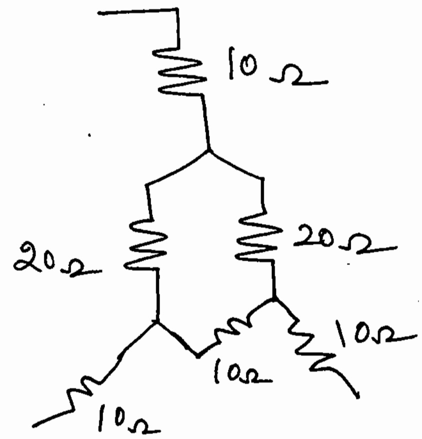
Dec 2014

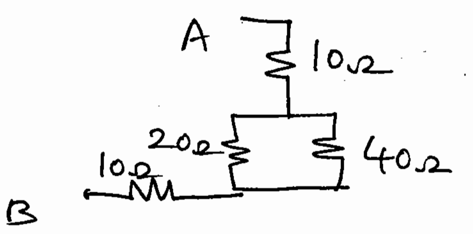
Find the equivalent resistance at AB using Y–Δ transformation method

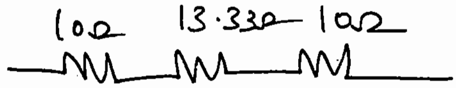




⇒

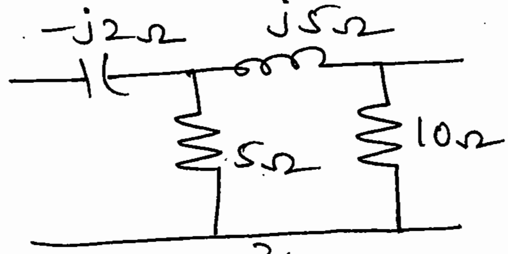


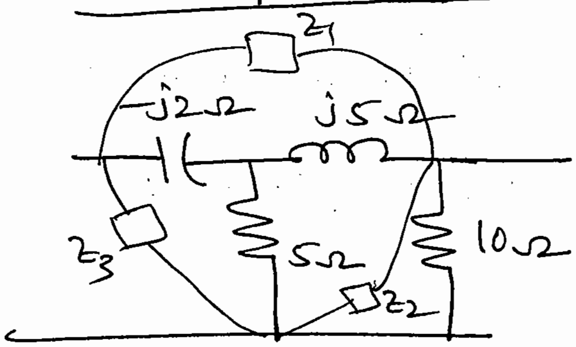




$$R = 33.33\Omega$$

obtain delta connected equivalent of the network shown





$Y-\Delta$

$$Z_1 = \frac{(-j2 \times j5) + (j5 \times 5) - (j2 \times 5)}{5}$$

$$Z_1 = 2 + j3\,\Omega$$

$$Z_2 = \frac{(-j2 \times j5) + (j5 \times 5) - (j2 \times 5)}{-j2}$$

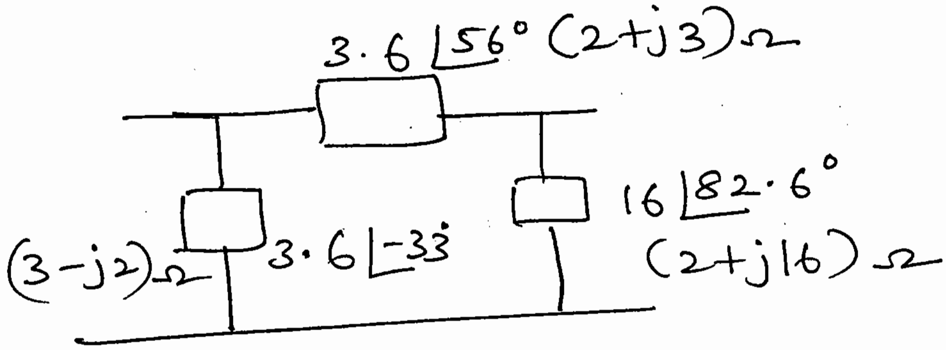
$$= -7.5 + j5$$

$$Z_3 = \frac{(-j2 \times 5j) + (j5 \times 5) - (j2 \times 5)}{j5}$$

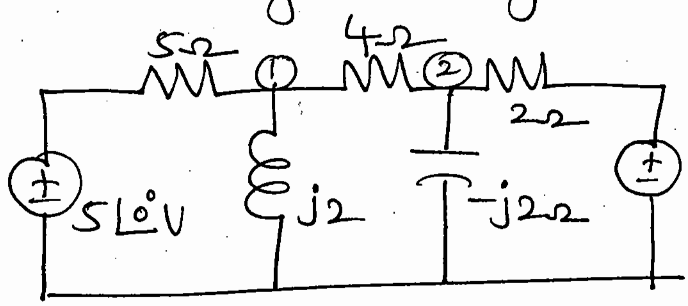
$$= 3 - 2j$$

$$\frac{Z_2 \times 10}{Z_2 + 10} = \frac{9\angle 146.2 \times 10\angle 0}{(-7.5 + j5 + 10)}$$

$$= 16\angle 82.6$$



For the network shown determine the node voltage using nodal method



KCL @ node 1

$$\frac{V_1 - V_2}{4} + \frac{V_1}{j2} + \frac{V_1}{5} = \frac{5\angle 0°}{5}$$

$$V_1\left[\frac{1}{4} + \frac{1}{5} + \frac{1}{2j}\right] - V_2\left[\frac{1}{4}\right] = 1$$

$$[0.45 - 0.5j]V_1 - 0.25V_2 = 1 \Rightarrow 1$$

KCL @ node 2

$$\frac{V_2 - V_1}{4} + \frac{V_2}{-j2} + \frac{V_2}{2} = \frac{5\angle 90}{2}$$

$$-V_1\left[\frac{1}{4}\right] + V_2\left[\frac{1}{4} + \frac{1}{-2j} + \frac{1}{2}\right] = 2.5\angle 90$$

$$-0.25V_1 + (0.75 + 0.5j)V_2 = 2.5j \Rightarrow 2$$

$$\Delta = \begin{bmatrix} (0.45-0.5j) & -0.25 \\ -0.25 & (0.75+0.5j) \end{bmatrix}$$

$$\Delta = 0.525 - j\,0.15$$

$$\Delta_1 = \begin{bmatrix} 1 & -0.25 \\ 2.5j & (0.75+0.5j) \end{bmatrix} = 0.75 + 1.125j$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{0.75 + 1.125j}{0.525 - j0.15} = 2.47 \angle 72.25^\circ \text{ V}$$

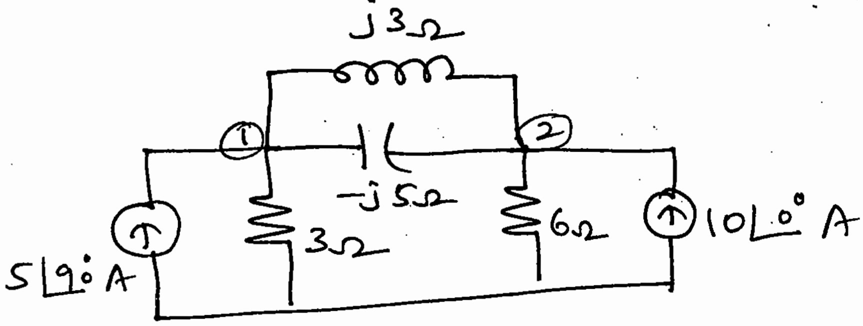
$$\Delta_2 = \begin{bmatrix} (0.45-0.5j) & 1 \\ -0.25 & 2.5j \end{bmatrix}$$

$$= 1.5 + 1.125j \qquad \therefore V_2 = \frac{\Delta_2}{\Delta}$$

$$V_2 = 2.07 + j\,2.7 \text{ V}$$

$$\boxed{V_2 = 3.43 \angle 52.8^\circ \text{ V}}$$

use nodal analysis to find V_2



KCL @ node 1

$$\frac{V_1}{3} + \frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j3} = 5\angle 90^\circ$$

$$V_1\left(\frac{1}{3} + \frac{1}{-j5} + \frac{1}{3j}\right) - V_2\left(\frac{1}{j3} + \frac{1}{-j5}\right) = 5\angle 90^\circ$$

$$V_1(0.333 - 0.133j) - V_2(-0.133j) = 5\angle 90^\circ$$

KCL @ node 2

$$\frac{V_2}{6} + \frac{V_2 - V_1}{-j5} + \frac{V_2 - V_1}{j3} = 10\angle 0^\circ$$

$$-V_1\left(\frac{1}{j3} + \frac{1}{-j5}\right) + V_2\left(\frac{1}{6} + \frac{1}{-j5} + \frac{1}{j3}\right) = 10\angle 0^\circ$$

$$0.1333j\, V_1 + V_2(0.1666 - 0.1333j) = 10\angle 0$$

$$\Delta = \begin{bmatrix} 0.333 - 0.133j & +0.133j \\ 0.1334j & (0.1666 - 0.1333j \end{bmatrix}$$

$$= 0.08959\angle -48$$

$$\Delta_1 = \begin{bmatrix} 5\angle 90^\circ & 0.1334j \\ 10\angle 0^\circ & 0.1666 - 0.133j \end{bmatrix}$$

$$= 0.8334\angle -36.89$$

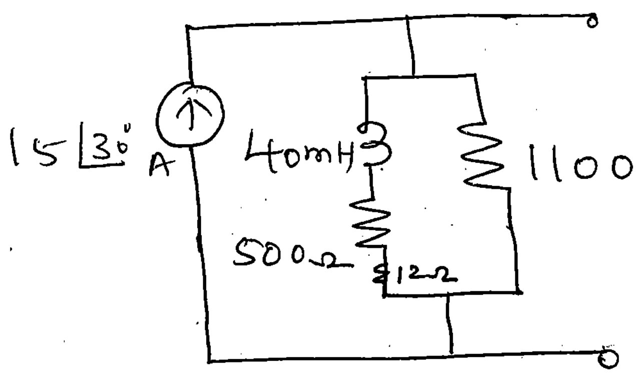
$$\Delta_2 = 4.2683 \angle -38.66$$

$$\Delta_2 = \begin{bmatrix} 0.333 - 0.133j & 5\angle 9^\circ \\ (0.1334j) & 10\angle 0^\circ \end{bmatrix}$$

$$V_1 = \frac{\Delta_1}{\Delta} = 9.302 \angle 11.11 \text{ Volts}$$

$$V_2 = \frac{\Delta_2}{\Delta} = 47.64 \angle 9.34 \text{ Volts}$$

Reduce the network shown into a single source network $W = 300$ rad/sec



$$X_L = WL = (40 \times 10^{-3})(300)$$

$$jX_L = j12\,\Omega$$

since $12\,\Omega$ is in series with $500\,\Omega$

$$R_S = Z_1 \parallel Z_2$$

$$Z_1 = 500 + 12j = 500 \angle 1.37^\circ\,\Omega$$

$$Z_2 = 1100 + 0j = 1100 \angle 0^\circ\,\Omega$$

$$R_s = Z_{eq} = \frac{500 \angle 1.37 \times 1100}{500 + 12j + 1100}$$

$$= \frac{55015 4 \angle 1.37}{1600 + 12j} = \frac{550154 \angle 1.37}{1600 \angle 0.429}$$

$$Z_{eq} = 343.83 \angle 0.941$$

$$= 343.78 + 5.646j$$

$$V = I \cdot Z_{eq}$$

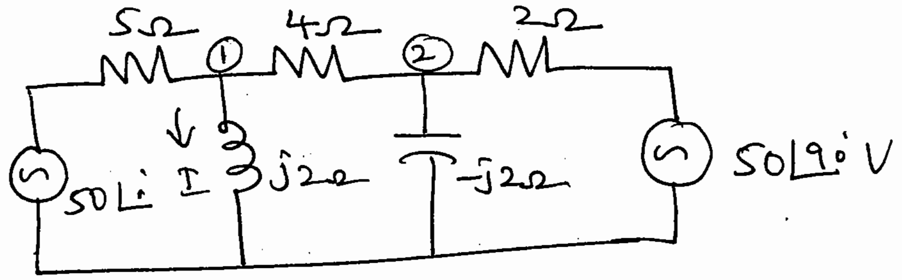
$$= 15 \angle 30^\circ \times 343.83 \angle 0.944$$

$$= 5157.45 \angle 30.94$$

$$\boxed{V = 5157.45 \angle 30.94}$$

$$\boxed{V = 4423.39 + 2651.9j}$$

using Nodal analysis find the current I for the network shown



KCL @ node 1

$$V_1\left[\frac{1}{5} + \frac{1}{4} - j\frac{1}{2}\right] - V_2\left[\frac{1}{4}\right] = 10$$

$$V_1\left[0.45 - j0.5\right] - V_2\left[\frac{1}{4}\right] = 10$$

$$\boxed{V_1\left[0.45 - j0.5\right] - 0.25 V_2 = 10} \rightarrow (1)$$

KCL @ node 2

$$\frac{V_1 - V_2}{4} - \frac{V_2}{-j2} - \frac{V_2 - 50\angle 90}{2} = 0$$

$$0.25 V_1 + V_2[-0.75 - j0.5] = -j25$$

$$\Delta = \begin{bmatrix} 0.45 - j0.5 & -0.25 \\ 0.25 & -0.75 - j0.5 \end{bmatrix}$$

$$= -0.3375 + j0.375 - j0.225 - 0.25 + 0.0625$$

$$= -0.525 + j0.15$$

$$\boxed{\Delta = 0.546 \angle 164.05}$$

$$\Delta_1 = \begin{bmatrix} 10 & -0.25 \\ -j25 & -0.75 - j0.5 \end{bmatrix}$$

$$= -7.5 - j5 - j6.25$$

$$= -7.5 - j11.25$$

$$= 13.52 \angle -123.69^\circ$$

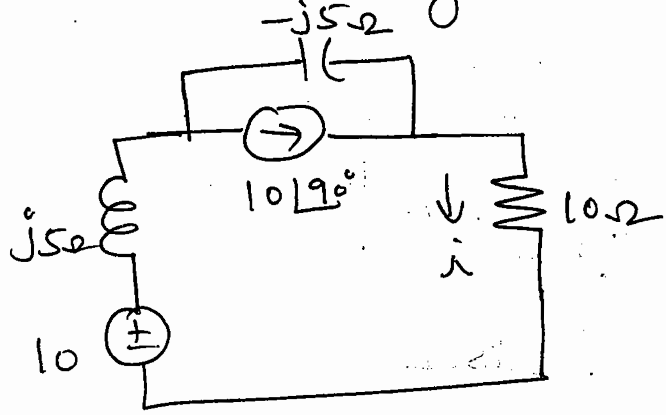
$$V_1 = \frac{\Delta_1}{\Delta} = \frac{13.52 \angle -123.69}{0.546 \angle 164.05}$$

$$= 24.762 \angle 40.36^\circ \text{ V}$$

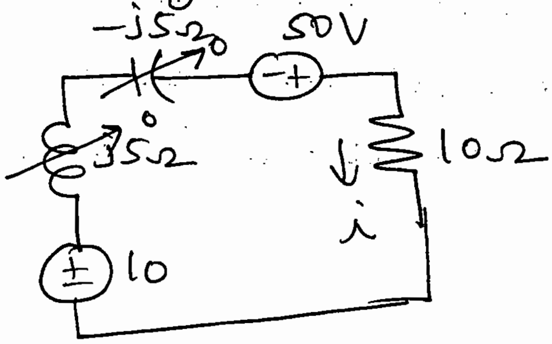
$$I = \frac{V_1}{j2} = \frac{24.762 \angle 40.36}{2 \angle 90}$$

$$\boxed{I = 12.381 \angle -49.64^\circ \text{ A}}$$

Reduce the network & find the current through 10Ω (63)



converting current source to voltagesource



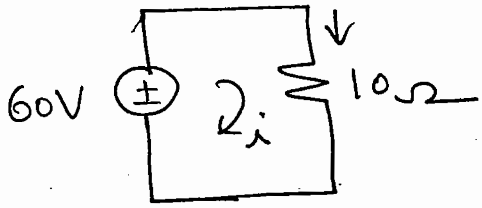
$V = I Z$

$= j\cdot 10 \times -j5$

$= -j^2 \cdot 50$

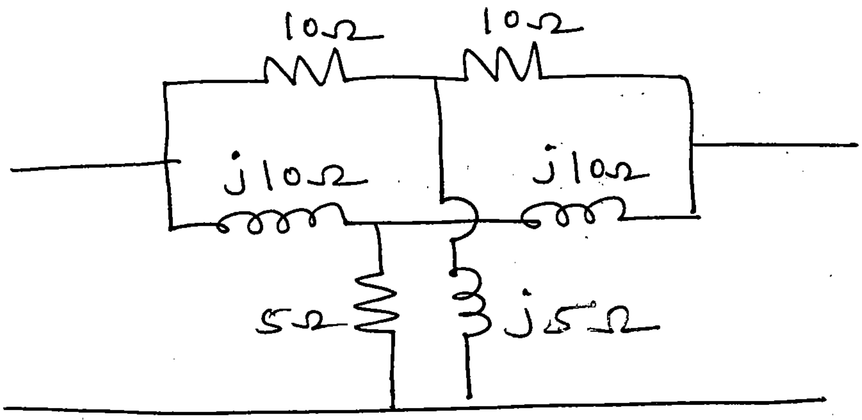
$V = (-1) \times -(50)$

$V = 50V$

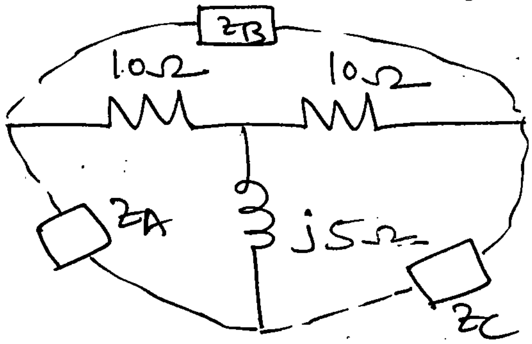


$i = \frac{60}{10} = 6A$

Find a single delta connected equivalent for the network shown



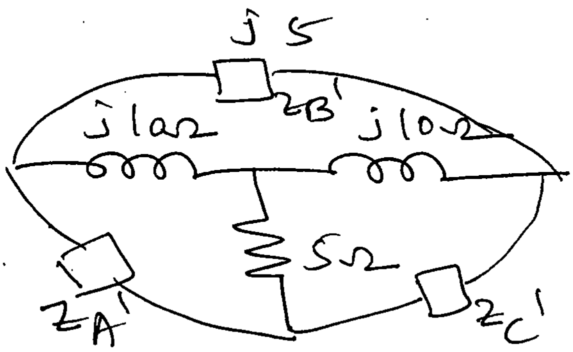
There are 2 star connected impedances convert the star impedances to equivalent delta



$$Z_A = Z_C = \frac{(10)(j5) + (j5)(10) + (10)(10)}{10}$$

$$= \frac{100 + j100}{10} = 10 + j10\,\Omega$$

$$Z_B = \frac{100 + j100}{j5} = 20 - j20\,\Omega$$

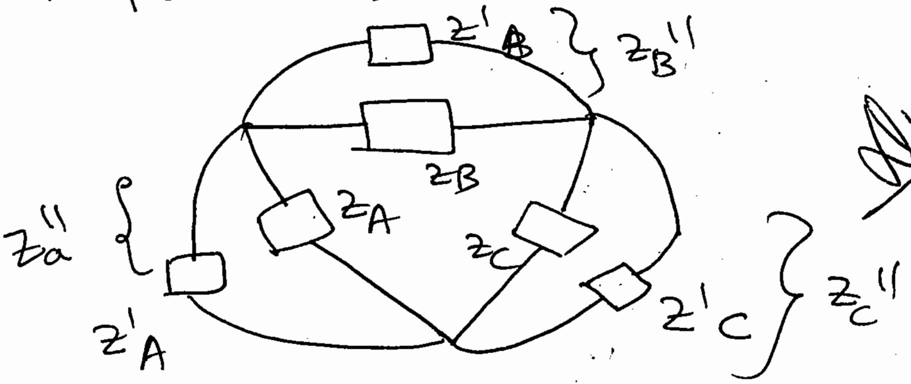


$$Z_A' = Z_C' = \frac{(5)(j10) + 5(j10) + j(10)(j10)}{j10}$$

$$= \frac{-100 + j100}{j10} = 10 + j10\,\Omega$$

$$Z_B' = \frac{-100 + j100}{5} = -20 + j20\,\Omega$$

2 delta connected equivalent impedances are in parallel



DIVAKARA B C.

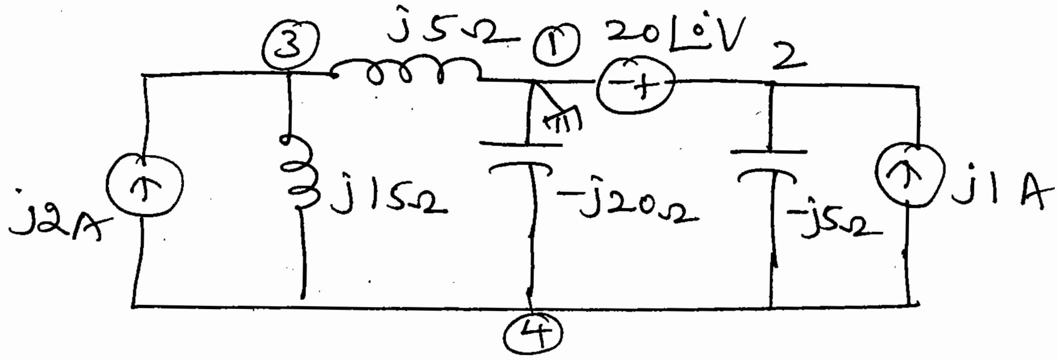
$$Z_a'' = Z_A \parallel Z_A' = \frac{(10+j10)(10+j10)}{20+j20}$$

$$= 5 + j5 = Z_c''$$

$$Z_B'' = \frac{(20-j20)(-20+j20)}{0}$$

$$= \infty \text{ (open circuit)}$$

Find the voltage across the capacitor of 20Ω using nodal analysis



Choosing Node 1 as reference node

$V_2 = 20$ volts

apply KCL @ node 3

$$\frac{V_3}{j15} + \frac{V_3 - V_4}{j15} = j2$$

$$\frac{3V_3 + V_3 - V_4}{j15} = j2$$

$$\boxed{4V_3 - V_4 = -30} \Rightarrow 1$$

KCL @ node 4

$$\frac{V_4}{-j20} + \frac{V_4 - V_3}{j15} + \frac{V_4 - 20}{-j5} = -j2 - j1$$

$$\frac{-3V_4 + 4V_4 - 4V_3 - 12V_4 + 240}{j60} = -j3$$

$$-4V_3 - 11V_4 = -60 \Rightarrow (2)$$

In matrix form

$$\begin{bmatrix} 4 & -1 \\ -4 & 11 \end{bmatrix} \begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} -30 \\ -60 \end{bmatrix}$$

to find the voltage across the capacitor of 20Ω find V_4

$$V_4 = \frac{\begin{bmatrix} 4 & -30 \\ -4 & -60 \end{bmatrix}}{\Delta}$$

$$\Delta = \begin{bmatrix} 4 & -1 \\ -4 & -11 \end{bmatrix} = -44 - 4$$
$$= -48$$

$$V_4 = \frac{-240 - 120}{-48} = 7.5 \text{ volts}$$

$\therefore$ Voltage across capacitor of 20Ω is

$$V_4 = 7.5 \text{ volts}$$

module : 2

Network Theorems

DIVAKARA S C 1

Loop analysis and Nodal analysis are applicable to simple electrical networks. For complex & large networks these methods of analysis become tedious, hence analysis of such networks is done using network theorems.

Network theorems provide simpler and more easier methods for analysis of complex electrical network

Superposition theorem is applicable for a circuit consisting of RLC but not for the circuit consisting of active devices such as diodes and transistors because they are neither a Linear nor a Bilateral network elements

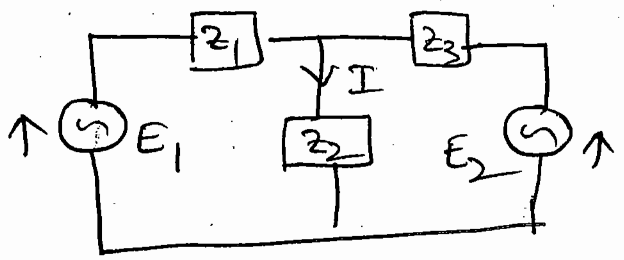
Statement ⇒ The response in any element of a Linear bilateral active network containing more than one source is the sum of the responses obtained by each source acting independently with all other sources set equal to zero

If a current source is set equal to zero it is replaced by open circuit when a voltage source is to be set equal to zero it is replaced by an internal impedance or a short circuit

superposition principle is applied to find the currents and Node voltages of a linear, bilateral active network

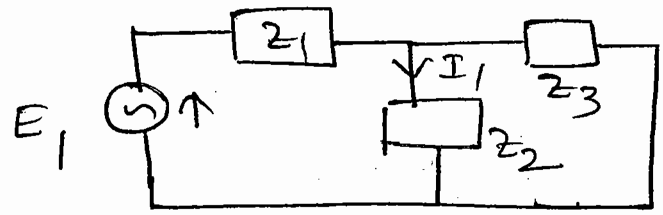
Explanation

consider a network as shown in figure



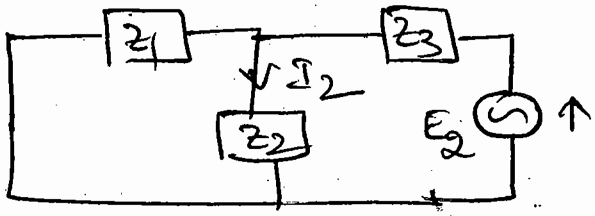
Let I be the response in the impedance Z_2 when both sources E_1 & E_2 are present

case I ⇒ with only E_1 source alone



E_2 is short circuited

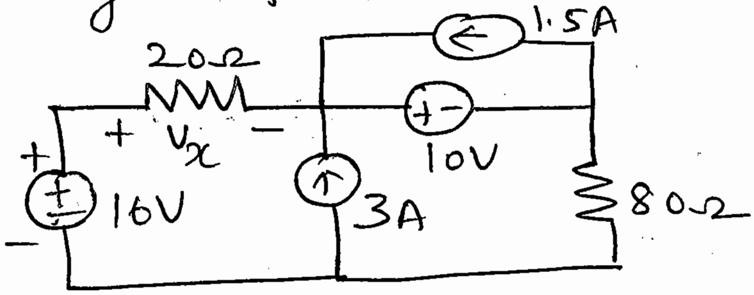
case II ⇒ with only E_2 acting alone



by superposition theorem

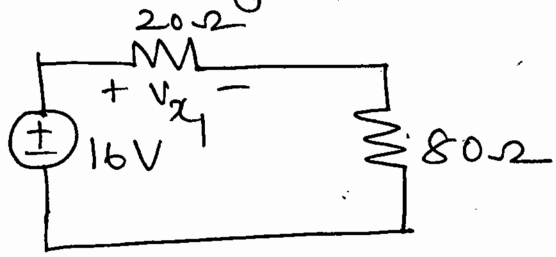
Total current $I = I_1 + I_2$

For the circuit shown find V_x using superposition theorem



case (a)

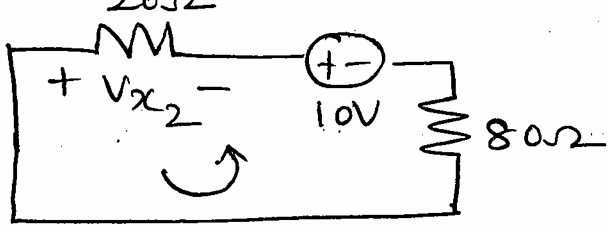
with only 16V source acting alone



$$I = \frac{16}{100} = 0.16\ A$$

$$V_{x_1} = I(20) = 0.16 \times 20$$
$$= 3.2 \text{ Volts}$$

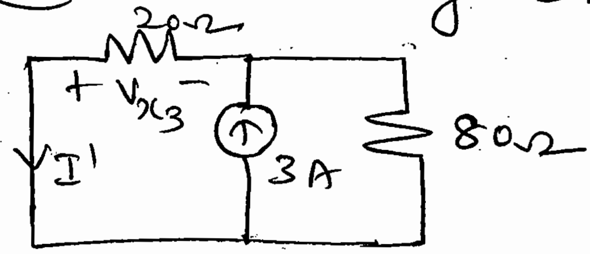
case (b) with only 10V source alone



$$I = \frac{10}{100} = 0.1 \text{ amp}$$

$$V_{x_2} = -I_x \cdot 20$$
$$= -0.1 \times 20 = -2 \text{ Volts}$$

case © with only 3 A current source



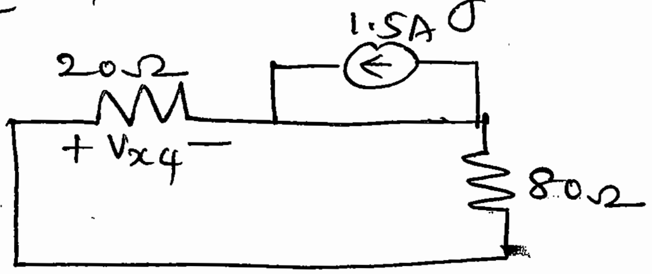
using current divider rule

$$I' = \frac{3 \times 80}{100} = 2.4 \text{ amp}$$

$$V_{x3} = -I_y(20) = -2.4 \times 20$$

$$= -48 \text{ volts}$$

cased ⇒ with only 1.5 A current source



Since 1.5 A current source is short circuited the current through 20Ω resistor is zero ∴ $V_{x4} = 0$ volts

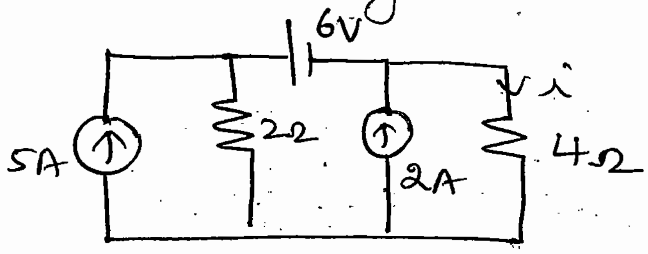
By superposition theorem.

$$V_x = V_{x_1} + V_{x_2} + V_{x_3} + V_{x_4}$$

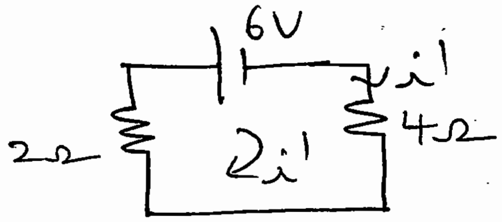
$$= 3.2 - 2 - 48 + 0$$

$$\boxed{V_x = -46.8 \text{ volts}}$$

Find the current through 4Ω resistor using superposition principle 3

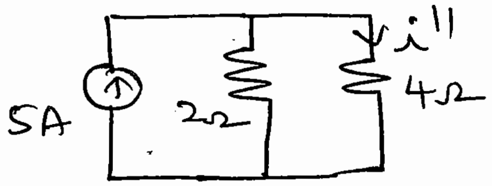


case ① ⇒ consider 6V source only



$$i' = \frac{-6}{6} = -1 \text{ Amp}$$

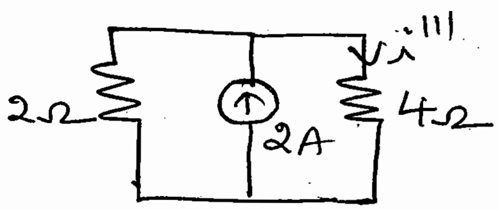
case ② ⇒ consider 5A current source



using current divider rule

$$i'' = \frac{5 \times 2}{6} \Rightarrow 1.666 \text{ amp}$$

case ③ ⇒ consider 2A source alone



using current divider rule

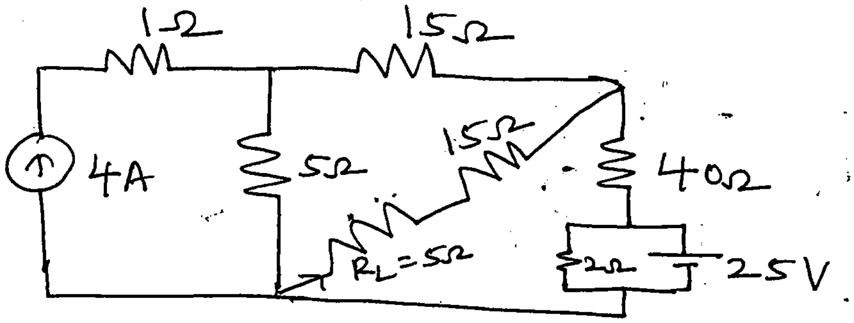
$$i''' = \frac{2 \times 2}{6} = 0.666\text{A}$$

$$\therefore \quad I_{4\Omega} = i' + i'' + i'''$$

$$= -1 + 1.666 + 0.666\text{A}$$

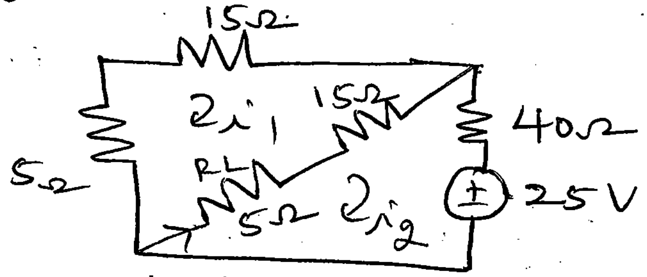
$$\boxed{I_{4\Omega} \Rightarrow 1.332 \text{ amperes}}$$

Find the current through R_L



Consider 25V alone

2Ω is a redundant



KUL to loop (1)

$40 i_1 - 20 i_2 = 0 \rightarrow (1)$

KUL to loop (2)

$-20 i_1 + 60 i_2 = -25 \rightarrow (2)$

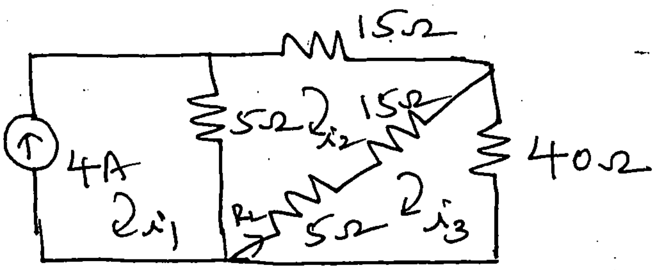
$i_1 = -0.25A \quad i_2 = -0.5A$

$i'_{R_L} = i_2 - i_1$

$= -0.25A$

Consider 4A current source

1Ω is a redundant element



$i_1 = 4A$

KVL to loop (2)

$$-5i_1 + 40i_2 - 20i_3 = 0$$

KVL to loop (3)

$$-20i_2 + 60i_3 = 0 \Rightarrow (1)$$

$$40i_2 - 20i_3 = 20 \Rightarrow (2)$$

$\boxed{i_2 = 0.6A}$ $\boxed{i_3 = 0.2A}$

$$\therefore \quad I''_{R_L} = i_3 - i_2$$

$$= -0.4A$$

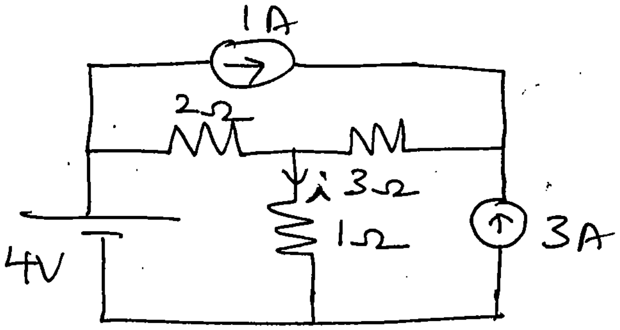
$$\therefore \quad I_{R_L} = i'_{R_L} + I''_{R_L}$$

$$= -0.25 - 0.4$$

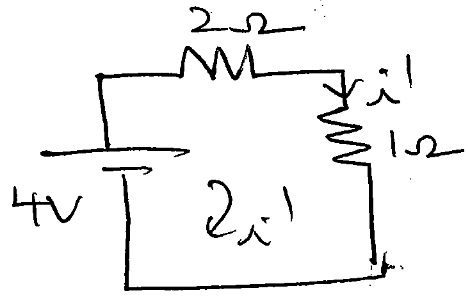
$\boxed{I_{R_L} = 0.65 \text{ amperes}}$

A

Find the current through 1Ω resistor using superposition theorem



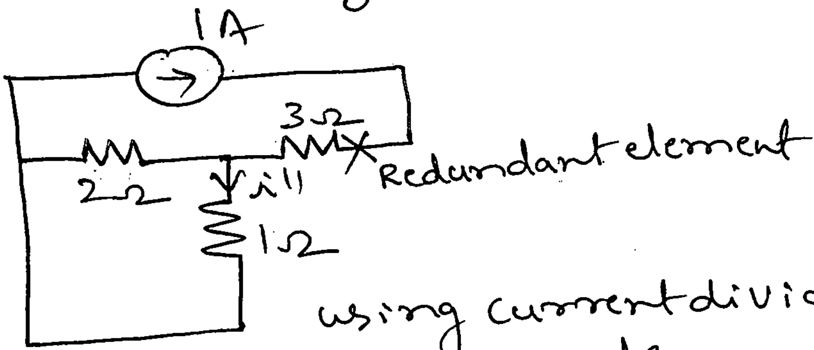
case 1) considering 4V alone



$i' = \frac{4}{3}$ Amp

$= 1.33$ amperes

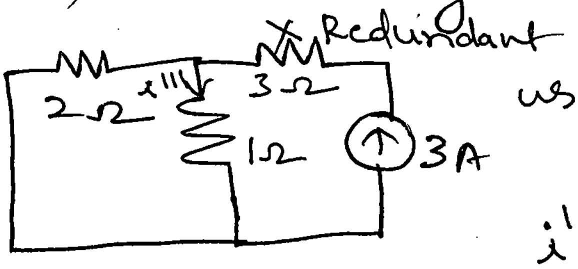
case 2) considering 1A current source



using current divider rule

$i'' = \frac{1 \times 2}{3} = \frac{2}{3} = 0.666$ Amperes

case 3) considering 3A source



using current divider rule

$i''' = \frac{3 \times 2}{2+1} = 2A$

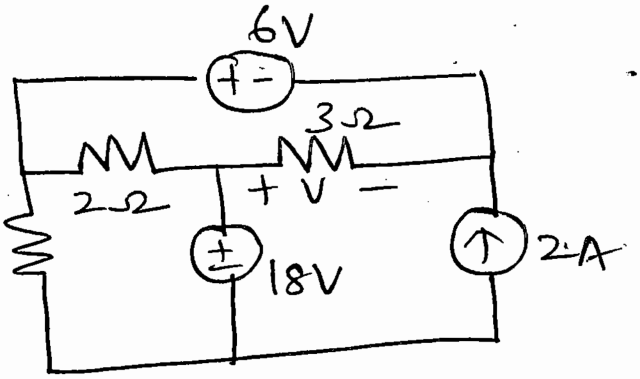
$\therefore \quad i = i' + i'' + i'''$

$= 1.33 + 0.666 + 2A$

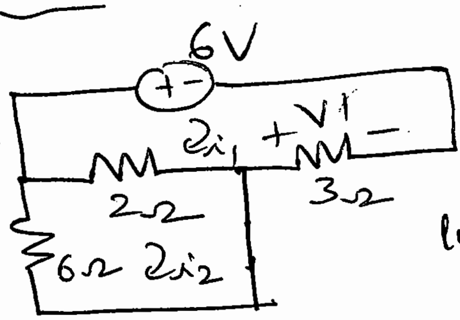
$\boxed{i = 4 \text{ amperes}}$

Jan 2014

Find the voltage 'V' using superposition theorem across 3Ω resistor



Case ① consider 6V source alone



KVL to loop ①

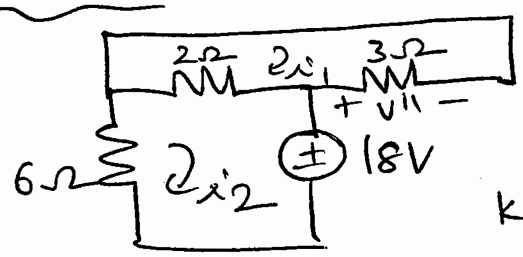
$5i_1 - 2i_2 = -6 \rightarrow ①$

loop ② $\quad -2i_1 + 8i_2 = 0 \rightarrow ②$

$i_1 = -1.33 A, \quad i_2 = -0.33A$

$\boxed{V'_{3\Omega} = 3(-i_1) = 3[-(-1.33)] = 4V}$

Case 2 consider 18V source



KVL to loop ①

$5i_1 - 2i_2 = 0 \rightarrow ①$

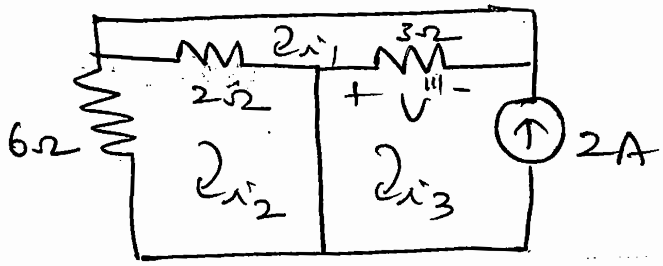
KVL to loop ②

$-2i_1 + 8i_2 = -18 \rightarrow ②$

$i_1 = -1A$; $i_2 = -2.5A$

$$\boxed{V''_{3\Omega} = 3(-i_1) = 3-(-1) = 3V}$$

considering 2A current source only



$i_3 = -2A$

KVL to loop ①

$$5i_1 - 2i_2 - 3i_3 = 0$$

$$\boxed{5i_1 - 2i_2 = -6} \rightarrow ①$$

KVL to loop ②

$$-2i_1 + 8i_2 = 0 \rightarrow ②$$

$i_1 = -1.33A$, $i_2 = -0.33A$

$$V'''_{3\Omega} = 3(i_3 - i_1)$$

$$\boxed{V'''_{3\Omega} = -2V}$$

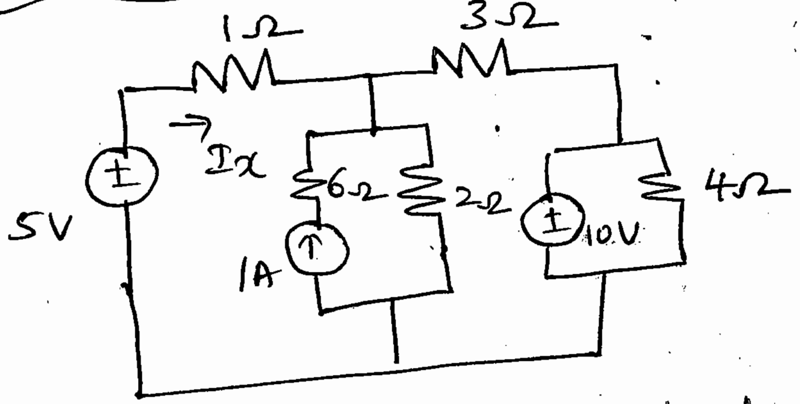
$$\therefore V_{3\Omega} = V' + V'' + V'''$$

$$= 4 + 3 - 2$$

$$\boxed{V = 5V}$$

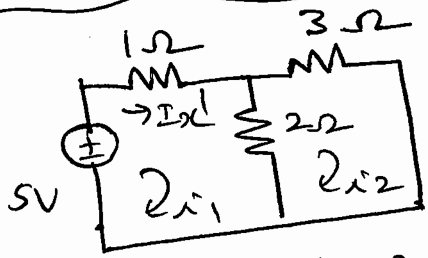
Jan 2015

Use superposition theorem to find I_x of the network shown



6Ω & 4Ω are redundant elements

Consider 5V source alone



KVL to loop ①

$$3i_1 - 2i_2 = 5 \rightarrow ①$$

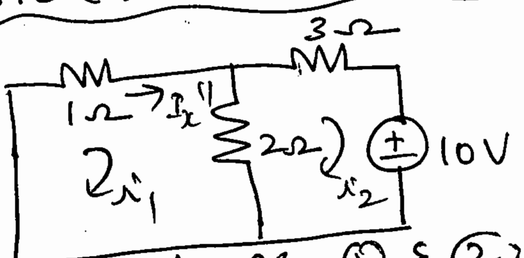
KVL to loop ②

$$-2i_1 + 5i_2 = 0$$

$$\boxed{I_x' = i_1 = 2.272\,A}$$

$$i_2 = 0.909\,A$$

Consider 10V source alone



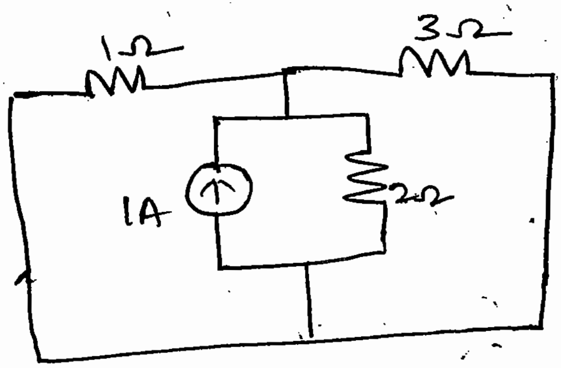
KVL to loops ① & ②

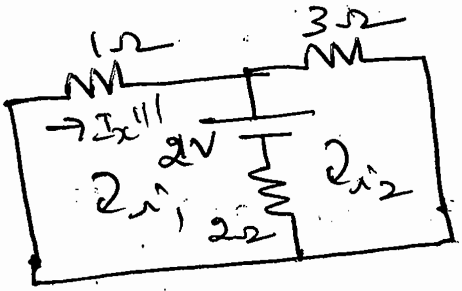
$$3i_1 - 2i_2 = 0 \rightarrow ①$$

$$-2i_1 + 5i_2 = -10 \rightarrow ②$$

$$\boxed{I_x'' = i_1 = -1.818\,A} \qquad i_2 = -2.727\,A$$

Case (3) :- consider 1 A source alone





KVL to loop (1)

$$3i_1 - 2i_2 = -2 \rightarrow (1)$$

KVL to loop 2

$$-2i_1 + 5i_2 = 2 \rightarrow (2)$$

$$\boxed{I_x''' = i_1 = -0.545\,A}$$

$$i_2 = 0.181\,A$$

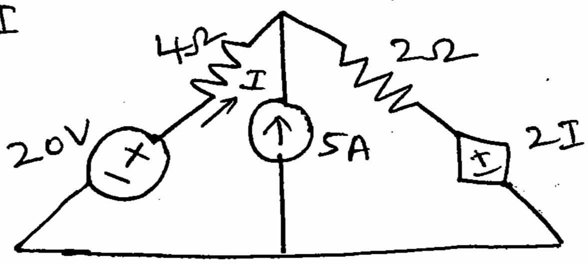
$$\therefore \quad I_x = I_x' + I_x'' + I_x'''$$

$$= 2.272 - 1.818 - 0.545$$

$$\boxed{I_x = -0.091\,A}$$

While solving the numericals using superposition theorem, if there is a dependent source in the given circuit retain the dependent source as it is

Using superposition theorem find the current I



Consider 20V source alone

$i_1 = I$

KVL to loop

$20 - 4i_1 - 2i_1 - 2I = 0$

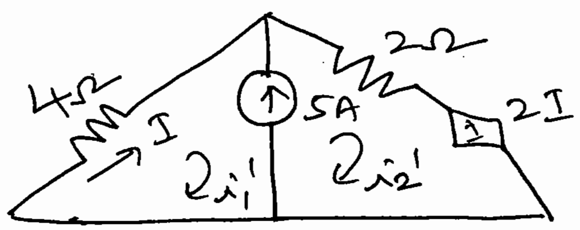
$20 - 4I - 2I - 2I = 0$

$8I = 20$

$\therefore \boxed{I = i_1 = 2.5A}$

DINAKAR B.C

Consider 5A current source only



$I = i_1'$

$$i_2' - i_1' = 5 \rightarrow (1)$$

loops (1) & (2) forms a supermesh

KVL to super loop

$$-4i_1' - 2i_2' - 2i_1' = 0$$

$$-6i_1' - 2i_2' = 0 \rightarrow (2)$$

$$\therefore i_1' = -1.25\,A \qquad i_2' = 3.75\,A$$

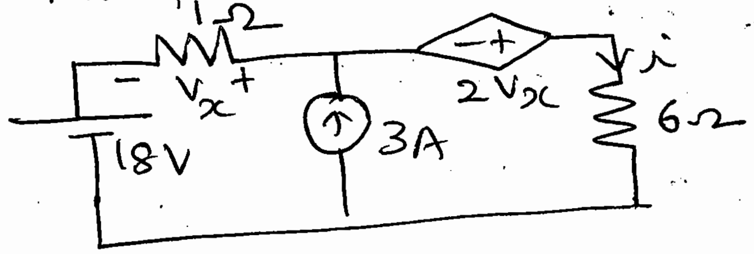
$$\therefore I = i_1' = -1.25\,A$$

$$\therefore I = i_1 + i_1'$$

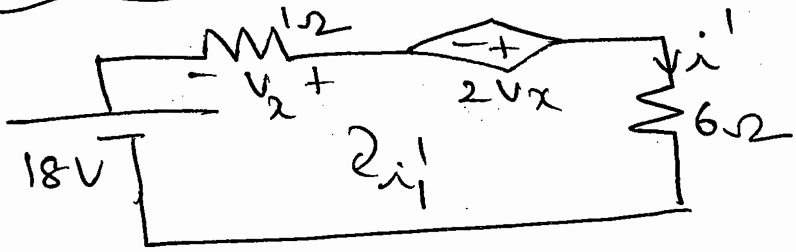
$$= 2.5 - 1.25 = 1.25\,A$$

$$\boxed{I = 1.25\,A}$$

July 2017 Find the current in 6Ω resistor



Consider 18 V alone



$V_x = -1 i_1'$ $\qquad\qquad$ $i_1' = i'$

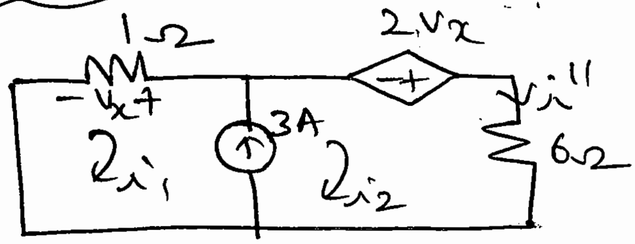
KVL to loop

$$18 - i_1' + 2V_x - 6i_1' = 0$$

$$18 - i_1' - 2i_1' - 6i_1' = 0$$

$$i_1' = \frac{18}{9} = 2\,A$$

Case 2 :- consider 3 A source alone



① & ② forms a superloop

$$i_2 - i_1 = 3 \rightarrow ①$$

$V_x = -i_1$

KVL to super loop

$$-i_1 + 2V_x - 6i_2 = 0$$

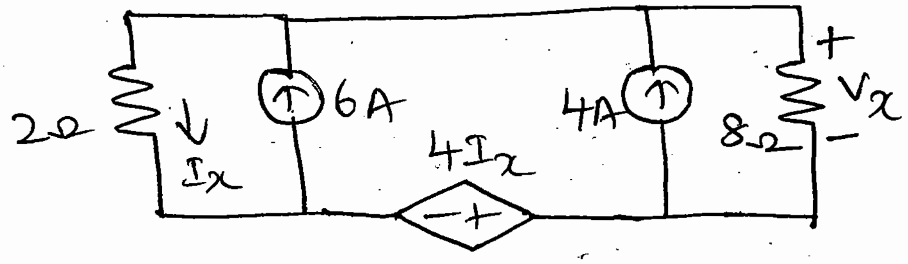
$$-i_1 + 2(-i_1) - 6i_2 = 0$$

$$-3i_1 - 6i_2 = 0 \rightarrow ②$$

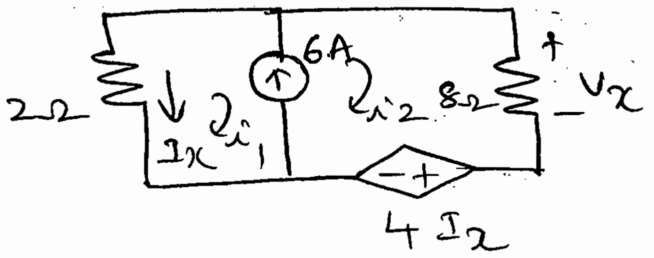
$i_1 = -2\,A$ $\qquad$ $\boxed{i_2 = i'' = 1\,A}$

$\therefore$ $\boxed{I_{6\Omega} = i_1' + i'' = 2 + 1 = 3\,A}$

using superposition theorem find the voltage V_x for the circuit shown



① consider 6 A source alone



$I_x = -i_1$

① & ② forms a super loop

$i_2 - i_1 = 6 \rightarrow ①$

KVL to super loop

$-2i_1 - 8i_2 - 4I_x = 0$

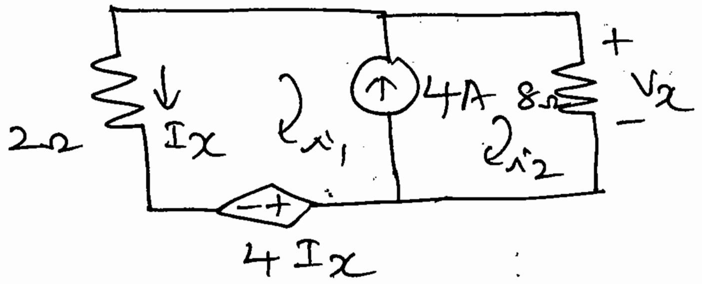
$-2i_1 - 8i_2 - 4(-i_1) = 0$

$\boxed{2i_1 - 8i_2 = 0} \rightarrow ②$

$i_1 = -8\text{ A}, \quad i_2 = -2\text{ A}$

$\therefore \boxed{V_x' = 8(-2) = -16\text{ V}}$

Case ② consider 4A source alone



① & ② forms a superloop

$$i_2 - i_1 = 4 \rightarrow ①$$

apply KVL to superloop

$$-2i_1 - 8i_2 - 4(-i_1) = 0$$

$$2i_1 - 8i_2 = 0 \rightarrow ②$$

$$i_1 = -5.33\,A, \quad i_2 = -1.33\,A$$

$$V_x'' = 8i_2 = 8(-1.33) = -10.64\,V$$

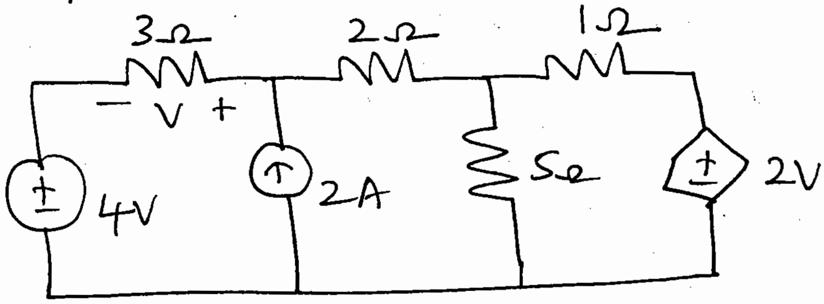
$$\therefore \quad V_x = V_x' + V_x''$$

$$= -16 - 10.64$$

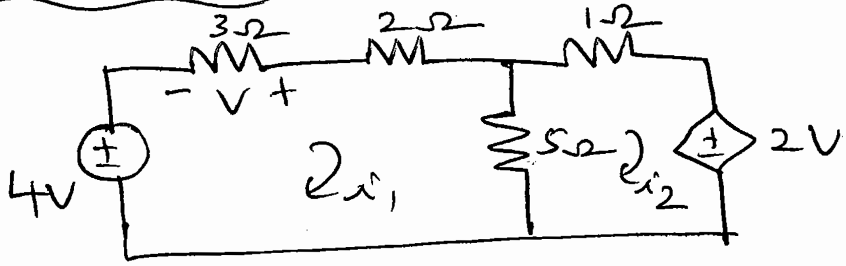
$$\boxed{V_x = -26.64\,V}$$

2011 Aug

Find V. using the principle of superposition
48 88



Consider 4V source alone



$$V = -3i_1$$

KVL to loop ①

$$\boxed{10i_1 - 5i_2 = 4} \rightarrow ①$$

KVL to loop ②

$$-5i_1 + 6i_2 = -2V$$

$$-5i_1 + 6i_2 = -2(-3i_1)$$

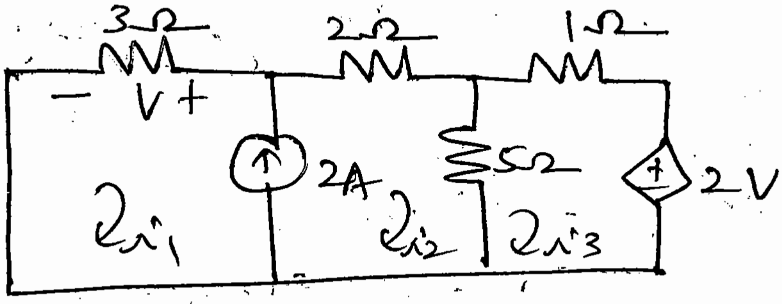
$$\boxed{-11i_1 + 6i_2 = 0} \Rightarrow ②$$

$$i_1 = 4.8\ A \qquad i_2 = 8.8\ A$$

$$\therefore\ V' = -3i_1 = -3 \times 4.8$$

$$\boxed{V' = -14.4\ \text{volts}}$$

consider 2A current source



$$\boxed{i_2 - i_1 = 2} \rightarrow (1)$$

supermesh equation

$$\boxed{3i_1 + 7i_2 - 5i_3 = 0} \rightarrow (2)$$

KVL to loop (3)

$$-5i_2 + 6i_3 = -2V$$

but $V = -3i_1$

$$\therefore \quad -5i_2 + 6i_3 = -2(-3i_1)$$

$$\boxed{-6i_1 - 5i_2 + 6i_3 = 0} \Rightarrow (3)$$

$i_1 = -6.8\,A$, $i_2 = -4.8\,A$ $i_3 = -10.8\,A$

$$V'' = -3i_1 = 20.4\,V$$

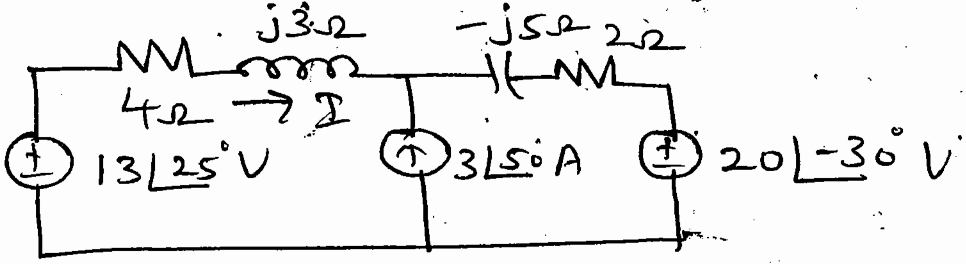
$$\therefore \quad V = V' + V''$$

$$= -14.4 + 20.4$$

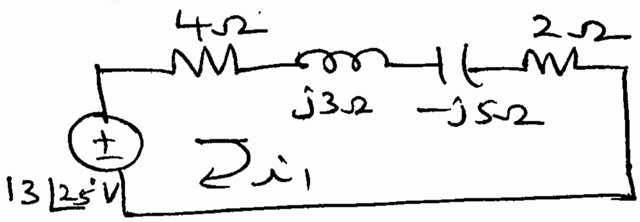
$$\boxed{V = 6V}$$

DIVAKAR. B. C

By using superposition principle find the current through $(4+j3\,\Omega)$ impedance



Consider $13\angle 25^\circ V$

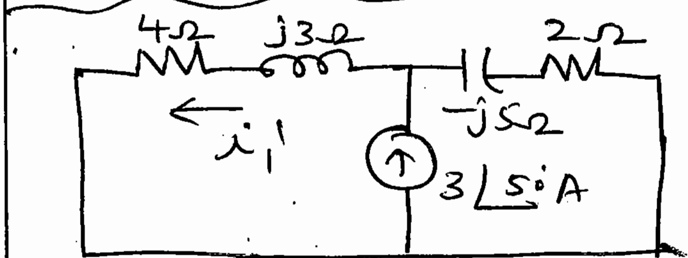


$$i_1 = \frac{13\angle 25}{4+j3-j5+2} = \frac{13\angle 25^\circ}{6-j2}$$

$$i_1 = \frac{13\angle 25^\circ}{6.3245\angle -18.43^\circ} = 2.055\angle 43.43^\circ$$

$$= 1.4927 + j1.413 \text{ Amp}$$

Consider current source alone



using current divider rule

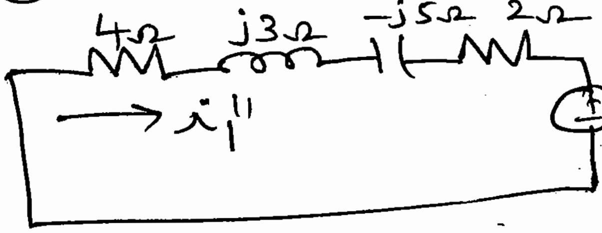
$$i_1' = \frac{3\angle 50 \times (2-j5)}{2-j5+4+j3}$$

$$= \frac{3\angle 50 \times 5.3852\angle -68.19}{6.3245\angle -18.43^\circ}$$

$= 2.5544 \angle 0.24^\circ A$

$= 2.5543 + j\,0.0107\,A \leftarrow$

case ③ $\Rightarrow$ consider $20\angle -30^\circ$ V alone



$$i_1'' = \frac{20\angle -30}{4+j3-j5+2} = \frac{20\angle -30}{6.3245\angle -18.43}$$

$= 3.1623 \angle -11.57^\circ A$

$= 3.098 - j\,0.6342A \leftarrow$

$\therefore\ I = i_1 + i_1' + i_1''$

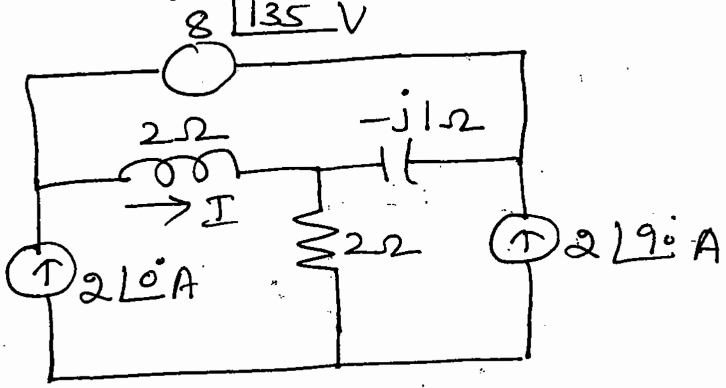
$= (1.4927 + j1.4131) - (2.5543 + j\,0.0107)$

$- (3.098 - j\,0.6342)$

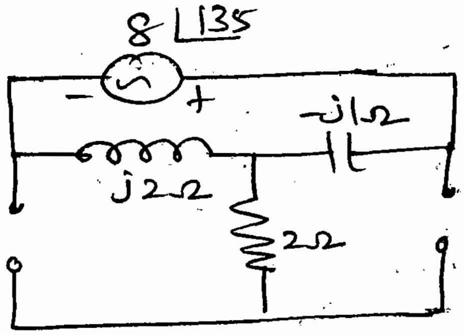
$= -4.1596 + j2.0366\,A$

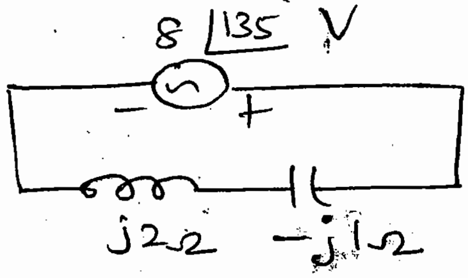
$$\boxed{I = 4.6314 \angle 153.913^\circ\ A \rightarrow}$$

using superposition theorem obtain the response I for the network shown



case ① consider $8\angle 135^\circ$ V

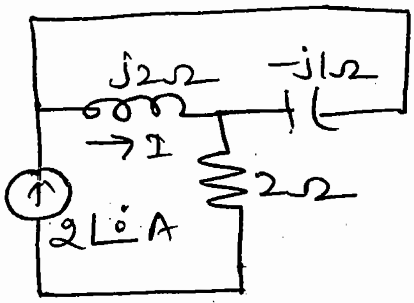




$$I' = \frac{8\angle 135^\circ}{-j1} = 8\angle 225^\circ$$

$$= \frac{8\angle 135}{1\angle -90^\circ} = -5.6568 - j5.6568\,A$$

case 2 consider $2\angle 0^\circ$ current source

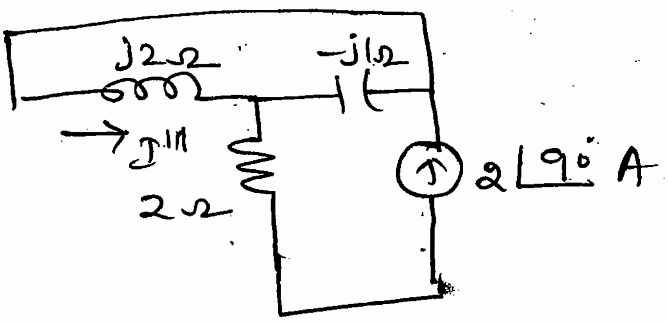


using current divider rule

$$I'' = 2\angle 0^\circ \left[\frac{-j1}{j2-j1}\right]$$

$$= 2\angle 0 \left(\frac{-j1}{-j1}\right) = -2A$$

case 3 consider $2\angle 90^\circ$ A source alone



using current divider rule

$$I''' = 2\angle 90\left(\frac{-j1}{j2-j1}\right)$$

$$= (j2)(-1) = -j2\,A$$

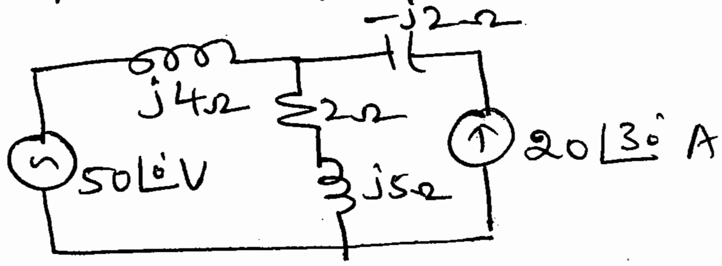
$$\therefore\ I = I' + I'' + I'''$$

$$= (-5.6568 - j5.6568) + (-2) + (-j2)$$

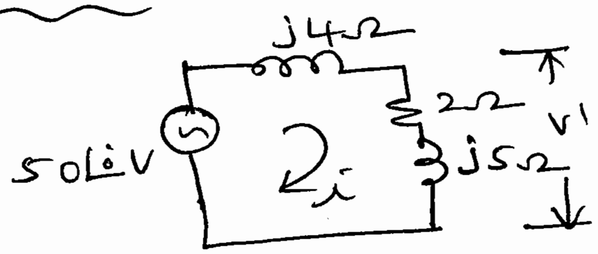
$$= (-7.6568 - j7.6568)$$

$$\boxed{I = 10.8283\angle -135^\circ\ A}$$

Determine voltage across $(2+j5)\Omega$ using superposition principle



case ① Consider $50\angle 0^\circ$ V source

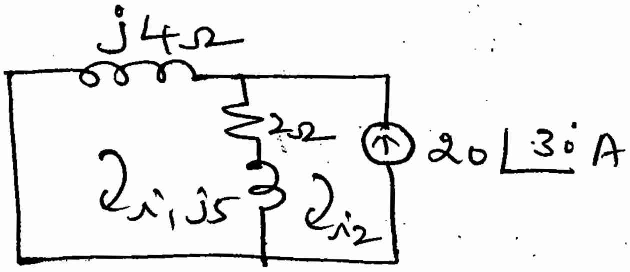


$$V' = \frac{50\angle 0^\circ \times (2+j5)}{2+j5+j4}$$

$$V' = \frac{50\angle 0^\circ (2+j5)}{2+j9} = 28.82 - 4.705j$$

$$= 29.2\angle -9.2726\,V$$

Case 2 Consider $20\angle 30^\circ$ A current source



$i_2 = -20\angle 30^\circ$ A

KVL to loop ①

$$(2+j5+j4)i_1 - (2+j5)i_2 = 0$$

$$(2+j9)i_1 = (2+j5)(-20\angle 30^\circ)$$

$$= (5.38\angle 68.19)(-20\angle 30^\circ)$$

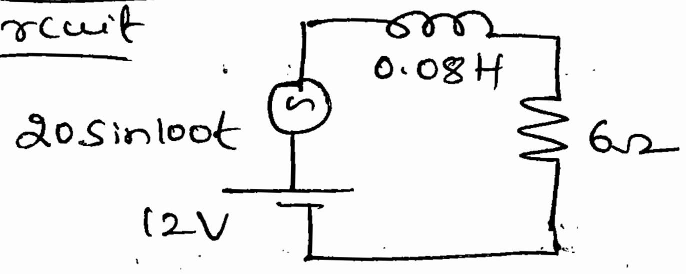
$$(9.219\angle 77.47)i_1 = -107.6\angle 98.19$$

$$\therefore i_1 = \frac{-107.6\angle 98.19}{9.219\angle 77.47}$$

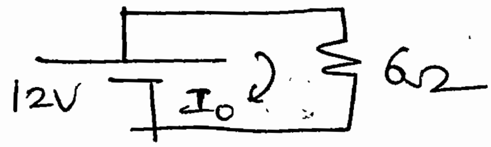
$$i_1 = -11.67\angle 20.72$$

$$V_{(2+j5)} = (2+j5)(i_1 - i_2)$$

Using superposition theorem obtain the expression for current, rms value of current & power consumed by the circuit 13



Case ① when only 12V dc voltage source acting alone



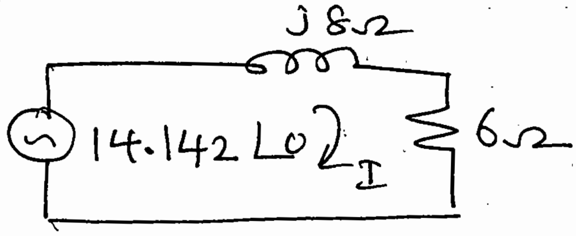
$I_0 = \frac{12}{6} = 2$ amperes

Inductor acts as a short circuit as the frequency of dc supply is zero

Case 2 ⇒ considering 20 sin 100t

$V = 20 \sin 100t$ $V = \frac{20}{\sqrt{2}} \angle 0$ volts

$X_L = \omega L = 100 \times 0.08 = j8\Omega$



DIWAKAR

$I = \frac{14.142}{6+j8} = \frac{14.142}{10\angle 53.13} = 1.414 \angle -53.13$

expression for current

$$i = \sqrt{2}\ I \sin(100t - 53.13)$$

$$= \sqrt{2} \times \sqrt{2} \sin(100t - 53.13)$$

$$i = 2\sin(100t - 53.13) \text{ amperes}$$

$$i_T = i + I_0$$

$$= 2 + 2\sin(100t - 53.13) \text{ amperes}$$

rms value of current.

$$I_T = \sqrt{I_0^2 + \frac{1}{2} I^2}$$

$$= \sqrt{2^2 + \frac{1}{2}(2)^2} = 2.45 \text{ amp}$$

$$\text{power} = I_T^2 . R = (2.45)^2 \times 6$$

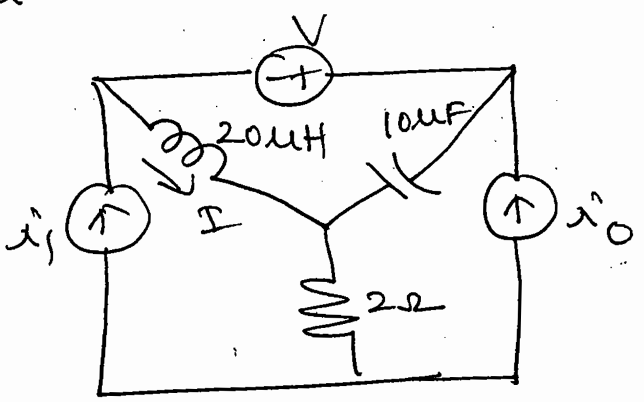
$$= 36 \text{ watts}$$

use superposition theorem obtain the response I (rms value) for the network shown

$$V = 8\cos(10^5 t + 45°) \text{ Volts}$$

$$i_1 = 2\sqrt{2} \sin 10^5 t \text{ amp}$$

$$i_2 = 2\sqrt{2} \cos 10^5 t \text{ amp}$$



$V = 8\cos(10^5 t + 45^\circ)$

$V = 8\sin(10^5 t + 135^\circ)$

$= V_m \sin(\omega t + \theta)$

rms value of supply voltage

$V = \frac{8}{\sqrt{2}} \angle 135 = 5.656 \angle 135^\circ$ volts

$i_1 = 2\sqrt{2} \sin 10^5 t$

$= I_m \sin \omega_m t$

rms value of current

$I_1 = \frac{2\sqrt{2}}{\sqrt{2}} = 2\angle 0$ amp

$i_2 = 2\sqrt{2} \cos 10^5 t$

$= 2\sqrt{2} \sin(10^5 t + 90)$ amp

$= I_m \sin(\omega t + \phi)$

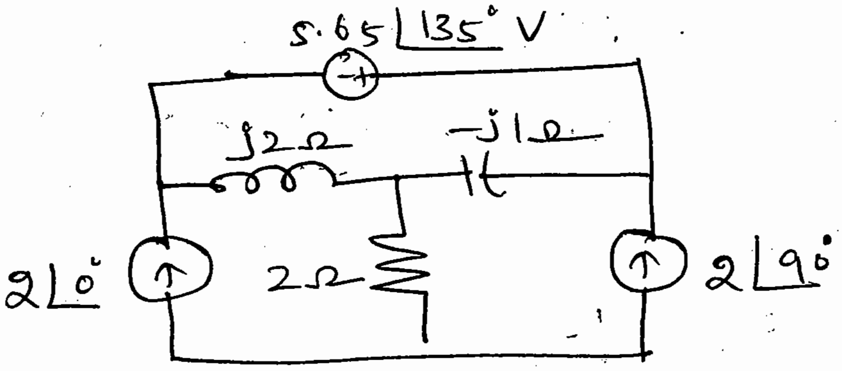
rms value of current

$I_2 = \frac{2\sqrt{2}}{\sqrt{2}} \angle 90 = 2\angle 90$ amperes

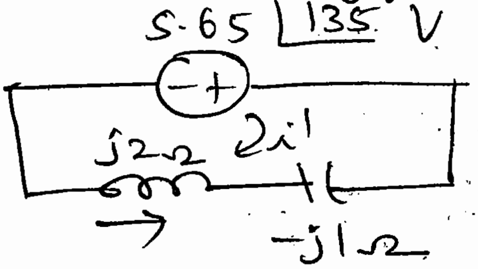
Inductive reactance $X_L = \omega L$

$X_L = 10^5 \times 20 \times 10^{-6} = j2\,\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{10^5 \times 10 \times 10^{-6}} = -j1\,\Omega$



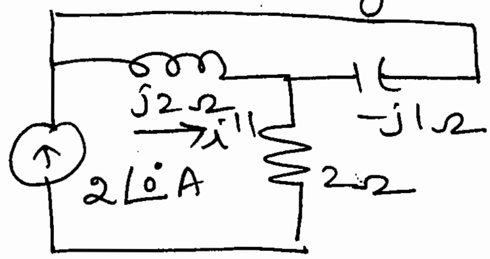
case 1 with only $5.65\angle 135^\circ$ V



$$i' = \frac{-5.65\angle 135}{j2 - j1}$$

$$= \frac{-5.65\angle 135}{1\angle 90} = -5.65\angle 45^\circ$$

case with only $2\angle 0^\circ$ A source

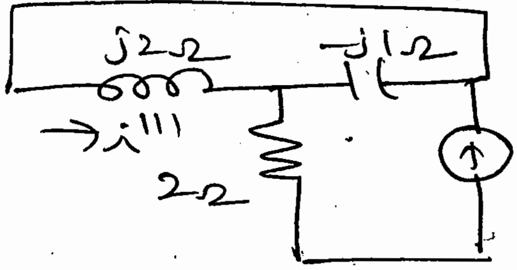


$$i'' = \frac{2\angle 0\,(-j1)}{j2 - j1}$$

$$= \frac{(2\angle 0)\,1\angle -90}{1\angle 90}$$

$$= 2\angle -180^\circ \text{ A}$$

case 3 with only $2\angle 90^\circ$ A



$$i''' = \frac{(2\angle 90)(-j1)}{(j2 - j1)}$$

$$= \frac{(2\angle 90)(1\angle -90)}{1\angle 90}$$

$= 2\angle -90^\circ$ amperes

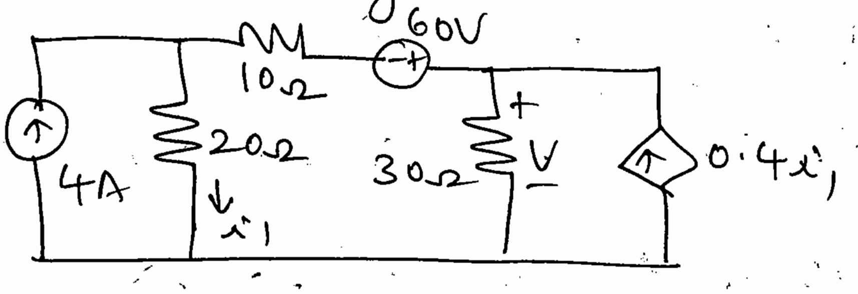
$$I = I_1' + i'' + i'''$$

$$= -5.65\angle 45 + 2\angle -180 + 2\angle -90$$

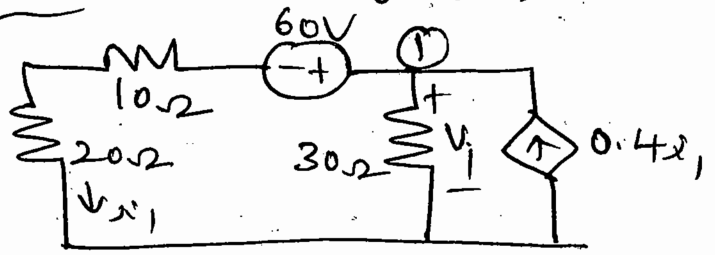
$$= -4 - j4 - 2 - j2 = -6 - j6$$

$$= 8.48\angle -135^\circ \text{ amperes}$$

Find V using superposition theorem



Case ① with 60V source alone



$$i_1 = \frac{V_1 - 60}{30}$$

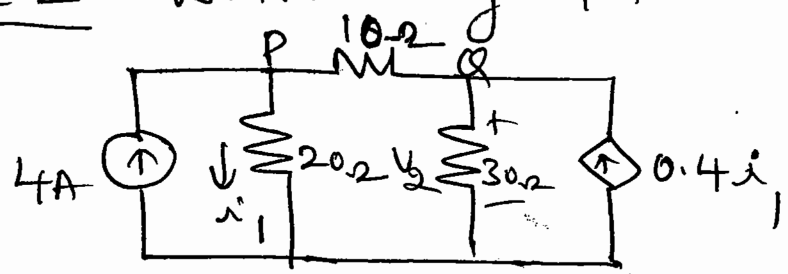
KCL @ node 1

$$\frac{V_1 - 60}{30} + \frac{V_1}{30} = 0.4 i_1$$

$$\frac{V_1 - 60}{30} + \frac{V_1}{30} = 0.4\left(\frac{V_1 - 60}{30}\right)$$

$$V_1 = 22.5 \text{ volts} = V_A$$

case 2 with only 4A source



$$i_1 = \frac{V_P}{20} \qquad \therefore \frac{V_P}{20} + \frac{V_P - V_Q}{10} = 4$$

$$\boxed{3V_P - 2V_Q = 80} \rightarrow ①$$

KCL @ node 2

$$\frac{V_Q - V_P}{10} + \frac{V_Q}{30} = 0.4\, i_1$$

but $i_1 = \frac{V_P}{20}$ $\therefore$ $-6V_P + 8V_Q = 1.2V_P$

$$\boxed{-7.2V_P + 8V_Q = 0} \rightarrow (2)$$

$V_Q = 60$ volts $= V_2$

$\therefore$ $V = V_1 + V_2 = 22.5 + 60$

$$\boxed{V = 82.5 \text{ volts}}$$

Reciprocity Theorem

The source E in one branch produces the current I in the second branch and the same source E in the second branch produces the same current I. in the first branch

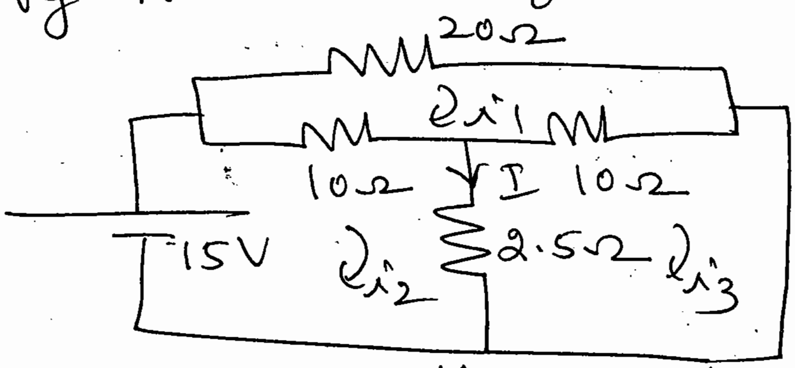
or

Statement ⇒ In any linear bilateral single source network the ratio of excitation to the response remains same even when the positions of excitation and response are interchanged.

Limitations

1. Applicable only for a single source network
2. cannot be applicable for the circuit consisting of dependent sources

* verify the truth of reciprocity theorem



Before the application of Reciprocity Theorem

loop 1 ⇒ $40i_1 - 10i_2 - 10i_3 = 0$

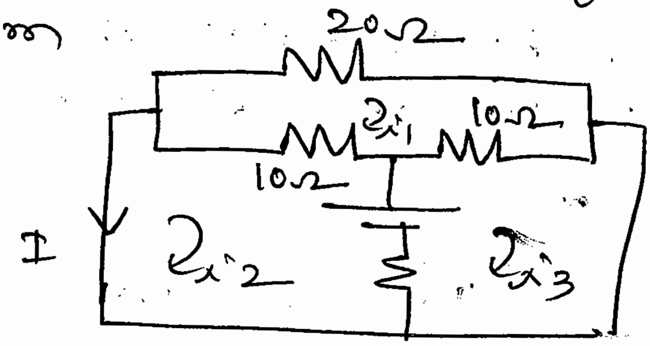
loop 2 ⇒ $-10i_1 - 12.5i_2 - 2.5i_3 = 15$

loop 3 ⇒ $-10i_1 - 2.5i_2 + 12.5i_3 = 0$

$i_1 = 0.75A, \; i_2 = 2A, \; i_3 = 1A$

$$\boxed{I = i_2 - i_3 = 1A}$$

after the application of Reciprocity theorem



applying KVL

loop ① $\Rightarrow$ $40i_1 - 10i_2 - 10i_3 = 0$

loop ② $\Rightarrow$ $-10i_1 + 12.5i_2 - 2.5i_3 = -15$

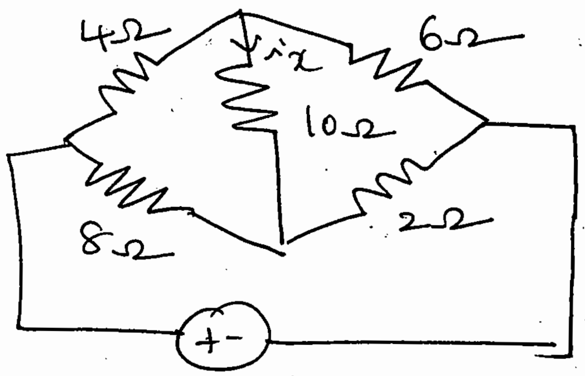
loop ③ $\Rightarrow$ $-10i_1 - 2.5i_3 + 12.5i_3 = 15$

$i_1 = 0A \; ; \; i_2 = -1A, \; i_3 = 1A$

$I = -i_2 \quad \therefore \quad \boxed{I = 1A}$

hence verified

② Find i_x using Reciprocity theorem



apply KVL

$i_1 = 1.1714A$

$i_2 = 0.8857A$

$i_3 = 2.1143A$

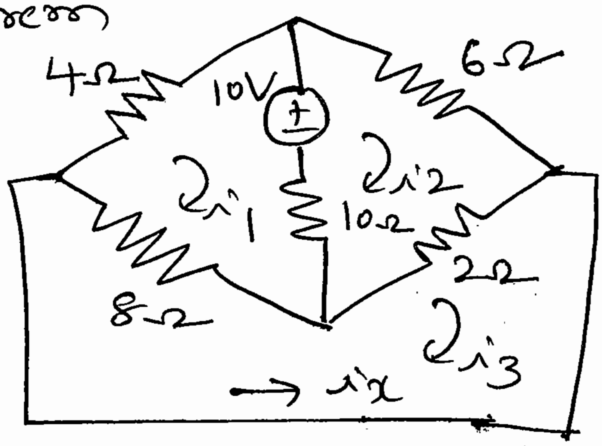
loop ① $\Rightarrow$ $22i_1 - 10i_2 - 8i_3 = 0$

loop ② $\Rightarrow$ $-10i_1 + 18i_2 - 2i_3 = 0$

loop ③ $\Rightarrow$ $-8i_1 - 2i_2 + 10i_3 = 10$

$i_x = i_1 - i_2 = 0.2857\ A$

After the application of Reciprocity theorem



apply KVL to loops

loop ① ⇒ $22i_1 - 10i_2 - 8i_3 = -10$

loop ② ⇒ $-10i_1 + 18i_2 - 2i_3 = 10$

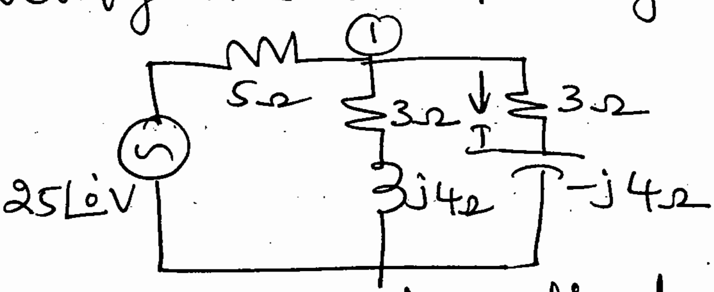
loop ③ ⇒ $-8i_1 - 2i_2 + 10i_3 = 0$

$i_1 = -0.4285\ A$; $i_2 = 0.2857\ A$

$i_3 = -0.2857$

$\therefore \boxed{i_x = -i_3 = 0.2857}$ verified

Verify the Reciprocity theorem



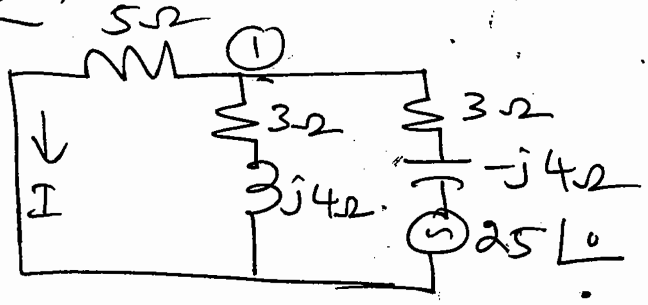
using nodal method

$$V_1\left(\frac{1}{5} + \frac{1}{3+j4} + \frac{1}{3-j4}\right) = \frac{25\angle 0°}{5}$$

$V_1 = 11.3636\ V$

$$i = \frac{11.3636}{3-j4} = \frac{V_1}{3-j4} = 2.264\angle 53.079 \text{ A}$$

after the application of Reciprocity theorem



KCL @ node 1

$$V_1\left[\frac{1}{3+j4} + \frac{1}{5} + \frac{1}{3-j4}\right] = \frac{25\angle 0}{3-j4}$$

$$V_1 = \frac{25\angle 0}{(3-j4)(0.44)} \quad \therefore V_1 = 11.3636\angle 53.13 \text{ V}$$

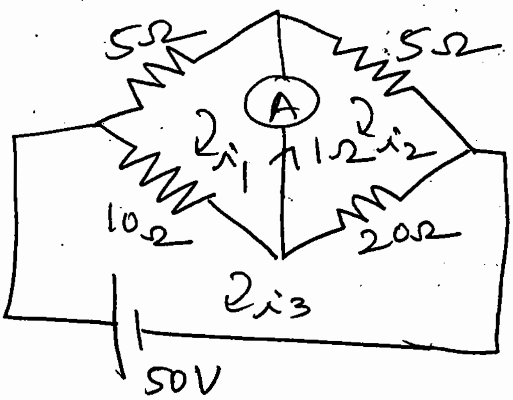
$$= 6.818 + 9.09j$$

$$i = \frac{V_1}{5} = 1.3636 + 1.818j$$

$$\boxed{i = 2.27\angle 53.13 \text{ A}}$$

hence verified

Find the current through ammeter and verify reciprocity theorem



Before the application of Reciprocity theorem

apply KVL to loops 1 2 & 3

loop 1 $\Rightarrow$ $16i_1 - i_2 - 10i_3 = 0$

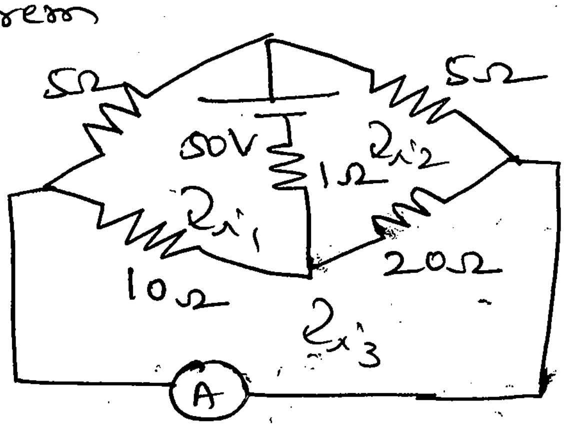
Loop 2 $\Rightarrow$ $-i_1 + 26i_2 - 20i_3 = 0$

loop 3 $\Rightarrow$ $-10i_1 - 20i_2 + 30i_3 = 50$

$i_1 = 4.59\,A$; $i_2 = 5.4\,A$, $i_3 = 6.8\,A$

$I = i_2 - i_1 = 0.81\,A$

After the application of Reciprocity theorem



KVL to loops

loop 1 $\Rightarrow$ $16i_1 - i_2 - 10i_3 = -50 \rightarrow (1)$

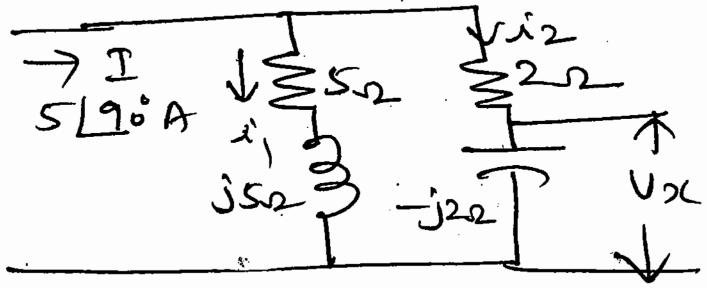
Loop 2 $\Rightarrow$ $-i_1 + 26i_2 - 20i_3 = 50 \rightarrow (2)$

Loop 3 $\Rightarrow$ $-10i_1 - 20i_2 + 30i_3 = 0 \rightarrow (3)$

$i_1 = -2.459\,A$, $i_2 = 2.459\,A$

$$\boxed{i_3 = I = 0.81\,A}$$

Verify the Reciprocity theorem



to find i_2 using current divider rule

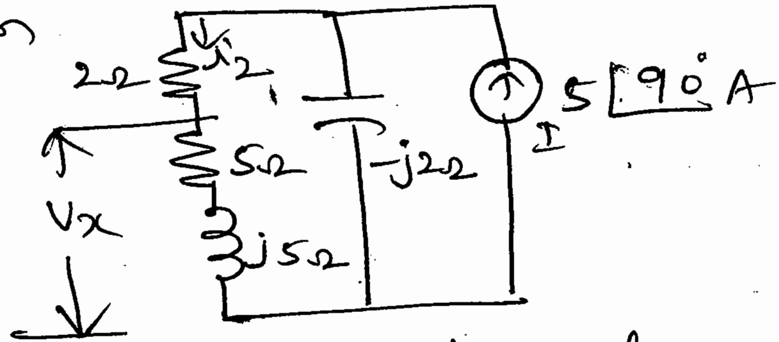
$$i_2 = \frac{5\angle 90 \times (5+j5)}{(5+j5)+(2-j2)} = \frac{(5\angle 90) \times 7.07\angle 45}{7+j3}$$

$$i_2 = 4.642\angle 111.8° \text{ A}$$

$$V_x = i_2(-j2) = (4.642\angle 111.8°)(2\angle -90°)$$

$$\boxed{V_x = 9.2847\angle 21.8° \text{ V}}$$

After the application of Reciprocity theorem



using current divider rule

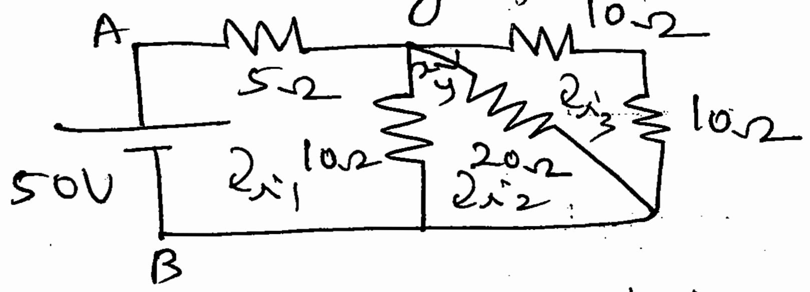
$$i_2 = \frac{I(-j2)}{2+5+j5-j2} = \frac{(5\angle 90)(-j2)}{7+3j}$$

$$i_2 = 1.313\angle -23.198° \text{ A}$$

$$V_x = i_2(5+j5) = (1.313\angle -23.13)\, 7.07\angle 45$$

$$V_x = 9.284\angle 21.8°$$

Show the validity of the Reciprocity theorem for the circuit shown between the points xy & AB ports.



KVL to loop ① $15i_1 - 10i_2 = 50 \rightarrow (1)$

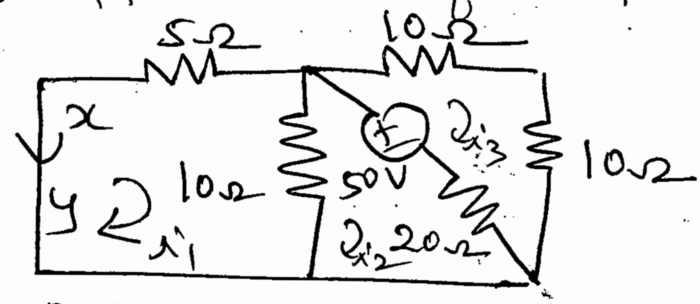
Loop 2 $\Rightarrow$ $-10i_1 + 30i_2 - 20i_3 = 0 \rightarrow (2)$

Loop 3 $\Rightarrow$ $-20i_2 + 40i_3 = 0 \rightarrow (3)$

$i_1 = 5A$, $i_2 = 2.5A$, $i_3 = 1.25A$

$$\boxed{i_{xy} = i_2 - i_3 = 1.25\,A}$$

After the application of Reciprocity theorem



KVL to loop ① $15i_1 - 10i_2 = 0 \rightarrow (1)$

Loop ② $-10i_1 + 30i_2 - 20i_3 = -50 \rightarrow (2)$

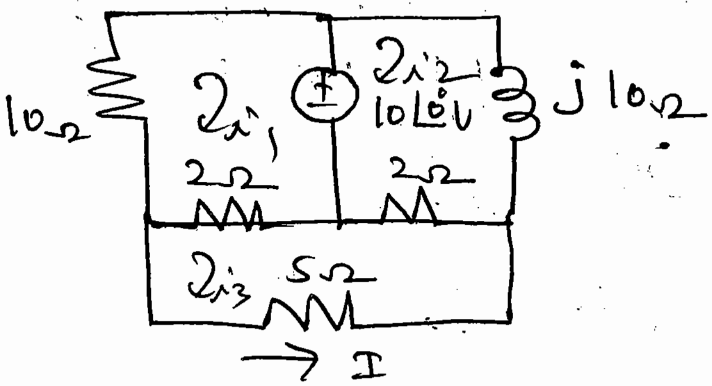
Loop ③ $-20i_2 - 40i_3 = 50 \rightarrow (3)$

$i_1 = -1.25A$ $i_2 = -1.875A$, $i_3 = 0.312A$

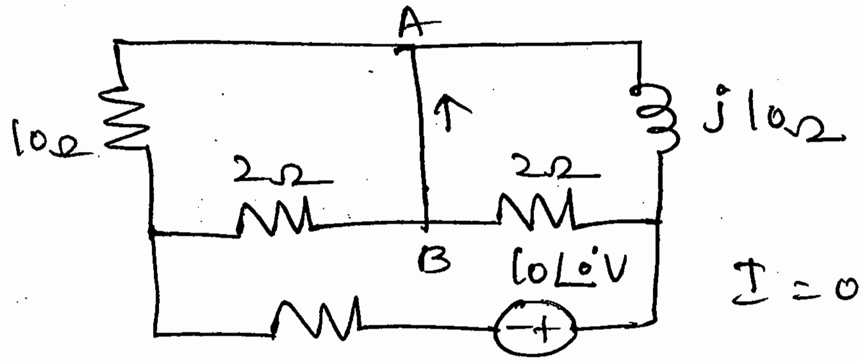
$$\boxed{i_1 = -i_{xy} = 1.25\,A}$$

hence verified

Verify Reciprocity theorem



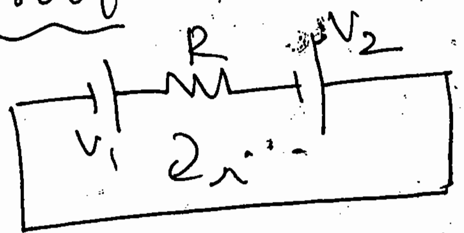
$I = 0.268 \angle 53.75$ amp



$I = 0.268 \angle 53.75$ amp

Superposition theorem

proof

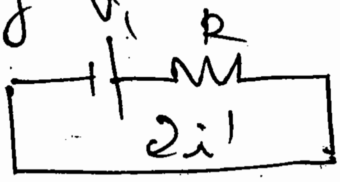


Before the application $i = \frac{V_1 + V_2}{R}$

after applying Superposition theorem

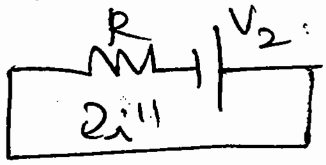
considering each sources separately

case(i)



$i' = \frac{V_1}{R}$

case<ii> consider V_2 source alone



$$i'' = \frac{V_2}{R}$$

$$\therefore \; i = i' + i'' = \frac{V_1}{R} + \frac{V_2}{R}$$

$$\therefore \; \boxed{i = \frac{V_1 + V_2}{R}}$$ hence verified

Reciprocity theorem proof

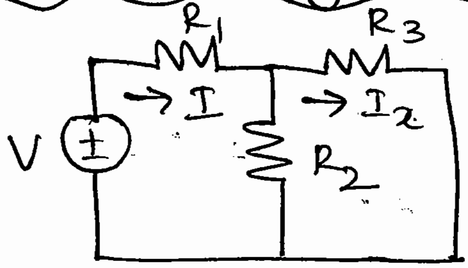


fig ①

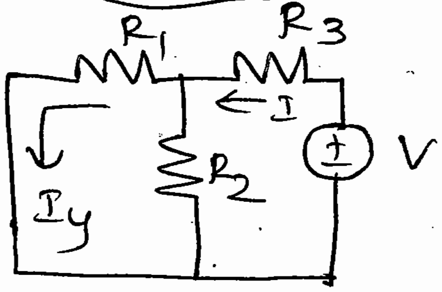


fig ②

from fig ①

$$R_{eq} = R_2 \parallel R_3 + R_1$$

$$R_{eq} = \left(\frac{R_2 R_3}{R_2 + R_3}\right) + R_1$$

but $I = \dfrac{V}{R_{eq}}$

$$I = \frac{V}{\left(\dfrac{R_2 R_3}{R_2 + R_3}\right) + R_1}$$

using current divider rule

$$I_x = \frac{I \cdot R_2}{R_2 + R_3}$$

$$I_x = \frac{V\cancel{(R_2+R_3)}}{R_2R_3+R_1(R_2+R_3)} \cdot \frac{R_2}{\cancel{(R_2+R_3)}}$$

$$\boxed{I_x = \frac{V R_2}{R_1R_2+R_1R_3+R_2R_3}}$$

From fig (2) $R_{eq} = R_2 \parallel R_1 + R_3$

$$R_{eq} = \left(\frac{R_1R_2}{R_1+R_2}\right) + R_3$$

but $I = \dfrac{V}{R_{eq}} = \dfrac{V}{\left(\dfrac{R_1R_2}{R_1+R_2}\right)+R_3}$

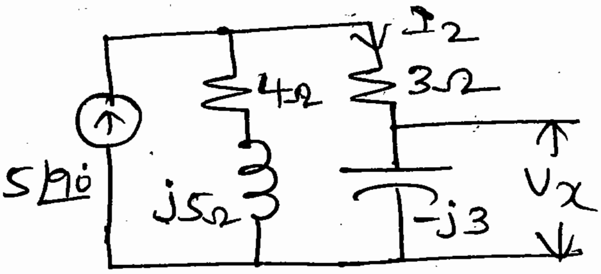
using current divider rule

$$I_y = \frac{I \cdot R_2}{R_1+R_2} = \frac{V\cancel{(R_1+R_2)}}{R_1R_2+R_3(R_1+R_2)} \cdot \frac{R_2}{\cancel{(R_1+R_2)}}$$

$$\boxed{I_y = \frac{V \cdot R_2}{R_1R_2+R_2R_3+R_1R_3}}$$

since $I_x = I_y$ theorem is verified

Find V_x & verify Reciprocity theorem



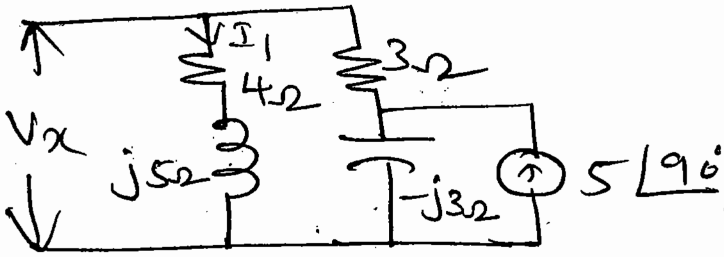
$$V_x = i_2(-j3)$$

$$i_2 = \frac{5\angle 90 \times 4+j5}{7+j2} = 4.4\angle 125.39°$$

$$V_x = 4.4\angle 125.39 \times 3\angle -90$$

$$= 13.2\angle 35.39° \text{ V}$$

After the application of Reciprocity theorem



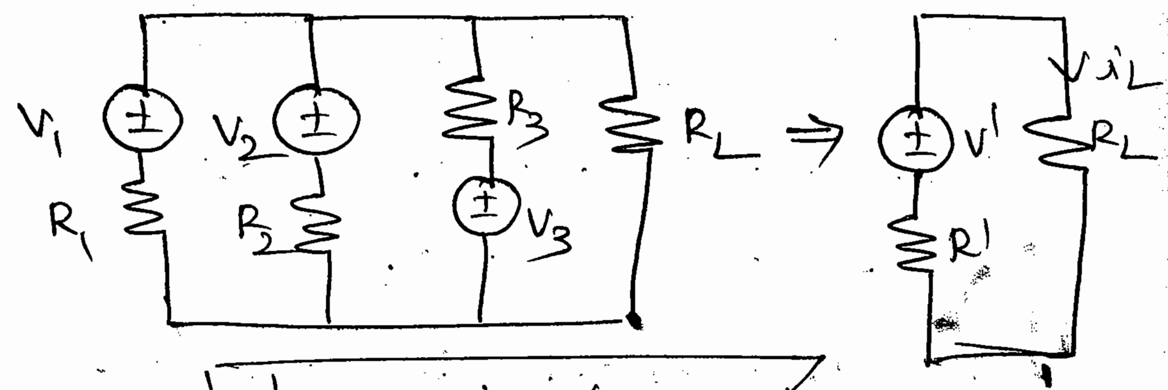
$$I_1 = \frac{5\angle 90 \times -j3}{7+j2} = 2.06\angle -15.95°$$

$$V_x = (4+j5)\, I_1$$

$$= 13.2\angle 35.39° \text{ V}$$

MILLMAN'S THEOREM

Millman's theorem states that a number of voltage sources with internal impedances in series connected in parallel may be replaced by a single voltage source with an impedance in series



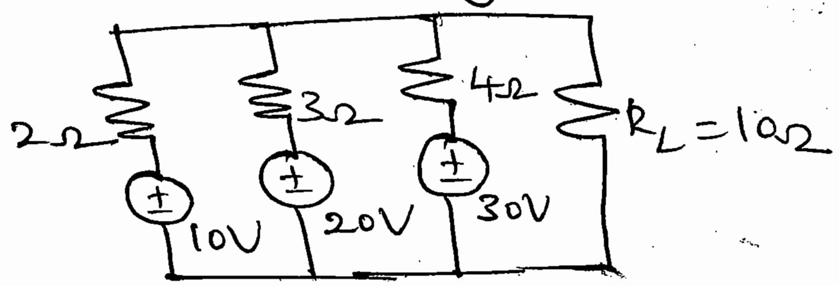
Where $V' = \dfrac{\Sigma V_N G_N}{\Sigma G_N}$ &

$$R' = \frac{1}{\Sigma G_N}$$

$$i_L = \frac{V'}{R' + R_L}$$

$G \Rightarrow$ Reciprocal of resistance

Find the current through 10Ω resistor using millman's theorem



$$V' = \frac{\Sigma V_N G_N}{\Sigma G_N} = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

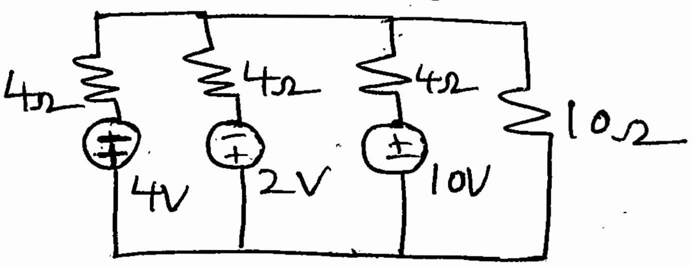
$$= \frac{10\left(\frac{1}{2}\right) + 20\left(\frac{1}{3}\right) + 30\left(\frac{1}{4}\right)}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}$$

$$\boxed{V' = \frac{19.1667}{1.0833} = 17.6923\,V}$$

$$\boxed{R' = \frac{1}{\Sigma G_N} = \frac{1}{1.0833} = 0.9231\,\Omega}$$

$$I_L = \frac{V'}{R' + R_L} = \frac{17.692}{0.923 + 10} \Rightarrow 1.6197\,A$$

* using millman's theorem find the current through 10Ω resistor



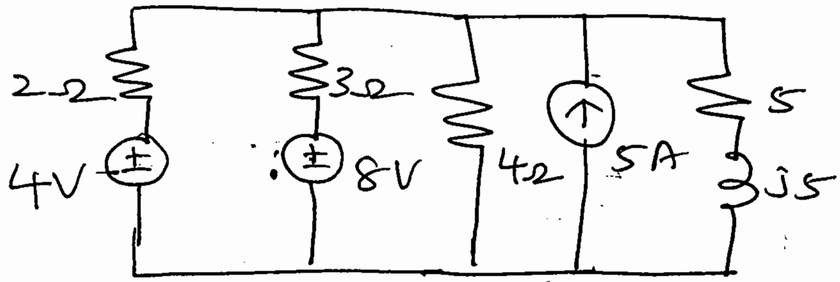
$$V' = \frac{-V_1 G_1 - V_2 G_2 + V_3 G_3}{G_1 + G_2 + G_3}$$

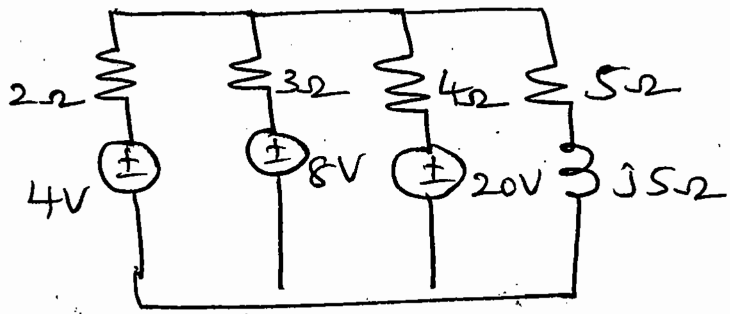
$$= \frac{-\left(\frac{4}{4}\right) - \left(\frac{2}{4}\right) + \frac{10}{4}}{\frac{1}{4} + \frac{1}{4} + \frac{1}{4}} = \frac{1}{0.75} = \underline{1.33\,V}$$

$$R' = \frac{1}{\Sigma G_N} = \frac{1}{0.75} = \underline{1.33\,\Omega}$$

$$I_L = \frac{V'}{R' + R_L} = \frac{1.33}{1.33 + 10} = \underline{0.1176\,A}$$

<u>using millman's theorem find the current through $(5+j5)\Omega$ impedance</u>





$$V' = \frac{\frac{4}{2} + \frac{8}{3} + \frac{20}{4}}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}} = \frac{9.6667}{1.0833} = \underline{\underline{8.923\,V}}$$

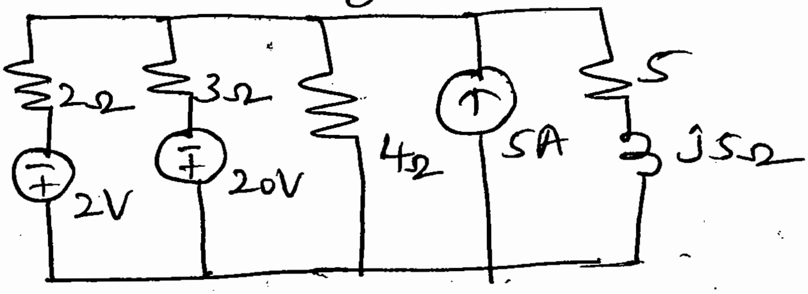
$$R' = \frac{1}{\Sigma G_N} = \frac{1}{1.083} = \underline{0.9231\,\Omega}$$

$$I_L = \frac{V'}{R' + R_L} = \frac{8.923}{0.9231 + 5 + j5}$$

$$\boxed{I_L = 1.151\ \angle -40.169^\circ\ A}$$

$$\boxed{I_L = 0.87965 - j0.7425}$$

For the circuit shown find the current through $(5+j5)\Omega$



$$V' = \frac{-\frac{2}{2} - \frac{20}{3} + \frac{20}{4}}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}} = \frac{-2.6667}{1.0833}$$

$$\boxed{V' = -2.4615\,V}$$

$$R' = \frac{1}{\Sigma G_N} = \frac{1}{1.0833} = \underline{\underline{0.9231\Omega}}$$

$$I_L = \frac{V'}{R' + R_L} = \frac{-2.4615}{0.9231 + 5 + j5}$$

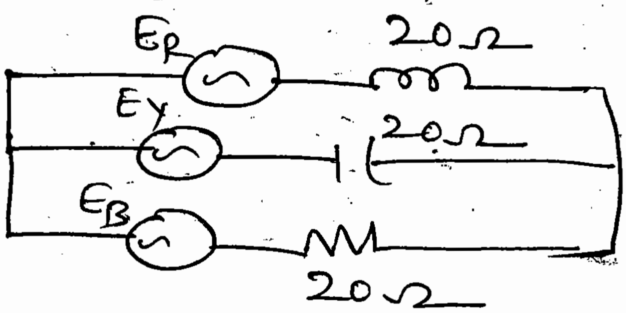
$$I = -0.2426 + 0.2j$$

$$\boxed{I = -0.317 \angle 139.83}$$

Determine, the voltage using millman's theorem for the circuit shown

$E_R = 230\angle 0°\ V$, $E_y = 230\angle -120°\ V$,

$E_B = 230\angle 120°\ V$



$$V' = \frac{\Sigma V_N G_N}{\Sigma G_N}$$

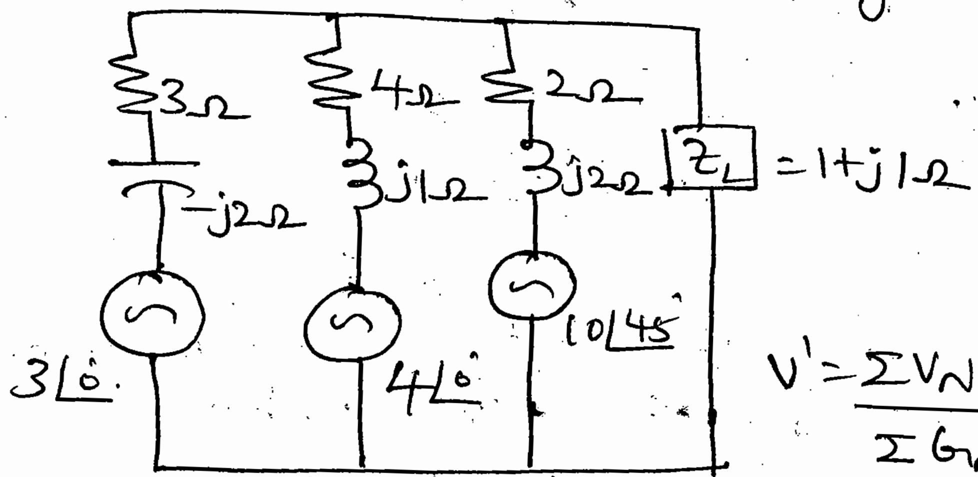
$$= \frac{\frac{230\angle 0}{20j} + \frac{230\angle -120}{-20j} + \frac{230\angle 120}{20}}{\left(\frac{1}{20j} + \frac{1}{-20j} + \frac{1}{20}\right)}$$

$$V' = \frac{4.209 - 7.2907j}{\frac{1}{20}}$$

$$V' = 84.18 - 145.8j$$

$$\boxed{V' = 168.37\angle -60^\circ \text{ V}}$$

Find the current through Z_L



$$V' = \frac{\Sigma V_N G_N}{\Sigma G_N}$$

$$V' = \frac{\frac{3\angle 0}{3-j2} + \frac{4\angle 0}{4+j} + \frac{10\angle 45}{2+j2}}{\left(\frac{1}{3-j2}\right) + \left(\frac{1}{4+j}\right) + \left(\frac{1}{2+j2}\right)}$$

$$V' = \frac{5.169 + 0.2262j}{0.71606 - 0.1549j}$$

$$V' = 6.83 + 1.793j$$

$$\boxed{V' = 7.062 \angle 14.712^\circ \ V}$$

$$Z' = \frac{1}{\Sigma G_N} = 1.3341 + 0.2886j$$

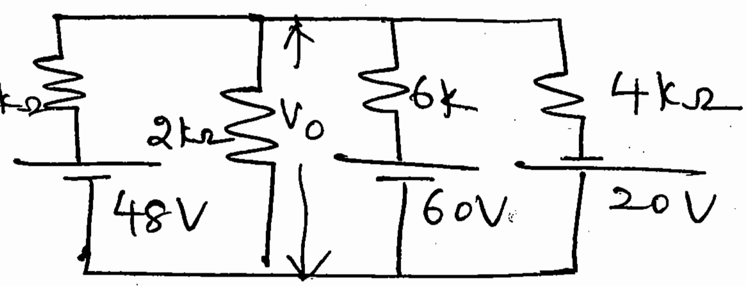
$$\boxed{Z' = 1.365 \angle 12.2^\circ \ \Omega}$$

$$i_L = \frac{V'}{Z' + R_L} = \frac{7.062 \angle 14.712}{1.365 \angle 12.2 + (1+j)}$$

$$i_L = 2.568 - 0.649j$$

$$\boxed{i_L = 2.648 \angle -14.19^\circ \ A}$$

using millman's theorem find the voltage across 2kΩ for the circuit shown



$$V' = \frac{\Sigma V_N G_N}{\Sigma G_N} = \frac{\frac{48}{12k} + \frac{60}{6k} - \frac{20}{4k}}{\frac{1}{12k} + \frac{1}{6k} + \frac{1}{4k}}$$

$$V' = \frac{9 \times 10^{-3}}{(0.5)10^{-3}} = 18V$$

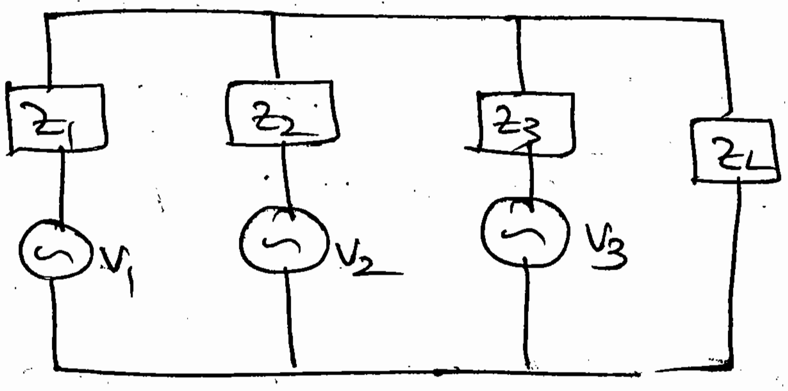
$$R' = \frac{1}{0.5 \times 10^{-3}} = 2\,k\Omega$$

$$I_L = \frac{V'}{R' + R_L} = \frac{1.8}{2k + 2k} = 4.5\,mA$$

$$V_0 = I_L (2k) = 4.5 \times 10^{-3} \times 2 \times 10^{3}$$

$$\boxed{V_0 = 9V}$$

Given $Z_L = 15 \angle -30^\circ \Omega$ find the current through Z_L



$Z_1 = 5 \angle 0^\circ \Omega$, $Z_2 = 10 \angle 30^\circ \Omega$, $Z_3 = 20 \angle -60^\circ \Omega$

$V_1 = 220 \angle 0^\circ$, $V_2 = 220 \angle -120^\circ$, $V_3 = 220 \angle 120$

$$V' = \frac{13.947 - 11j}{0.3116 - 6.698 \times 10^{-3} j}$$

$$V' = 45.4968 - j\,34.32$$

$$\boxed{V' = 56.99 \angle -37.03\ V}$$

$$R' = \frac{1}{\Sigma G_N} = 3.207 + 0.0689j$$

$$\boxed{R' = 3.208 \angle 1.23\ \Omega}$$

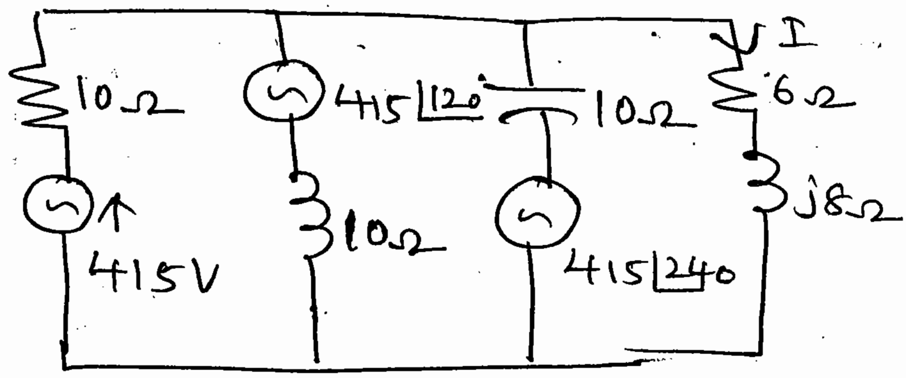
$$i_L = \frac{V'}{R' + R_L} = \frac{56.99 \angle -37.03}{3.208 \angle 1.23 + 15 \angle -30}$$

$$i_L = 3.123 - 0.686j$$

$$\boxed{i_L = 3.19 \angle -12.38^\circ \text{ A}}$$

Jan 2015

For the circuit shown find I using millman's theorem



$Z_1 = 10\Omega, \quad Z_2 = j10\Omega, \quad Z_3 = -j10\Omega$

$$Z_m = Z' = \frac{1}{Y_1 + Y_2 + Y_3} = \frac{1}{0.1 + j0.1 - j0.1}$$

$$\boxed{Z_m = 10\Omega}$$

DIVAKAR

$$V_m = V' = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3}{\Sigma G_N}$$

$$= \frac{(415 \times 0.1) + (415 \angle 120)(0.1 \angle -90) + 415 \angle 240 (0.1 \angle 90)}{0.1}$$

$$= 41.5 + 41.5 \angle 30^\circ + 41.5 \angle 330^\circ / 0.1$$

$$= \frac{41.5 + 35.94 + j\cancel{20}.75 + 35.94 - j\cancel{20.75}}{0.1}$$

$$V_m = \frac{113.38}{0.1} = 1133.8V$$

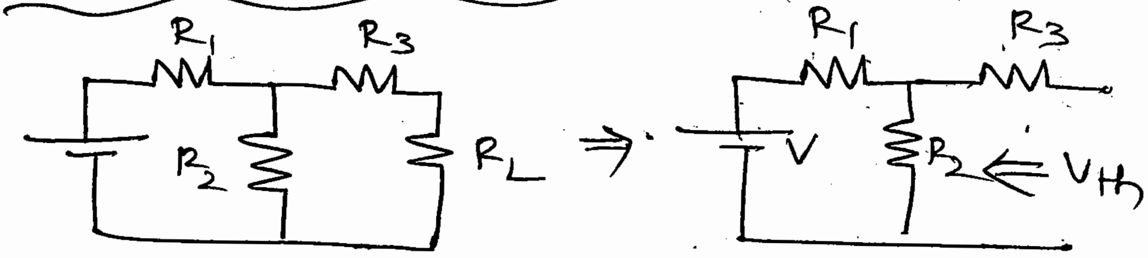
$$I = \frac{V'}{Z' + Z_L} = \frac{1133.8}{17.88 \angle 26.56}$$

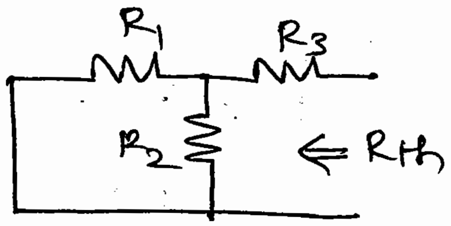
$Z_L = 6 + j8$, $Z_m = 10\Omega$

$\therefore \; Z' = 16 + j8 = 17.88 \angle 26.56$

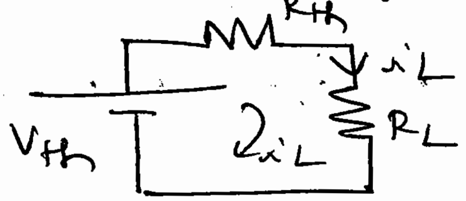
$$\boxed{I = 63.38 \angle -26.56^\circ \; A}$$

Thevenin's theorem





Thevenin's equivalent circuit



$$\boxed{V_{th} = i \cdot R_2}$$

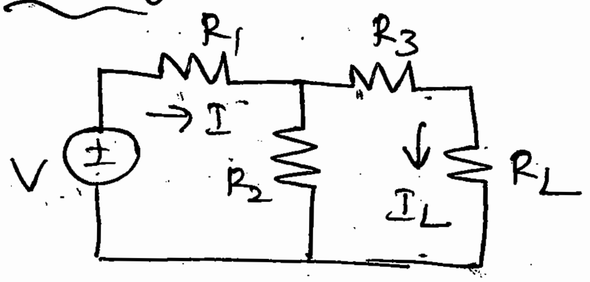
$$\boxed{R_{th} = (R_1 \parallel R_2) + R_3}$$

$$\boxed{i_L = \frac{V_{th}}{R_{th} + R_L}}$$

Statement ⇒ It states that "In any Linear, bilateral active network however it is complicated, it is represented by a single voltage source V_{th} (or) V_{oc} in series with Resistance R_{th} and the current through the Load resistor is given by $i_L = \frac{V_{th}}{R_{th} + R_L}$

where V_{th} is the open circuit voltage measured by removing R_L & R_{th} is the thevenin's equivalent resistance measured by removing the load resistance R_L replacing voltage source by short circuit considering internal resistances if any and replacing current sources by open circuit

Proof ⇒



$$I_L = \frac{I \cdot R_2}{R_2 + R_3 + R_L}$$

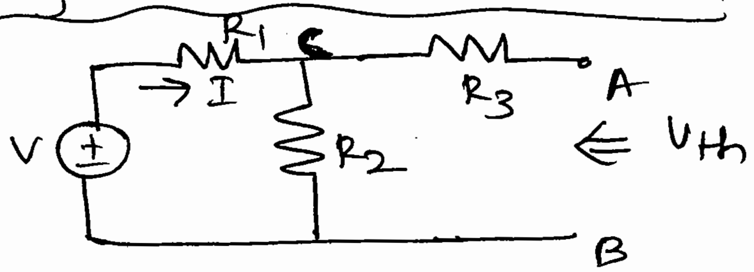
$$I = \frac{V}{R_1 + (R_3 + R_L) \parallel R_2} = \frac{V}{R_1 + \frac{(R_3+R_L)R_2}{R_3+R_L+R_2}}$$

$$I = \frac{V(R_3 + R_L + R_2)}{R_1R_3 + R_1R_L + R_1R_2 + R_2R_3 + R_LR_2}$$

$$I_L = \frac{R_2}{\cancel{R_2 + R_3 + R_L}} \times \frac{V\,\cancel{(R_3 + R_L + R_2)}}{R_1R_3 + R_1R_L + R_1R_2 + R_2R_3 + R_2R_L}$$

$$\boxed{I_L = \frac{V \cdot R_2}{R_1R_3 + R_1R_L + R_1R_2 + R_2R_3 + R_LR_2}}$$

By Thevenin's theorem

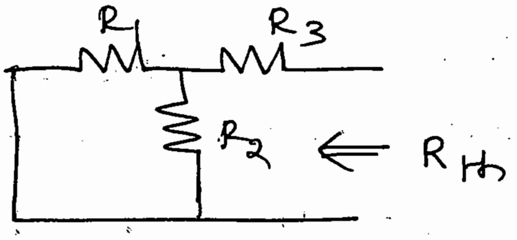


since drop across R_3 is zero

$$V_C = I \cdot R_2 = V_A$$

$$I = \frac{V}{R_1 + R_2} \qquad \therefore \; V_C = \frac{V \cdot R_2}{R_1 + R_2}$$

$$\boxed{V_{Th} = V_C = \frac{V \cdot R_2}{R_1 + R_2}}$$



$$R_{Th} = (R_1 \parallel R_2) + R_3$$

$$= \frac{R_1 R_2}{R_1 + R_2} + R_3$$

$$R_{Th} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1 + R_2}$$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{\frac{V \cdot R_2}{R_1 + R_2}}{\frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1 + R_2} + R_L}$$

$$I_L = \frac{\frac{V \cdot R_2}{\cancel{R_1 + R_2}}}{\frac{R_1 R_2 + R_1 R_3 + R_2 R_3 + R_1 R_L + R_2 R_L}{\cancel{R_1 + R_2}}}$$

$R_2 R_L$

$$\boxed{I_L = \frac{V \cdot R_2}{R_1 R_3 + R_1 R_2 + R_1 R_L + R_2 R_3 + R_2 R_L}}$$

hence proved

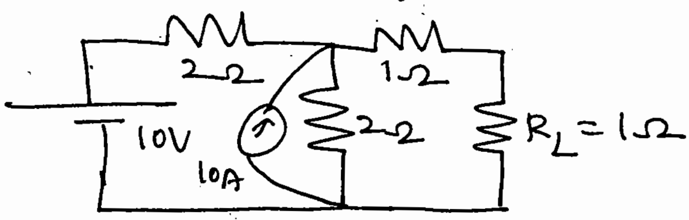
Steps for solving a network using Thevenin's theorem

1. Remove the load resistance R_L
2. Find the open circuit voltage V_{Th} across the 2 terminals
3. Find the resistance R_{Th} as seen from the 2 points with voltage source replaced by short circuit, current source replaced by open circuit

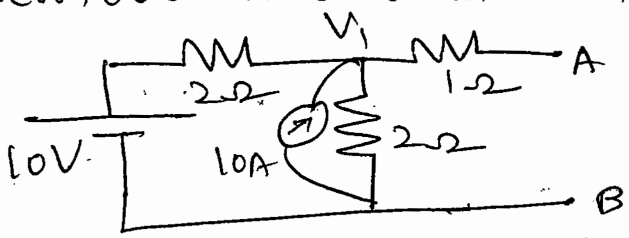
(4) Replace the network by voltage source V_{th} in series with Resistance R_{th}

(5) Find the current through the Load resistor $i_L = \frac{V_{th}}{R_{th}+R_L}$ A

using Thevenin's theorem find the load current



Remove the load resistance R_L



Since the terminals A & B are open no current flows through 1Ω ∴

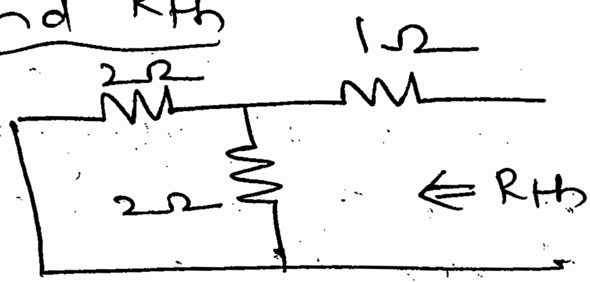
apply KCL @ node

$$\frac{V_1}{2} + \frac{V_1 - 10}{2} = 10$$

$V_1 = 10 + 5$ ∴ $\boxed{V_1 = 15\,V}$

$$\boxed{V_1 = V_{th} = 15\,V}$$

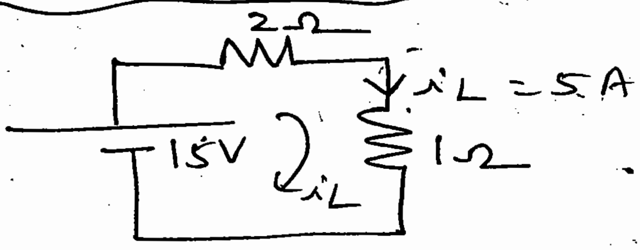
To find R_{th}



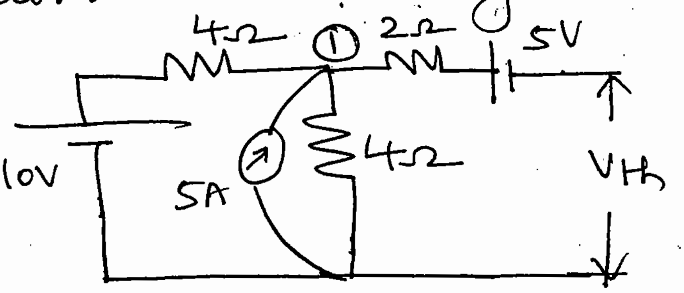
$$R_{th} = (2 \parallel 2) + 1$$
$$= 1 + 1 = 2\Omega$$

$$\therefore i_L = \frac{V_{th}}{R_{th} + R_L} = \frac{15}{2+1} = 5A$$

Equivalent circuit



using Thevenin's theorem find the current through R_L



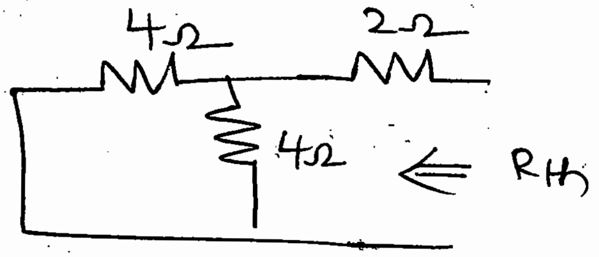
from the figure $V_{th} = V_1 - 5$

apply KCL @ node 1

$$\frac{V_1 - 10}{4} + \frac{V_1}{4} = 5 \Rightarrow V_1 = 15V$$

$$\therefore V_{th} = V_1 - 5 = 10V$$

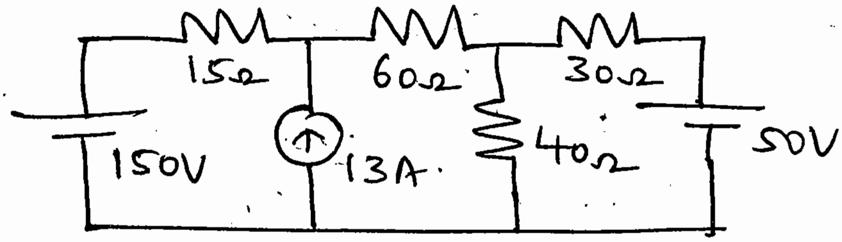
$R_{th} \Rightarrow$



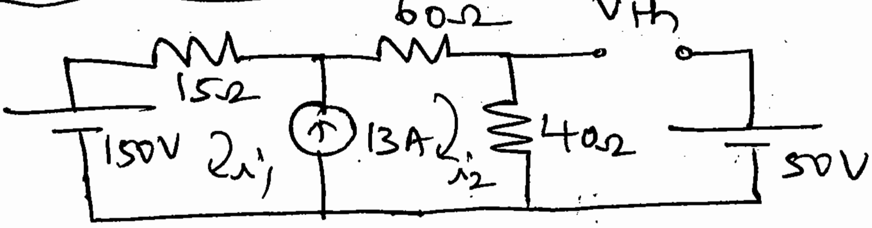
$$R_{th} = (4 \parallel 4) + 2 = 4\Omega$$
$$= 2 + 2$$

$$i_L = \frac{V_{Th}}{R_{Th}+R_L} = \frac{10}{4+6} = 1\text{ A}$$

using Thevenin's theorem find the current through 30Ω resistor



To find V_{Th}



$$i_2 - i_1 = 13 \rightarrow (1)$$

KVL to superloop

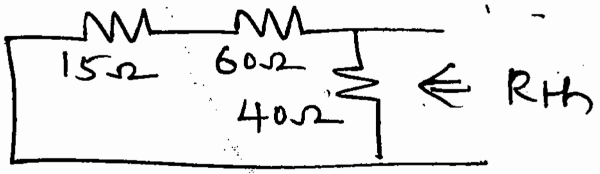
$$150 - 15i_1 - 60i_2 - 40i_2 = 0$$

$$15i_1 + 100i_2 = 150 \Rightarrow (2)$$

$$i_1 = -10\text{ A} ; \quad i_2 = 3\text{ A}$$

$$V_{Th} = 40i_2 - 50 = 40(3) - 50 = 70\text{V}$$

To find R_{Th}



$$R_{Th} = (15+60) \parallel 40$$

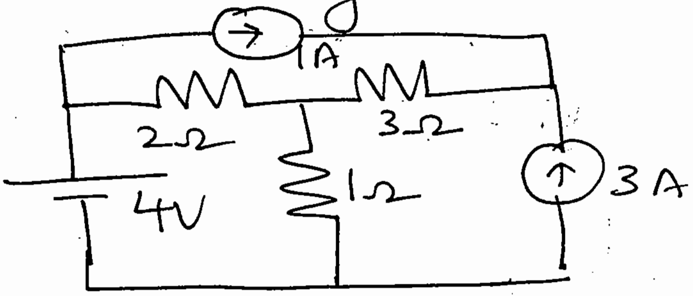
$$= \frac{75 \times 40}{75+40} = 26.08\Omega$$

$$i_L = \frac{V_{Th}}{R_{Th}+R_L}$$

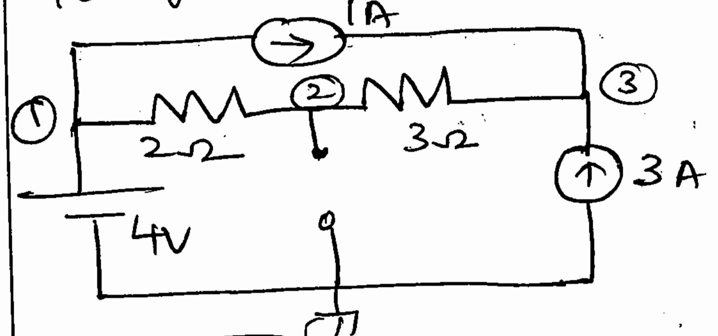
$$= \frac{70}{26.08+30}$$

$$\boxed{i_L = 1.248\text{A}}$$

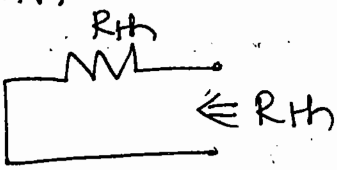
Find the current through 1 Ω resistor using Thevenin's theorem



To find V_{th}



$R_{th} = 2\Omega$



$\boxed{V_1 = 4V}$

KCL @ node 2

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{3} = 0$$

$$V_2(5/6) - \frac{V_3}{3} = 2$$

$$\boxed{0.833 V_2 - 0.333 V_3 = 2} \rightarrow (1)$$

KCL @ node 3

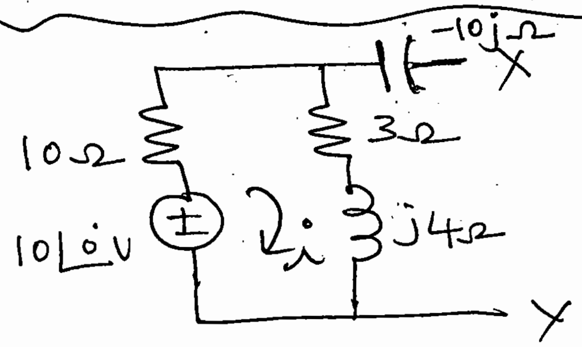
$$\frac{V_3 - V_2}{3} = 1 + 3 \quad \therefore \boxed{-V_2 + V_3 = 12} \rightarrow (2)$$

$$V_2 = 12V, \quad V_3 = 24V$$

$$\therefore \boxed{V_{th} = V_2 = 12V}$$

$$\boxed{i_L = \frac{V_{th}}{R_{th} + R_L} = \frac{12}{2+1} = \frac{12}{3} = 4A}$$

Obtain the Thevenin's equivalent for the circuit shown across the terminals X & Y



as the terminals X & Y are opened no current flows through $-10j\,\Omega$

To find V_{th}

$V_{th} = (3+j4)\,i$

apply KVL to loop ①

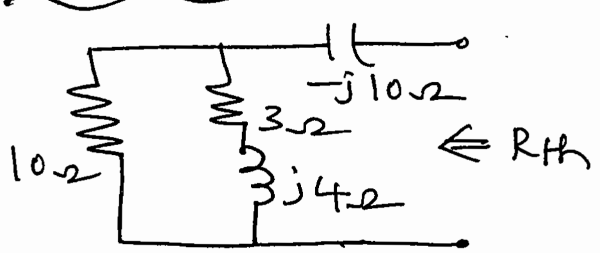
$10 - 10i - (3+j4)\,i = 0$

$i = \dfrac{10}{13+j4} = 0.7352\angle -17.102$ A

$V_{th} = (3+j4)\,(0.7352\angle -17.102)$

$V_{th} = 3.676\angle 36.02$ V

To find R_{th}



$Z_{th} = \left[(3+j4)\,\|\,10\right] - j10$

$= \dfrac{(3+j4)\,10}{13+j4} - j10$

$$Z_{Th} = 8.38 \angle -69.22\,\Omega$$

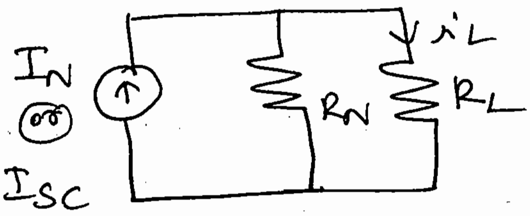
$$i_L = \frac{V_{Th}}{R_{Th}} = \frac{3.676 \angle 36.02}{8.38 \angle -69.22}$$

$$\boxed{i_L = 0.4385 \angle 105.24^\circ \text{ A}}$$

NORTON'S THEOREM

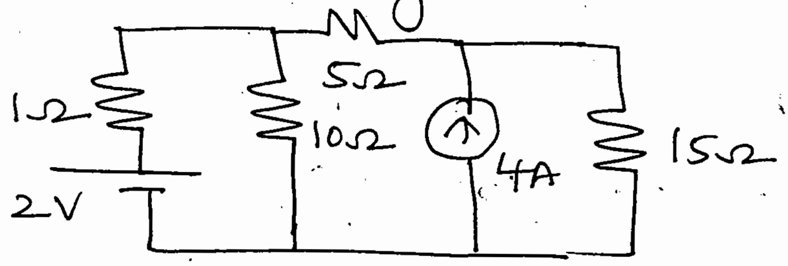
It states that " In any linear, bilateral active network, however it is complicated it is replaced by a single current source I_N or I_{SC}, in parallel with the resistor R_N, where I_N can be determined by replacing the load terminals by a short circuit & R_N can be determined by removing the load resistance replacing voltage sources by short circuit & current sources if any by open circuit, considering internal resistances if any

Norton's equivalent circuit

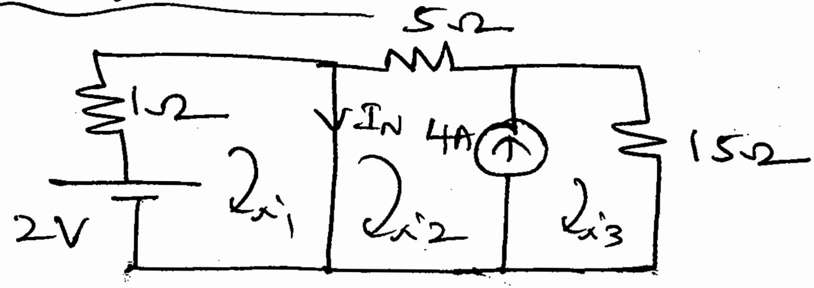


$$i_L = I_N\left(\frac{R_N}{R_N + R_L}\right) \text{amp}$$

using Norton's theorem find the current through 10Ω resistor



To find I_N



$$I_N = I_{SC} = i_1 - i_2$$

applying KVL loop ① $i_1 = \frac{2}{1} = 2A$

loops ② & ③ forms a supermesh

$\therefore$ $i_3 - i_2 = 4 \rightarrow$ ①

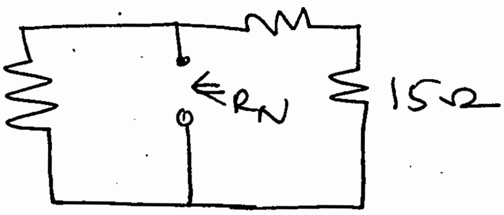
KVL to superloop

$5i_2 + 15i_3 = 0 \rightarrow$ ②

$\boxed{i_2 = -3A}$, $\boxed{i_3 = 1A}$

$$\boxed{I_N = 2-(-3) = 5A}$$

To find R_N



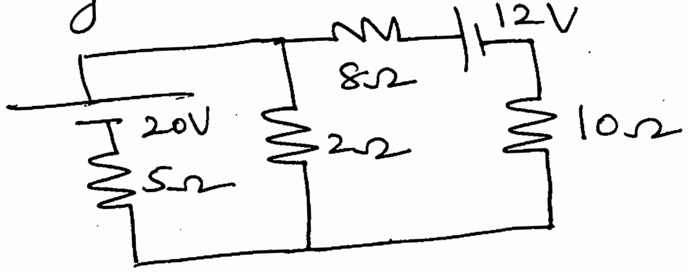
$R_N = (5+15) \parallel 1$

$= \frac{20 \times 1}{21}$

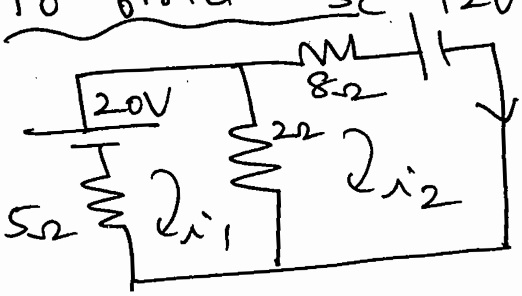
$= 0.952\Omega$

$$I_L = \frac{I_N \cdot R_N}{R_N + R_L} = \frac{5(0.952)}{10 + 0.952} = 0.4347\,A$$

Find the current through 10Ω resistor using Norton's theorem



To find I_{SC}



KUL to loop ①

$$7i_1 - 2i_2 = 20 \rightarrow ①$$

loop ②

$$-2i_1 + 10i_2 = -12 \rightarrow ②$$

$i_1 = 2.666$ amperes

$i_2 = -0.666$ amperes

$\therefore$ $\boxed{I_N = i_2 = -0.666 \text{ amperes}}$

To find R_N

$$R_N = (5 \| 2) + 8$$

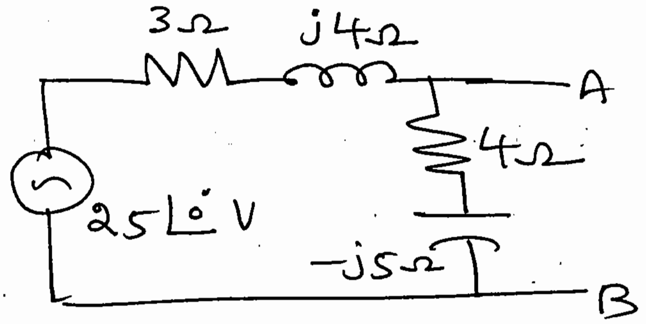
$$= \frac{10}{7} + 8$$

$$\boxed{R_N = 9.4285\,\Omega}$$

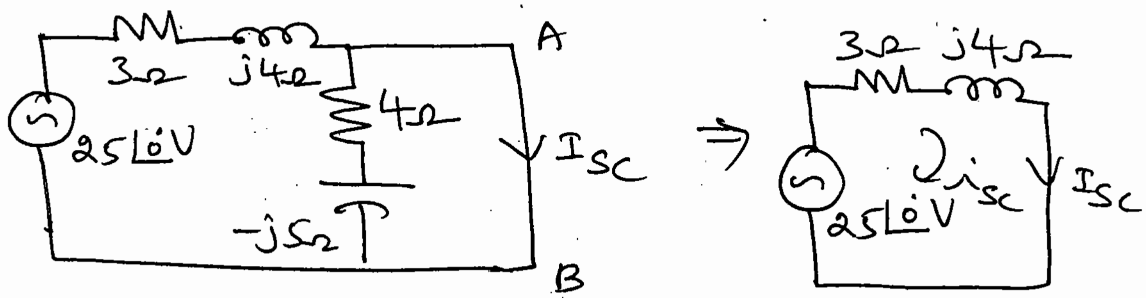
$$i_L = \frac{I_N \cdot R_N}{R_N + R_L} = \frac{(-0.666) \times 9.428}{19.4285}$$

$$\boxed{i_L = -0.32369 \text{ Amperes}}$$

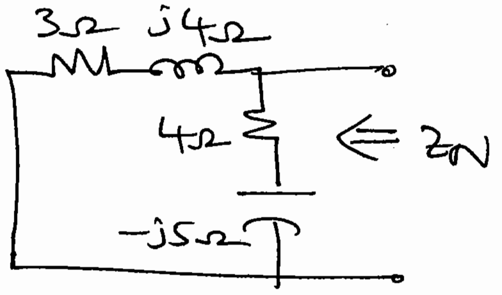
Obtain the Norton's equivalent for the network shown



To find I_{sc}

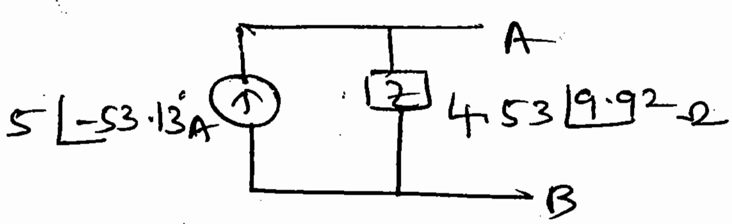


$$I_{sc} = \frac{25\angle 0}{3+j4} = \frac{25\angle 0}{5\angle 53.13} = 5\angle -53.13^\circ \text{ A}$$

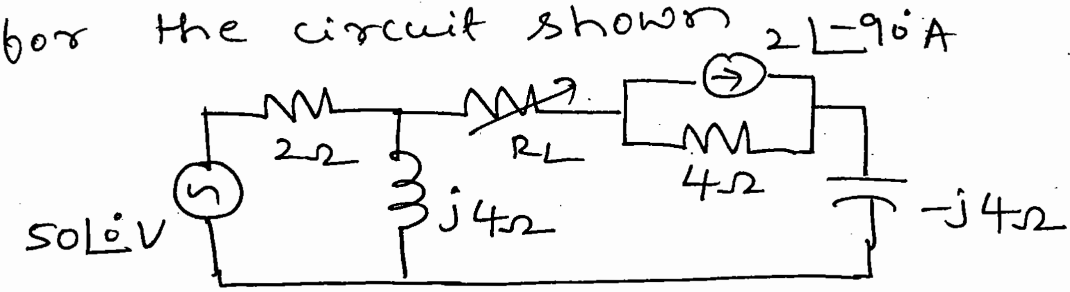


$$Z_N = \frac{(3+j4)(4-j5)}{3+j4+4-j5} = \frac{5\angle 53.13 \times 6.4\angle -51.34}{7.07\angle -8.13}$$

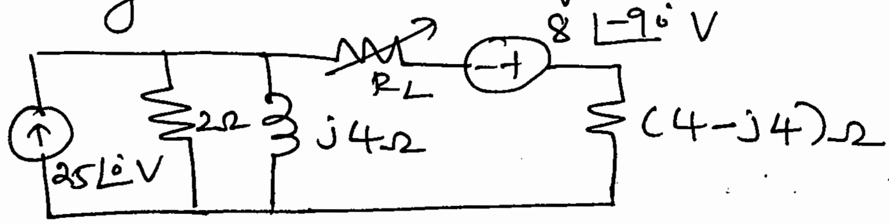
$$\boxed{Z_N = 4.53\angle 9.92}$$

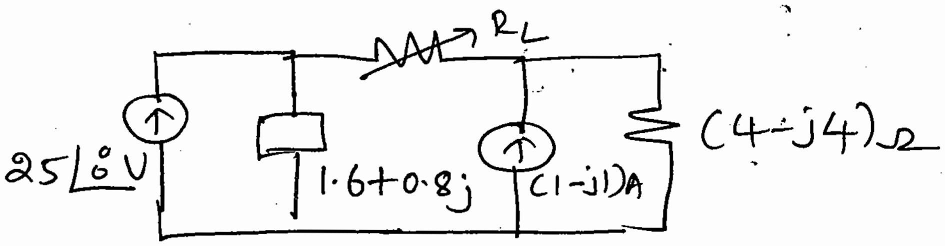


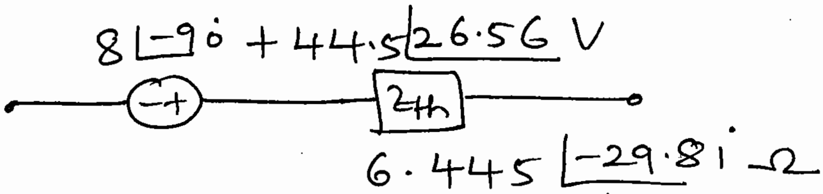
Q10) Obtain the Thevenin's equivalent 32
for the circuit shown

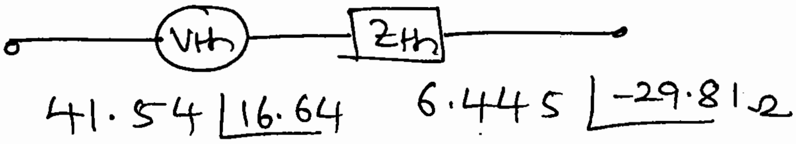


using source transformation

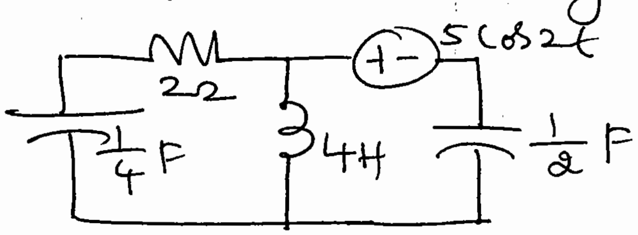








34 Determine the current through 4H for the circuit shown using Norton's theorem



Representing the given circuit in frequency domain [ω = 2 rad/sec]

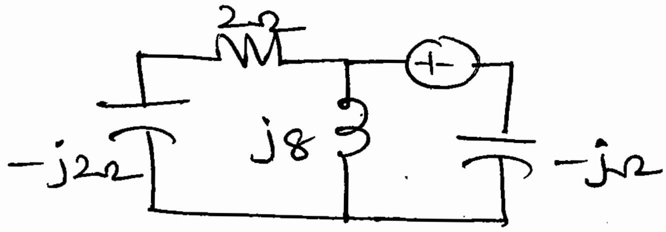
$X_L = j\omega L = j\,2(4) = j\,8\,\Omega$

$X_C = \dfrac{1}{j\omega C} = \dfrac{-j}{2\left(\frac{1}{4}\right)} = -j\,2\,\Omega$

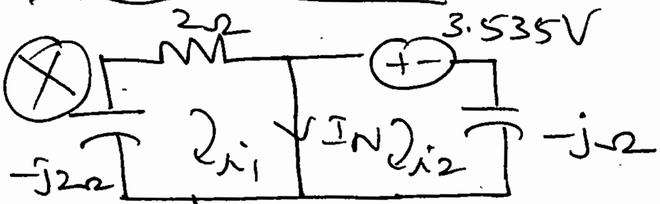
$X_C = \dfrac{-j}{2(1/2)} = -j\,\Omega$

$V = V_m \cos \omega t$

$V_{rms} = \dfrac{V_m}{\sqrt{2}} = \dfrac{5}{\sqrt{2}} = 3.535\,V$



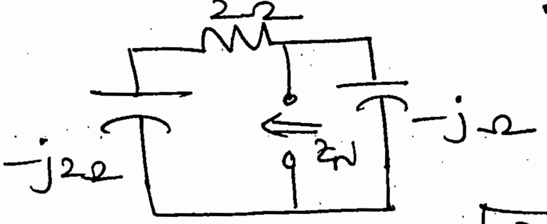
to find I_{sc}



shorted

$I_N = i_2 = \dfrac{3.535}{-j1} = 3.535\,\angle 90^\circ\,A$

TO find Z_N



$Z_N = (2-j2)\,||\,(-j\,\Omega)$

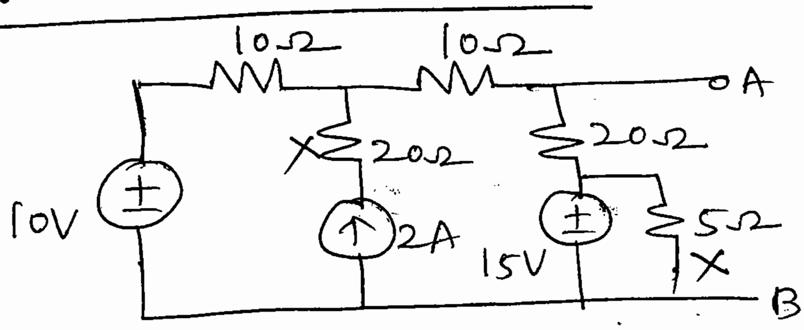
$= \dfrac{(2-j2)(-j)}{2-j2-j}$

$\boxed{Z_N = 0.7844\,\angle -78.69^\circ\,\Omega}$

$I_L = \dfrac{I_N \cdot Z_N}{Z_N + Z_L} = \dfrac{(3.53\angle 90)(0.7844\angle -78.69^\circ)}{0.7844\angle -78.69 + 8j}$

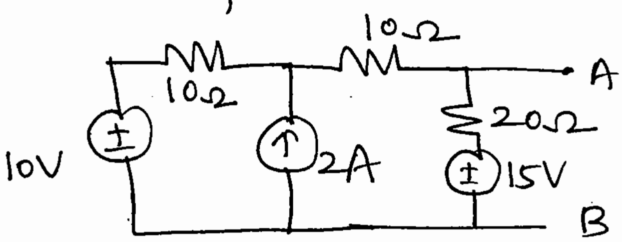
$\boxed{I_L = 0.3828\,\angle -77.47^\circ\,A}$

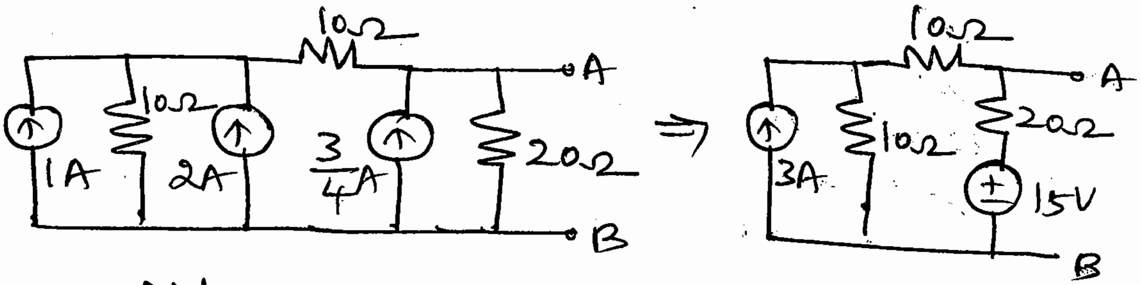
obtain the thevenin's equivalent for the circuit shown

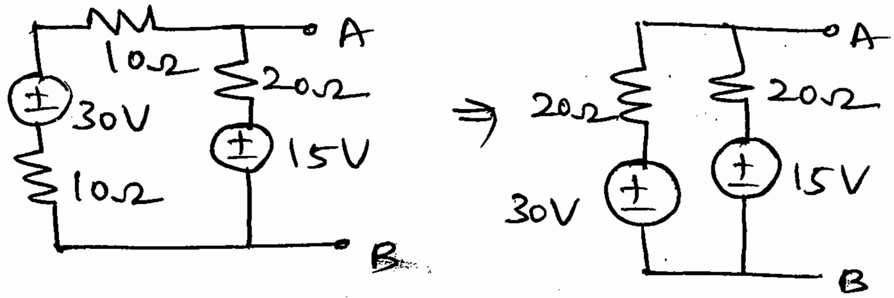


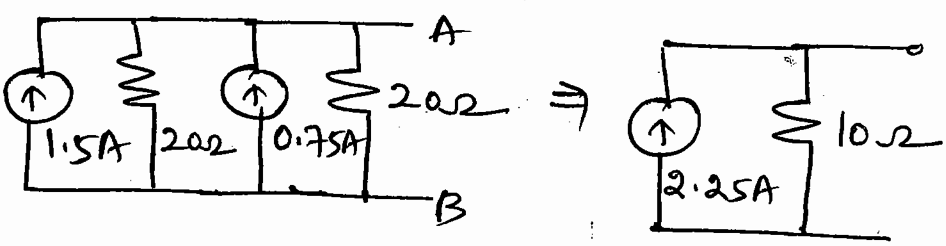
20Ω & 5Ω are redundant elements

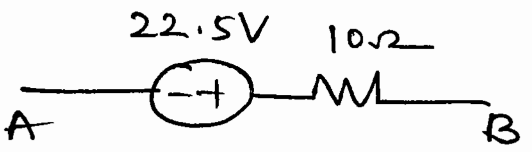
using source transformation





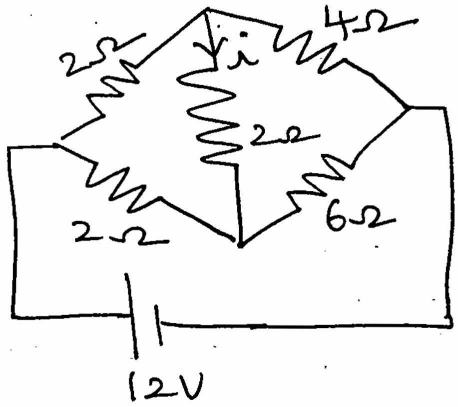




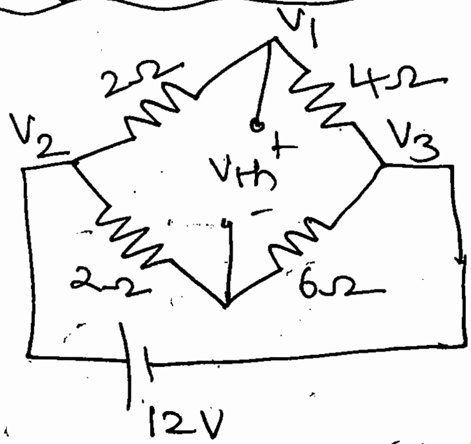


Thevenin's equivalent circuit

Determine the current through 2Ω resistor



To find V_{th}



Nodes ② & ③ forms a supernode

$$\boxed{V_2 - V_3 = 12} \rightarrow ①$$

KCL @ supernode

$$\frac{V_2}{2} + \frac{V_2 - V_1}{2} + \frac{V_3}{6} + \frac{V_3 - V_1}{4} = 0$$

$$\left(-\frac{3}{4}\right)V_1 + V_2 + V_3\left(\frac{5}{12}\right) = 0$$

$$\boxed{-0.75\,V_1 + V_2 + 0.4166\,V_3 = 0} \rightarrow ②$$

KCL @ node 1

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = 0$$

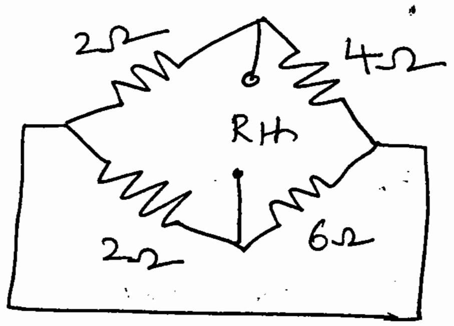
$$\left(\frac{3}{4}\right)V_1 - \frac{V_2}{2} - \frac{V_3}{4} = 0$$

$$\boxed{0.75\,V_1 - 0.5\,V_2 - 0.25\,V_3 = 0} \rightarrow ③$$

$V_1 = -1V, \quad V_2 = 3V, \quad V_3 = -9V$

$$\therefore \boxed{V_{th} = V_1 = -1\,V}$$

To find R_{th}



$$R_{th} = (2 \| 4) + (2 \| 6)$$

$$= \frac{2 \times 4}{2+4} + \frac{2 \times 6}{2+6}$$

$$= \frac{17}{6} = 2.833\Omega$$

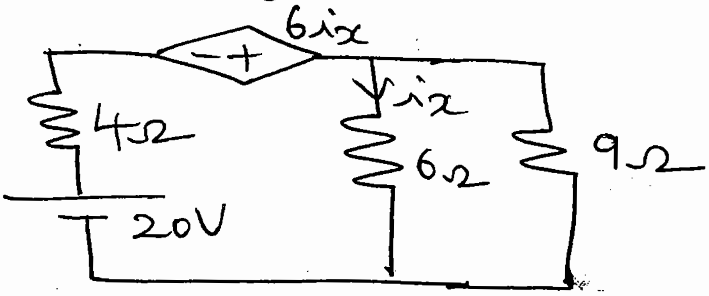
$$i_L = \frac{V_{th}}{R_{th} + R_L} = \frac{-1}{2.833 + 2} = -0.20689\,A$$

Thevenin's and Norton's theorem for the circuit with dependent sources

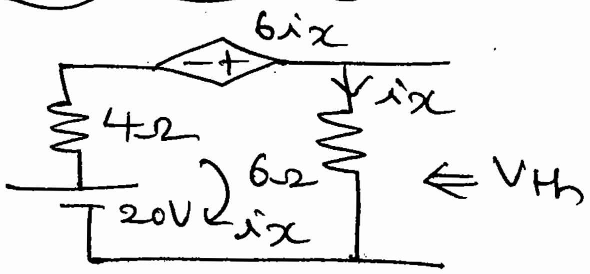
1) Identify and remove the load resistance
2) look in to the load terminals & calculate V_{th}
3) place a short circuit on the load terminals and calculate the Norton's current & determine $R_{th} = \dfrac{V_{th}}{I_{sc}} = \dfrac{V_{th}}{I_N}$

4) The load current $I_L = \dfrac{V_{th}}{R_{th} + R_L}$

Determine the current through 9Ω resistor using Thevenin's theorem



To find V_{Th}



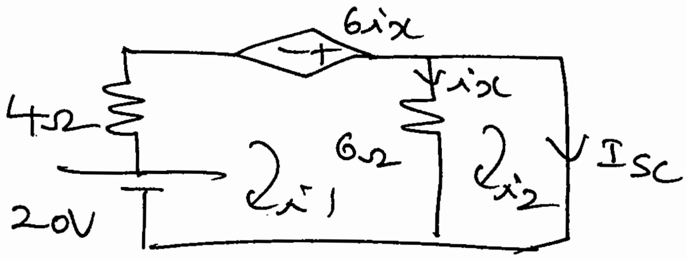
$V_{Th} = 6 i_x$

KVL to loop

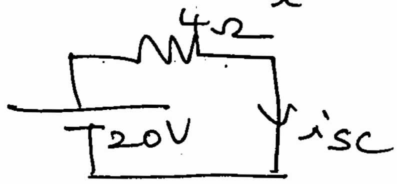
$$20 - 4 i_x + 6 i_x - 6 i_x = 0$$

$\boxed{i_x = 5\,A}$ $\therefore$ $\boxed{V_{Th} = 5 \times 6 = 30V}$

to find R_{Th} calculate I_{sc}



6Ω is in parallel with short circuit branch $I_x = 0$ $\therefore$ $6 i_x = 0$

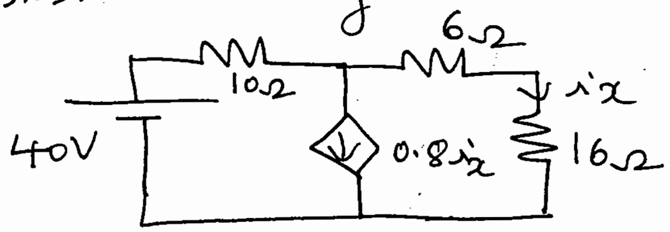


$$i_{sc} = \frac{20}{4} = 5A$$

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{30}{5} = 6\Omega$$

$$i_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{6+9} = 2A$$

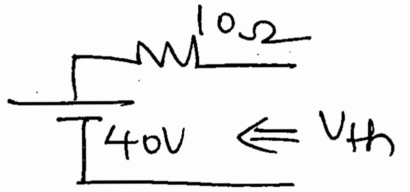
Determine the current through 16Ω 35 resistor using Thevenin's theorem



To find V_{th} remove R_L $\therefore i_x = 0$

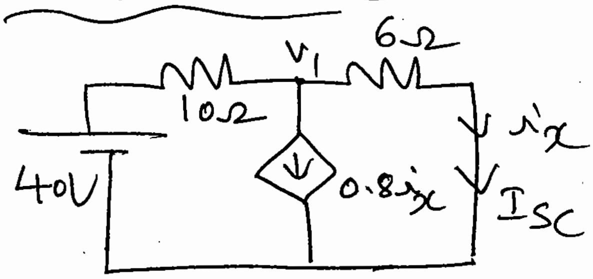
since $i_x = 0$ $0.8 i_x$ is also zero

$\therefore$ replace current source $(0.8 i_x)$ by zero



$V_{th} = 40V$

To find I_{sc}



KCL @ node 1

$$\frac{V_1 - 40}{10} + \frac{V_1}{6} = -0.8 i_x$$

$$\frac{V_1 - 40}{10} + \frac{V_1}{6} = -0.8 i_x$$

$$i_{sc} = i_x = \frac{V_1}{6}$$

$$\frac{V_1}{10} + \frac{V_1}{6} + \frac{4 V_1}{30} = 4$$

$$V_1 = \frac{4}{\frac{1}{10} + \frac{1}{6} + \frac{4}{30}}$$

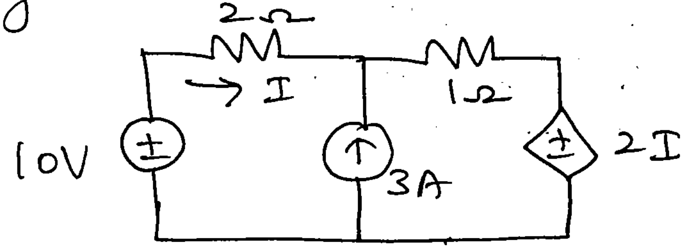
$\therefore V_1 = 10V$

$$i_{sc} = \frac{V_1}{6} = \frac{10}{6} = 1.667 A$$

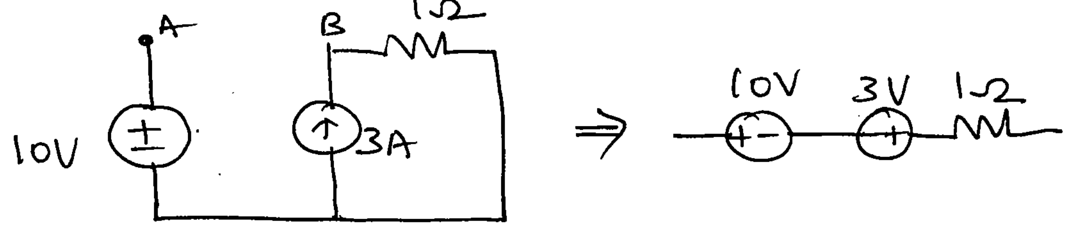
$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{40}{1.667} = 24Ω$$

$$i_L = \frac{V_{th}}{R_{th} + R_L} = \frac{40}{24+16} = 1 A$$

V.V Imp For the circuit shown obtain the value of the current I by using Thevenin's theorem

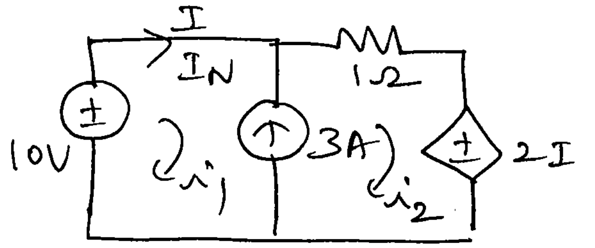


To find V_{th}



$$\boxed{V_{th} = 7V}$$

To find I_{sc}



$$\boxed{I = i_1 = I_N}$$

Loops ① & ② forms a superloop

$i_2 - i_1 = 3 \rightarrow ①$

KUL to superloop

$$\boxed{I_{sc} = i_1 = 2.33A}$$

$$10 - i_2 - 2i_1 = 0$$

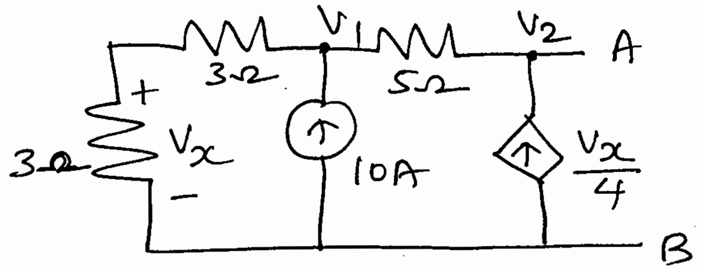
$$2i_1 + i_2 = 10$$

$$i_1 = 2.33A \;;\; i_2 = 5.33A$$

$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{7}{2.33} = 3\Omega$$

$$i_L = \frac{V_{th}}{R_{th} + R_L} = \frac{7}{3+2} = 1.4A$$

Find the Thevenin's equivalent for the network shown



using voltage divider rule

$$V_x = \frac{V_1(3)}{3+3} = \frac{3V_1}{6} = \frac{V_1}{2}$$

Apply KCL to the nodes

KCL @ node 1

$$\frac{V_1}{6} + \frac{V_1 - V_2}{5} = 10$$

$$V_1\left(\frac{1}{6}+\frac{1}{5}\right) - \frac{1}{5}V_2 = 10$$

$$V_1(0.1666 + 0.2)$$

$$\boxed{0.3666V_1 - 0.2V_2 = 10} \Rightarrow (1)$$

KCL @ node 2

$$\frac{V_2 - V_1}{5} = \frac{V_x}{4}$$

$$\frac{V_2 - V_1}{5} = \frac{V_x}{4} \Rightarrow -\left(\frac{1}{5}+\frac{1}{8}\right)V_1 + \frac{V_2}{5} = 0$$

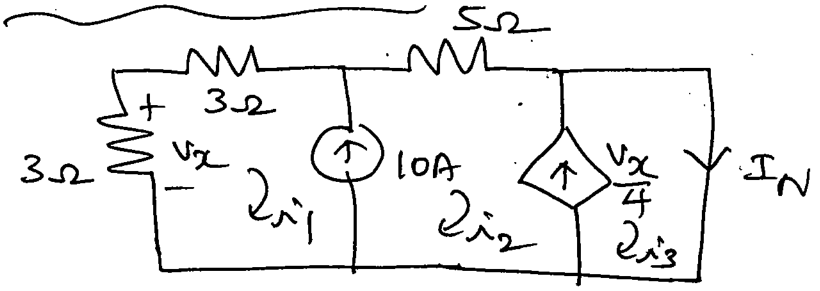
$$-(0.2 + 0.125)V_1 + 0.2V_2 = 0$$

$$\boxed{-0.325V_1 + 0.2V_2 = 0} \Rightarrow (2)$$

$V_1 = 240V$, $V_2 = 390V$

$V_{Th} = 390V$

To find I_{sc}



$$\boxed{V_x = -3i_1}$$

Loops ① & ② forms a supermesh

$$i_2 - i_1 = 10 \rightarrow ①$$

loops ② & ③ forms a supermesh

$$i_3 - i_2 = \frac{V_x}{4}$$

$$i_3 - i_2 + \frac{3}{4} i_1 = 0 \rightarrow ②$$

KVL to supermesh

$$6i_1 + 5i_2 = 0 \rightarrow ③$$

$\boxed{i_1 = -4.54A}$ $\boxed{i_2 = 5.45A}$ $\boxed{i_3 = 8.86A}$

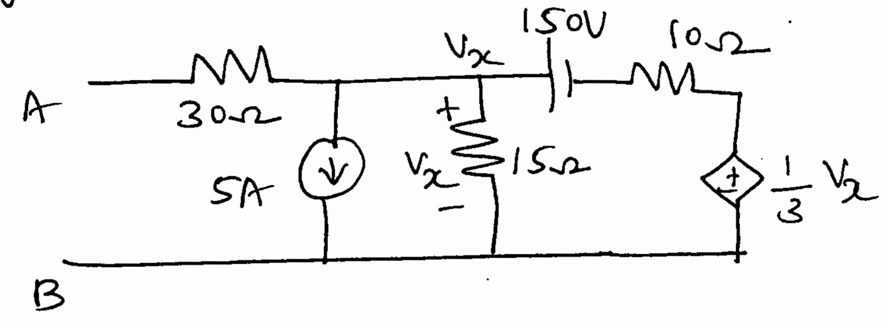
$$\boxed{I_N = i_3 = 8.86 \text{ Amp}}$$

$$R_{Th} = \frac{V_{Th}}{I_N} = 44\Omega$$

$$i_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{390}{44} = 8.86A$$

$$\boxed{i_L = 8.86A}$$

obtain the Thevenin's equivalent for the circuit shown



To find V_{Th} :- As the terminals A & B are open no current flows through 30Ω ∴ applying KCL @ node V_x

$$\frac{V_x}{15} + \frac{V_x - 150 - \frac{1}{3}V_x}{10} = -5$$

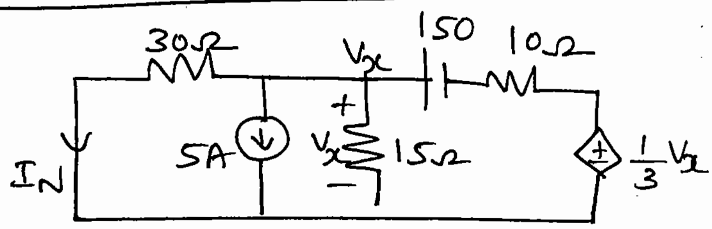
$$\frac{V_x}{15} + \frac{2}{30}V_x = -5 + 15$$

$$0.0666V_x + 0.06666\,V_x = 10$$

$$0.1333\,V_x = 10$$

$$\therefore \boxed{V_x = \frac{10}{0.1333} = 75\,V}$$

To find I_{sc}



KCL @ node

$$\frac{V_x}{15} + \frac{V_x}{30} + \frac{V_x - 150 - \frac{1}{3}V_x}{10} = -5$$

$$\frac{V_x}{15} + \frac{V_x}{30} + \frac{2}{30}V_x = -5 + 15$$

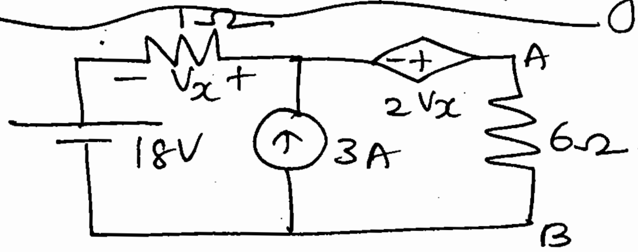
$$\frac{5V_x}{30} = 10 \Rightarrow \boxed{V_x = \frac{300}{5} = 60V}$$

$$I_{sc} = \frac{V_x}{I_{sc}} = \frac{60}{30} = 2A$$

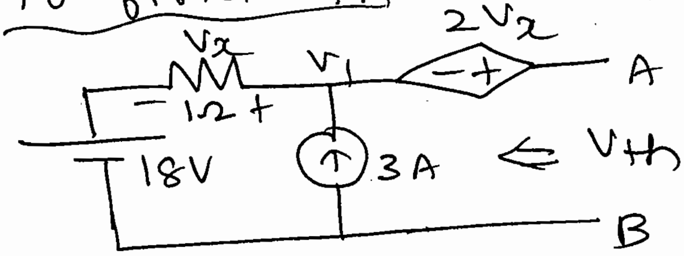
$$\therefore R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{75}{2} = 37.5\Omega$$

$$\boxed{R_{Th} = 37.5\Omega}$$

Obtain the Thevenin's equivalent & determine the current through 6Ω resistor



To find V_{Th}



$$\boxed{V_{Th} = V_1 + 2V_x}$$

KCL @ node ①

$$V_1 - 18 = 3$$

$$\boxed{V_1 = 21V}$$

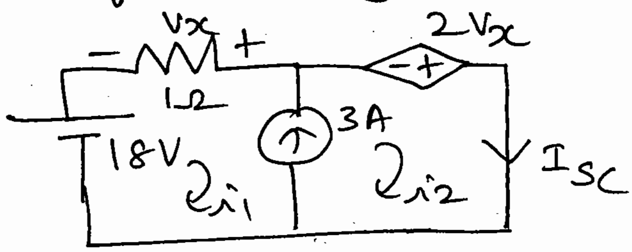
$$V_x = V_1 - 18 = 21 - 18 = 3V$$

$$\boxed{V_x = 3V}$$

$$\therefore V_{Th} = 21 + 2(3) = 27V$$

$$\boxed{V_{Th} = 27\,\text{Volts}}$$

To find I_{sc}



$$V_x = -i_1$$

loops 1 & 2 forms a super loop

$$\boxed{i_2 - i_1 = 3} \rightarrow 1$$

KVL to super loop

$$18 - i_1 + 2V_x = 0$$

$$18 - i_1 - 2i_1 = 0$$

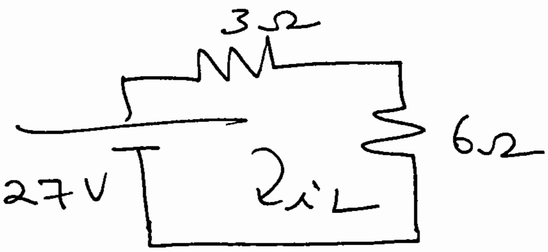
$$\boxed{i_1 = 6\,A}$$

$$\therefore i_2 = i_1 + 3 = 3+6 = 9\,A$$

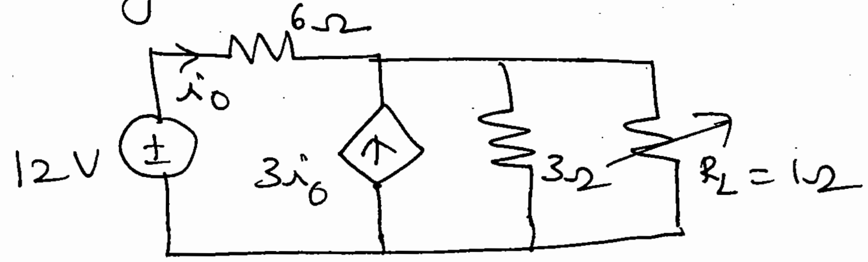
$$\boxed{i_2 = i_{sc} = 9\,A}$$

$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{27}{9} = 3\Omega$$

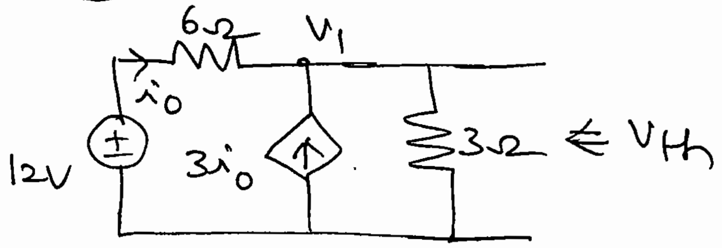
$$i_L = \frac{V_{th}}{R_{th} + R_L} = \frac{27}{3+6} = 3\,A$$



Find the current through R_L using Norton's theorem



To find V_{Th}



from the figure $V_{Th} = V_1$

KCL @ node V_1

$$\frac{V_1 - 12}{6} + \frac{V_1}{3} = -3i_0$$

$$\boxed{i_0 = \frac{V_1 - 12}{6}}$$

$$\frac{V_1 - 12}{6} + \frac{V_1}{3} = -\frac{V_1}{2} + 6$$

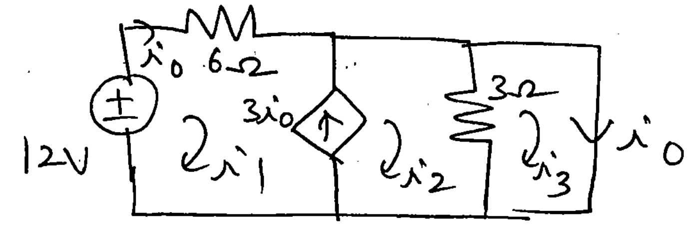
$$\frac{V_1}{6} + \frac{V_1}{3} + \frac{V_1}{2} = 8$$

$$\therefore V_1 [0.5 + 0.33 + 0.166] = 8$$

$$V_1 [1] = 8$$

$$\boxed{V_1 = 8\,V}$$

To find I_N



$\boxed{i_0 = i_1}$

loops ① & ② forms a superloop

$$i_2 - i_1 = 3i_0$$

$$i_2 - i_1 = 3i_1 \quad \therefore \boxed{-4i_1 + i_2 = 0} \Rightarrow ①$$

KVL to super mesh

$$12 - 6i_1 - 3(i_2 - i_3) = 0$$

$$\boxed{6i_1 + 3i_2 - 3i_3 = 12} \rightarrow ②$$

$$3(i_3 - i_2) = 0$$

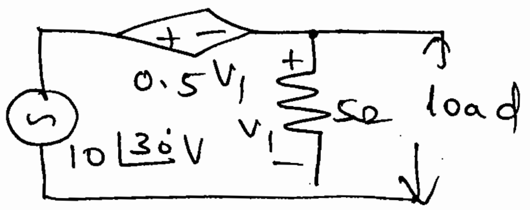
$$\boxed{-i_2 + i_3 = 0 \rightarrow ③}$$

$$i_1 = 2A, \quad i_2 = 8A, \quad i_3 = 8A$$

$$I_{SC} = i_3 = 8A \quad \therefore \quad R_{Th} = \frac{V_{Th}}{I_{SC}} = \frac{8}{8} = 1\Omega$$

$$i_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{8}{1+1} = 4A$$

Find the Thevenin's equivalent for the circuit shown

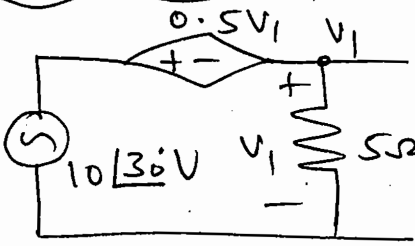


by apply KVL

$$-0.5V_1 - 5I_1 + 10\angle 30^\circ = 0$$

$$\boxed{V_1 = 5I_1}$$

To find V_{th}



$$-0.5 \times 5I_1 - 5I_1 + 10\angle 30^\circ = 0$$

$$-7.5I_1 = -10\angle 30^\circ$$

$$\boxed{I_1 = 1.33\angle 30^\circ \text{ A}}$$

$$\boxed{V_{th} = V_1 = 5I_1}$$

$$\boxed{V_{th} = 5 \times 1.333\angle 30 = 6.66\angle 30^\circ \text{ V}}$$

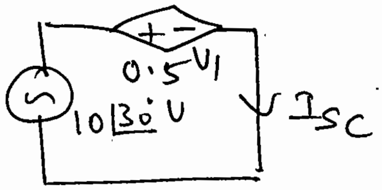
$V_{th} = V_1$

or

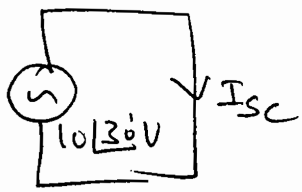
$$V_1 + 0.5V_1 - 10\angle 30 = 0$$

$$V_1 = \frac{10\angle 30}{1.5} = 6.667\angle 30^\circ = V_{th}$$

to find I_N



⇒



$$I_N = \frac{10\angle 30^\circ}{0} = \infty$$

$$Z_{th} = \frac{V_{th}}{I_{sc}} = \frac{6.667\angle 30^\circ}{\infty} = 0\,\Omega$$

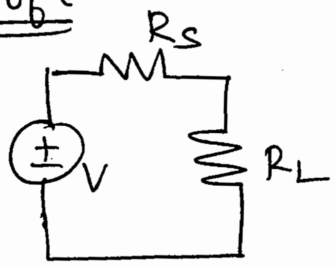
MAXIMUM POWER TRANSFER THEOREM

For DC circuit

maximum power transfer theorem states that a maximum power will be delivered from the source to the load when the Load resistance is equal to the source resistance

ie, $R_L = R_{Th} = R_S = R_{eq}$

Proof:-



$$I = \frac{V}{R_S + R_L}$$

$$P = I^2 \cdot R_L$$

$$P = \frac{V^2 \cdot R_L}{(R_S + R_L)^2} \rightarrow (1)$$

The condition for the maximum power is $\frac{dP}{dR_L} = 0$

$$\frac{dP}{dR_L} = \frac{V^2 (R_S + R_L)^2 - V^2 R_L \times 2(R_S + R_L)}{(R_S + R_L)^4}$$

$$\frac{du}{dv} = \frac{v \cdot \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

for $\frac{dP}{dR_L} = 0$ Numerator term should be equal to zero

$\therefore \quad V^2(R_S+R_L)^2 - 2V^2R_L(R_S+R_L) = 0$

or $\quad 2\cancel{V^2}R_L\cancel{(R_S+R_L)} = \cancel{V^2}(R_S+R_L)^{\cancel{2}}$

$$2R_L = R_S + R_L$$

$$2R_L - R_L = R_S$$

$$\therefore \boxed{R_L = R_S}$$

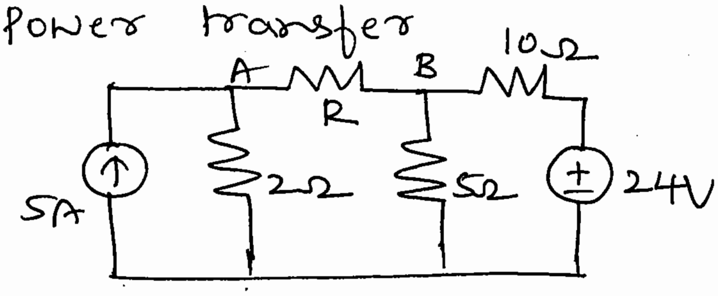
From (1)

$$P = \frac{V^2}{(R_S+R_S)^2} \times R_S$$

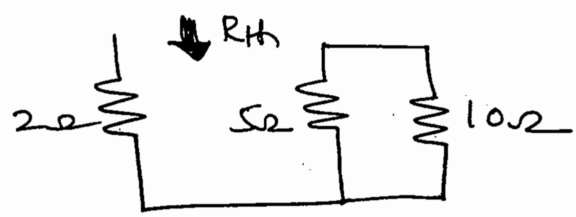
$$P = \frac{V^2}{4R_S} = \frac{V^2}{4R_L}$$

Exp

Determine the value of R for the circuit shown & find the maximum power transfer



Soln:- $\quad R_L = R_{th}$



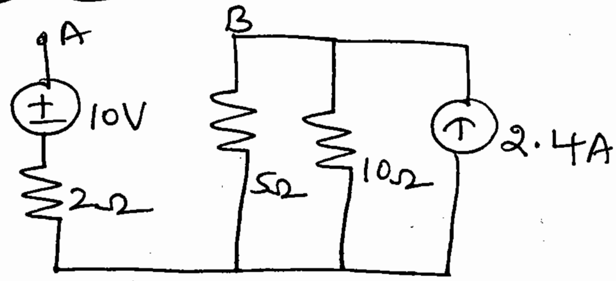
$$R_{th} = \frac{10 \times 5}{10+5} + 2$$

$$= \frac{50}{15} + 2$$

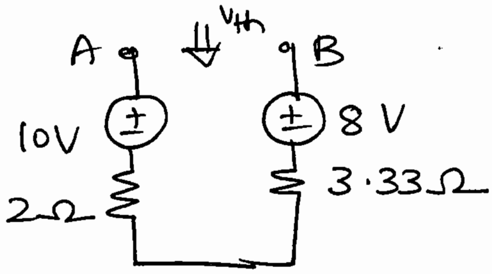
$$R_{th} = 3.33 + 2$$

$$= 5.33\,\Omega$$

To find V_{th}



$10 \parallel 5 = 3.33\Omega$



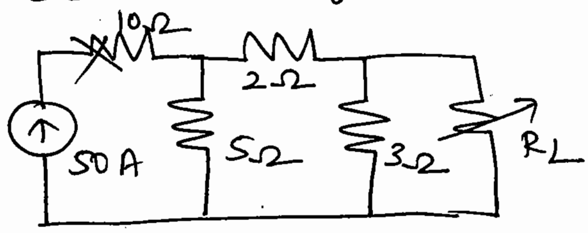
$V_{th} = 10 - 8 = 2V$

$$i_L = \frac{V_{th}}{R_{th} + R_L} = \frac{2}{5.33 + 5.33} = \frac{2}{10.66}$$

$$\boxed{i_L = 0.1876 \text{ Ampere}}$$

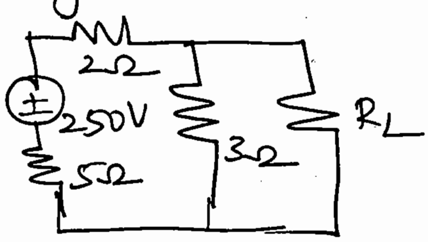
$$\therefore \boxed{P = i_L^2 \cdot R_L = 0.1877 \text{ Watts}}$$

Find the value of R_L & find the maximum power delivered for the circuit shown

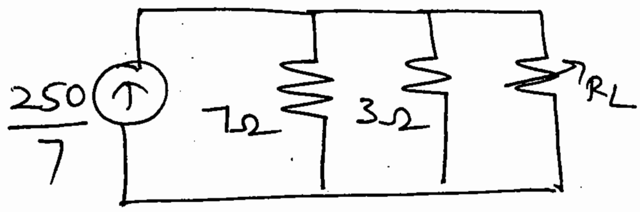


10Ω ⇒ Redundant element

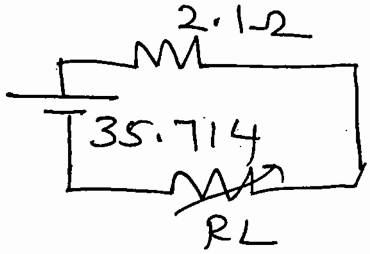
using source transformation

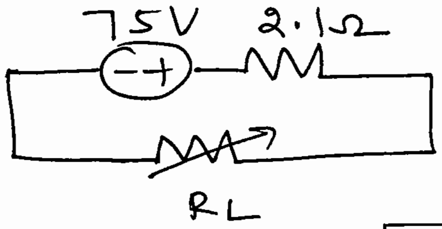


⇒



$$7 \parallel 3 = \frac{7 \times 3}{7+3} = 2.1\Omega$$





$\boxed{R_{Th} = 2.1\Omega}$, $\boxed{V_{Th} = 75V}$

$\therefore \; i_L = \dfrac{V_{Th}}{2R_{Th}} = 17.857\,A$

$P = i_L^2 \cdot R_L$

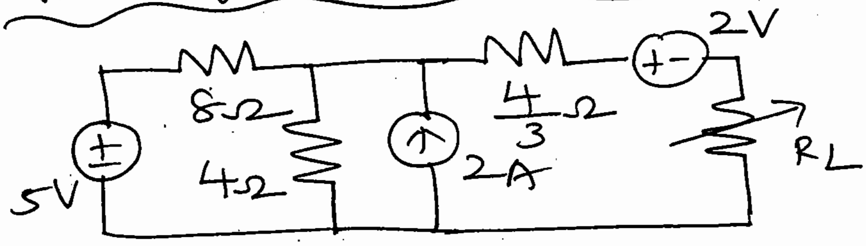
$= (17.857)^2 \times 2.1$

$= 318.87 \times 2.1 = 669.63$ watts

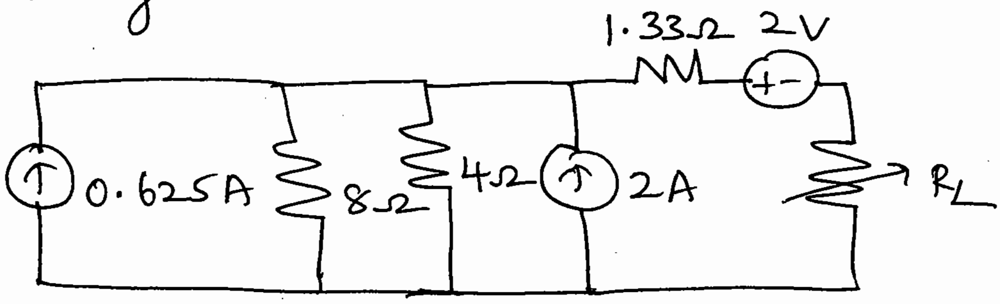
$\boxed{P = 669.63 \text{ watts}}$

Imp

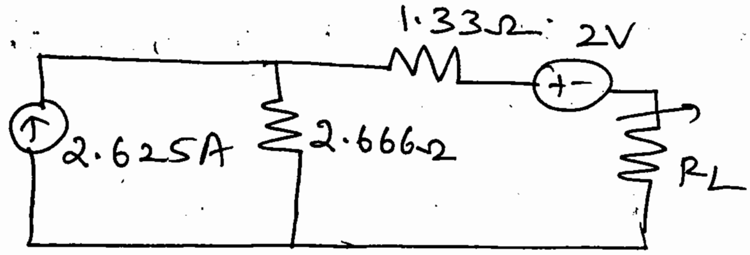
Q
using maximum power transfer theorem find the value of R_L and find maximum power

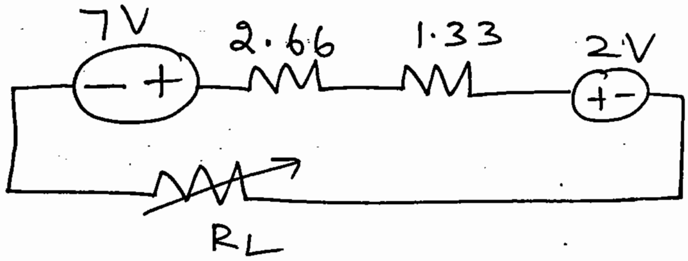


using source transformation

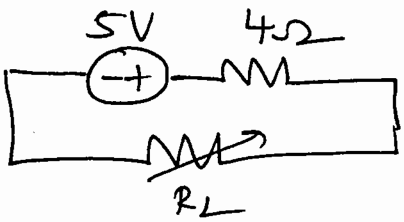


$\dfrac{8 \times 4}{8+4} = \dfrac{32}{12} = 2.666$





2 voltage sources have same polarity hence subtract $7 - 2 = 5V$



$$\boxed{V_{th} = 5V}$$

$$\boxed{R_{th} = 4\Omega}$$

$$i_L = \frac{V_{th}}{R_{th} + R_L} = \frac{5}{4+4} = \underline{0.625A}$$

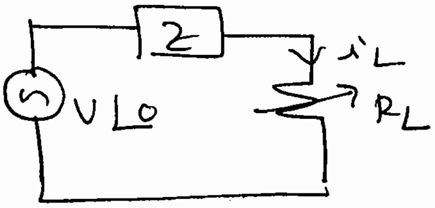
$$\boxed{P = i_L^2 \cdot R_L}$$

$$= (0.625)^2 \times 4 = \underline{0.3906 \times 4}$$

$$\boxed{P = 1.5625 \text{ Watts}}$$

Maximum power transfer theorem for ac source

Case (i) ⇒ AC source, source impedance being complex and load impedance being a pure resistive varying



$Z = R_0 + jX_0$

The current through R_L is

$$i_L = \frac{V\angle 0}{(R_0+R_L)+jX_0}$$

Since the power cannot be imaginary considering only the magnitude

$$|i_L| = \frac{V}{\sqrt{(R_0+R_L)^2+X_0^2}} \rightarrow (1)$$

but $P = |i_L|^2 \cdot R_L$

$$P = \frac{V^2 \cdot R_L}{(R_0+R_L)^2+X_0^2} \rightarrow (2)$$

differentiate (2) w.r.t R_L

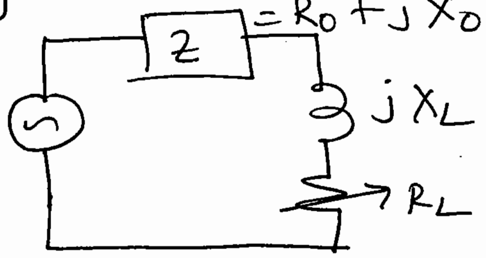
$$\frac{dP}{dR_L} = \frac{V^2\left[(R_0+R_L)^2+X_0^2\right]-V^2\left[R_L\left(2(R_0+R_L)\right)\right]}{\left[(R_0+R_L)^2+X_0^2\right]^2}$$

$$= V^2\left[R_0^2+R_L^2+\cancel{2R_0R_L}+X_0^2-\cancel{2R_0R_L}-2R_L^2\right]$$

$$R_0^2 - R_L^2 + X_0^2 = 0$$

$$R_L^2 = R_0^2 + X_0^2 \quad \therefore \boxed{R_L = |R_0 + jX_0|}$$

Case 2 ⇒ AC source, complex source impedance and complex load impedance only resistor load varying



$$i_L = \frac{V\angle 0}{R_o + jX_o + R_L + jX_L}$$

$$|i_L| = \frac{V}{(R_o+R_L)^2 + (X_o+X_L)^2}$$

$$\therefore P = |i_L|^2 . R_L = \frac{V^2 . R_L}{\left[(R_o+R_L)^2 + (X_o+X_L)^2\right]^2} \Rightarrow (1)$$

differentiating (1) w.r.t R_L ⇒

$$\frac{dp}{dR_L} = V^2\left[\frac{(R_o+R_L)^2 + (X_o+X_L)^2 - R_L\{2(R_o+R_L)\}}{\left[(R_o+R_L)^2 + (X_o+X_L)^2\right]^4}\right]$$

$$\frac{dp}{dR_L} = V^2\left\{\frac{R_o^2 + R_L^2 + \cancel{2R_oR_L} + X_o^2 + X_L^2 + 2X_oX_L - \cancel{2R_oR_L} - 2R_L^2}{\left[(R_o+R_L)^2 + (X_o+X_L)^2\right]^4}\right\}$$

$$R_o^2 - R_L^2 + X_o^2 + X_L^2 + 2X_oX_L = 0$$

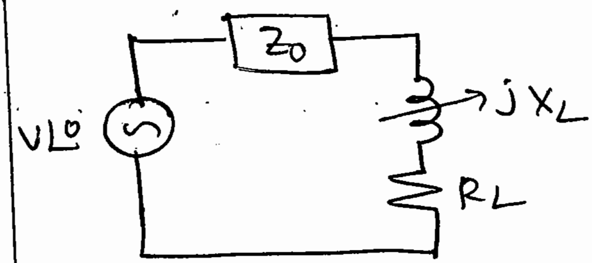
$$R_o^2 - R_L^2 + (X_o + X_L)^2 = 0$$

$$R_L^2 = R_o^2 + (X_L + X_o)^2$$

$$R_L = |R_o + j(X_L + X_o)|$$

$$\boxed{R_L = |Z_o + jX_L|\ \Omega}$$

Case 3 ⇒ AC source, complex source impedance, complex load impedance but only Load reactance varying



$$Z_0 = R_0 + jX_0$$

$$i_L = \frac{V\angle 0}{R_0 + jX_0 + R_L + jX_L}$$

$$|i_L| = \frac{V}{(R_0+R_L)^2 + (X_0+X_L)^2}$$

but $P = |i_L|^2 \cdot R_L$

$$P = \frac{V^2 \cdot R_L}{[(R_0+R_L)^2 + (X_0+X_L)^2]^2} \rightarrow (1)$$

differentiating (1) w.r.t. X_L

$$\frac{dP}{dX_L} = V^2 \left[\frac{(R_0+R_L)^2 + (X_0+X_L)^2 - R_L\{2(X_0+X_L)\}}{\{(R_0+R_L)^2 + (X_0+X_L)^2\}^4} \right]$$

(No X_L term)

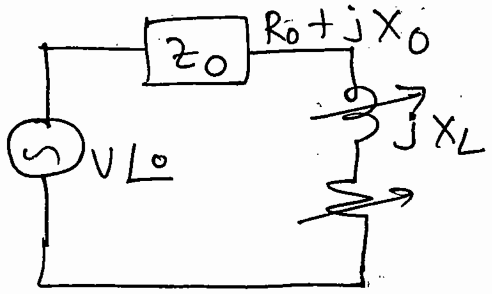
$$\therefore R_0^2 + \cancelto{0}{R_L^2} + 2R_0R_L + X_0^2 + \cancelto{0}{X_L^2} + 2X_0X_L - 2R_LX_0 - 2R_LX_L = 0$$

$$-2R_L(X_0 + X_L) = 0$$

$$X_0 + X_L = 0$$

$$\boxed{X_0 = -X_L}$$

Case 4 ⇒ AC source, complex source impedance and complex load impedance by varying both X_L & R_L



$$i_L = \frac{V\angle 0}{R_0 + R_L + jX_L + jX_0}$$

$$|i_L| = \frac{V}{\sqrt{(R_0+R_L)^2 + (X_L+X_0)^2}}$$

but $P = |i_L|^2 \cdot R_L$

$$P = \frac{V^2 \cdot R_L}{(R_0+R_L)^2 + (X_L+X_0)^2} \Rightarrow (1)$$

assuming R_L to be fixed and only X_L is varying. w.r.t maximum power is transfered to the load when $X_L = -X_0$ substituting this in equation (1)

$$P = \frac{V^2 \cdot R_L}{\left[(R_0+R_L)^2 + (X_L-X_L)^2\right]}$$

$$P = \frac{V^2 \cdot R_L}{(R_0+R_L)^2}$$ differentiating w.r.t R_L

$$\frac{dp}{dR_L} = \frac{V^2\left[(R_0+R_L)^2 - 2(R_0+R_L)R_L\right]}{(R_0+R_L)^4}$$

$$0 = \frac{V^2\left\{R_0^2 + R_L^2 + 2\cancel{R_0}R_L - 2\cancel{R_0}R_L - 2R_L^2\right]}{(R_0+R_L)^4}$$

$$R_0^2 - R_L^2 = 0$$

$$\Rightarrow \boxed{R_0 = R_L}$$

when both R_L & X_L are varied

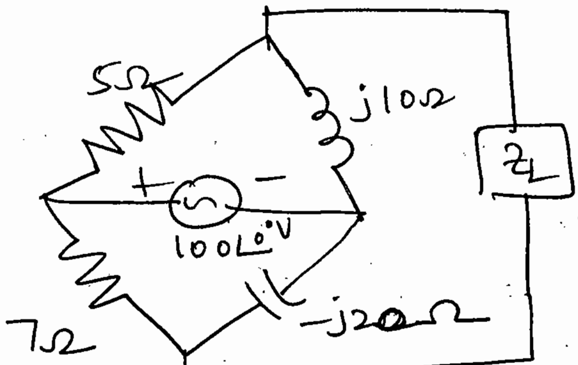
$R_L = R_0$ and $X_L = -X_0$

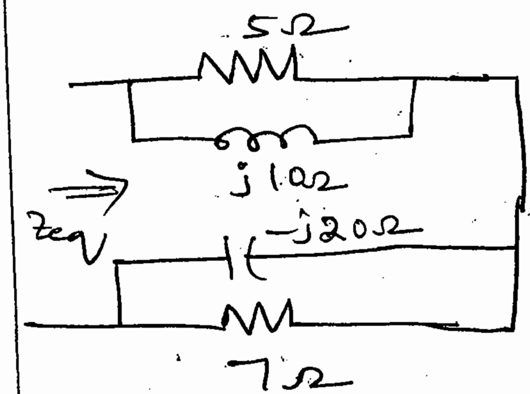
$$Z_L = R_L + jX_L$$

$$= R_0 - jX_0$$

$$Z_L = Z_{th}^{*} \quad * \Rightarrow \text{complex conjugate}$$

Find the value of Z_L for which maximum power is transfered to the load Z_L





$$Z_{eq} = 5 \overset{lel}{\|} j10 + 7 \overset{lel}{\|} -j20$$

$$\frac{5 \times j10}{5 + j10} = \frac{50\angle 90}{11.1803\angle 63.43}$$

$$= 4.4721\angle 26.57$$

$$7 \overset{lel}{\|} -j20 = \frac{7 \times -j20}{7 - j20} = \frac{140\angle -90}{21.189\angle -70.7}$$

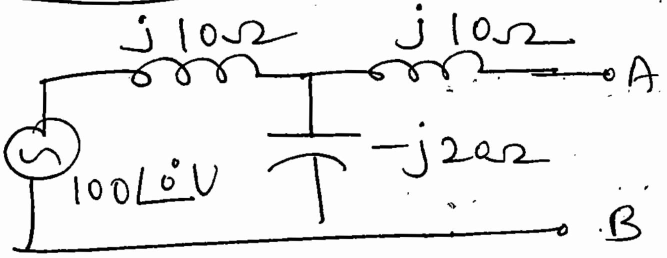
$$= 6.6072 \angle -19.29^\circ \ \Omega$$

$$\therefore \ Z_{eq} = 4 + j2 + 6.2362 - j2.1826$$

$$= 10.2362 - j0.1826 \ \Omega$$

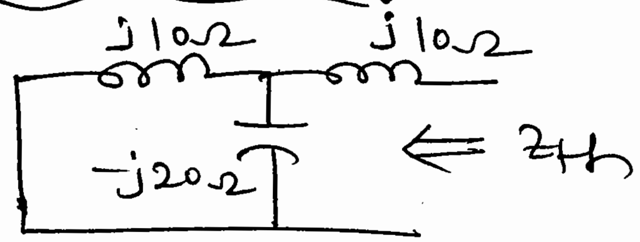
$$\therefore \ \boxed{Z_L = Z_{eq}^* = 10.2362 + j\,0.1826 \Omega}$$

What should be the value of pure resistive load to be connected across the terminals A and B for the network shown. Show that the maximum power is transfered to the load. What is the maximum power



If pure resistance is the load connected for P_{max}, $R_L = |Z_{eq}|$

To find Z_{eq}



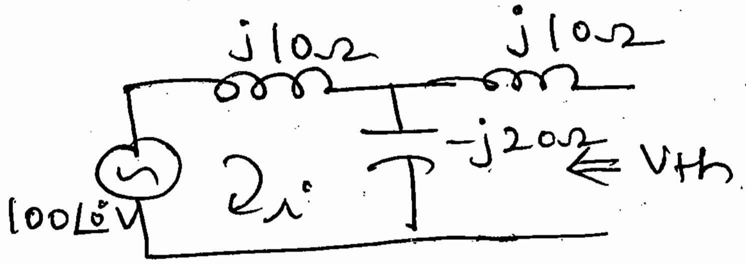
$R_{th} = 30 \Omega$

$$Z_{th} = Z_{eq} = \frac{j10 \times -j20}{j10 - j20} + j10$$

$$Z_{eq} = 30j \ \Omega \qquad \therefore \ R_{th} = |30j|$$

$$= 30 \Omega$$

To find V_{th}



$$i = \frac{100\angle 0}{j10 - j20} = \frac{100\angle 0}{-10j}$$

$$i = 10j$$

$$V_{th} = i_1 \times -j20 = 10j \times -j20$$

$$V_{th} = 200V$$

$$i_L = \frac{V_{th}}{Z_{th} + Z_L} = \frac{200}{30j + 30}$$

$$i_L = 4.714 \angle -45°$$

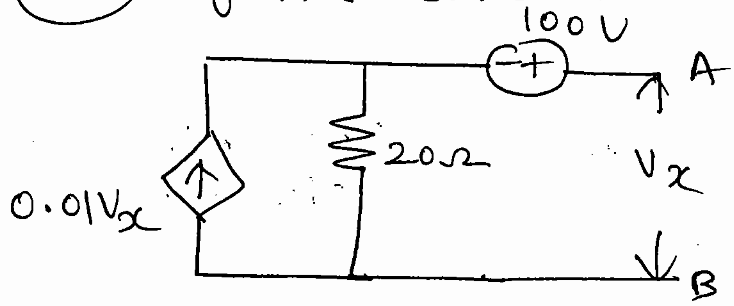
$$P_{max} = (i_L^2)(R_L)$$

$$= (4.714)^2 \times 30$$

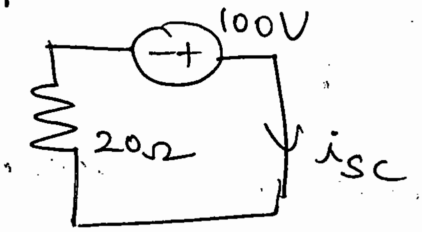
$$\boxed{P = 666.65 \text{ Watts}}$$

July 2017 QP (08m)

Obtain the Thevenin's equivalent of the circuit shown.

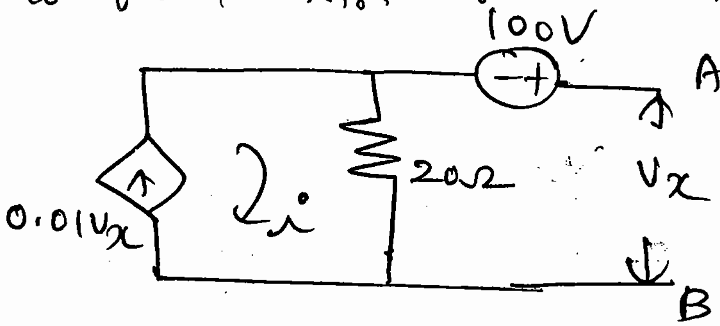


to find I_{sc} replace the terminals A & B by short circuit ∴ dependent current source acts as an open circuit as $V_x = 0$



$$i_{sc} = \frac{100}{20} = 5A$$

to find R_{th} find V_{th}



$i = 0.01 V_x$ [current source lies @ the perimeter]

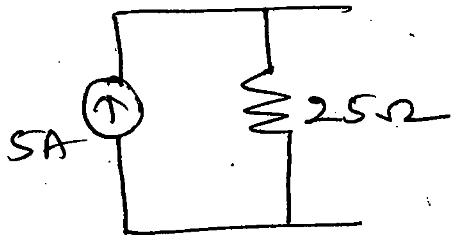
from the figure $V_{AB} = V_x$

$$= 100 + 20\, i$$

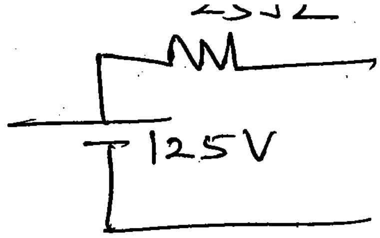
$$= 100 + 20(0.01 V_x)$$

$$V_x = \frac{100}{0.8} = 125 \text{ volts} = V_{th}$$

$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{125}{5} = 25\,\Omega$$

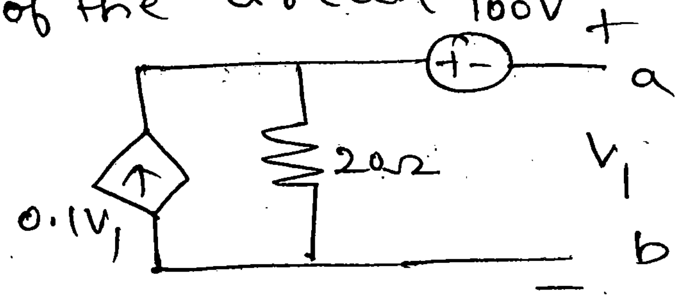


Norton's equivalent circuit

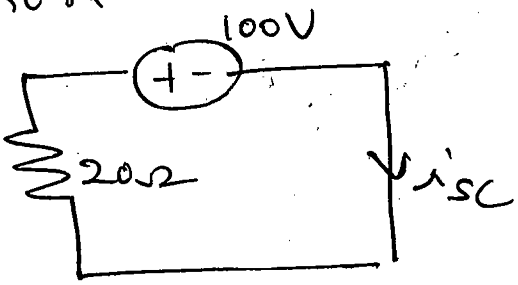


Thevenin's equt circuit

* obtain Thevenin's equivalent of the circuit



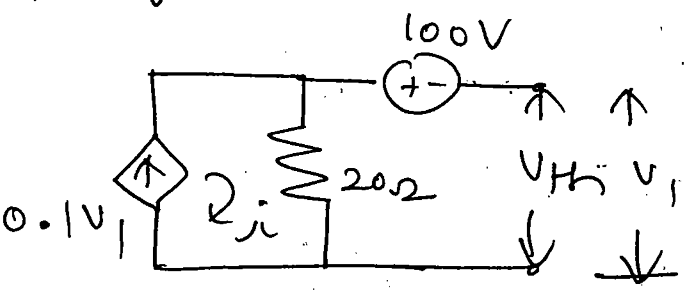
to find i_{sc} replace load terminals by short circuit as $V_1 = 0$ $0.1 V_1 = 0$



$$i_{sc} = \frac{-100}{20}$$

$$\boxed{i_{sc} = -5A}$$

To find V_{th}



$i = 0.1 V_1$ (current source lies @ the perimeter)

$$\therefore V_{Th} = V_{oc} = V_1 = -100 - 20i$$

$$V_1 = -100 - 20(0.1\,V_1)$$

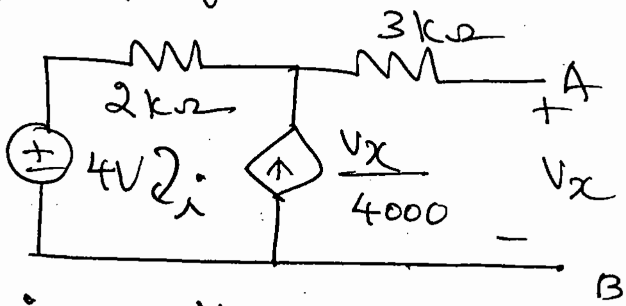
$$V_1 = -100 - 2V_1$$

$$-V_1 = -100$$

$$\boxed{V_1 = 100\,V} = V_{Th}$$

$$\therefore R_{Th} = \frac{V_{Th}}{I_{SC}} = \frac{100}{-5} = -20\,\Omega$$

obtain the Thevenin's equivalent circuit for the network shown



$$i = \frac{-V_x}{4000} \qquad \therefore V_{Th} = V_{AB}$$

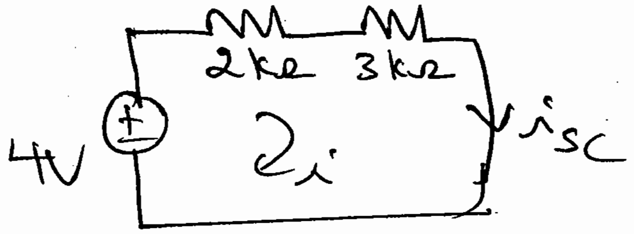
$$= -(2\times10^3)(i) + 4$$

$$= -(2\times10^3)\left(\frac{-V_x}{4000}\right) + 4 = 0.5V_x + 4$$

but $V_{AB} = V_x = 0.5\,V_x + 4$

$$\boxed{V_x = 8\ \text{Volts}}$$

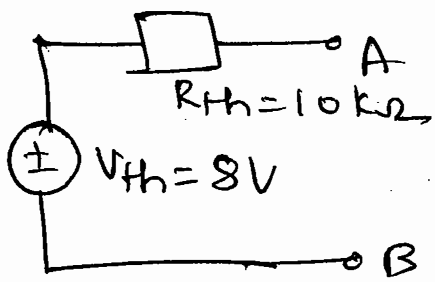
To find R_{Th}, find I_{sc}



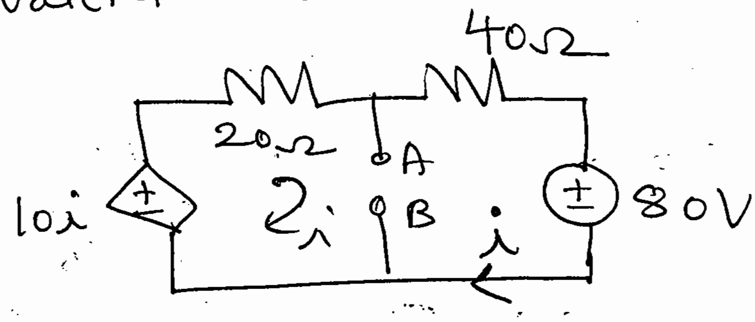
as $V_x = 0 \Rightarrow \frac{V_x}{4000} = 0$

$$i = \frac{4}{(2+3)\,10^3} = 0.8\,mA = i_{sc}$$

$$\therefore R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{8}{0.8 \times 10^{-3}} = 10\,k\Omega$$



obtain Thevenin's & Norton's equivalent across the terminals A & B



apply KVL

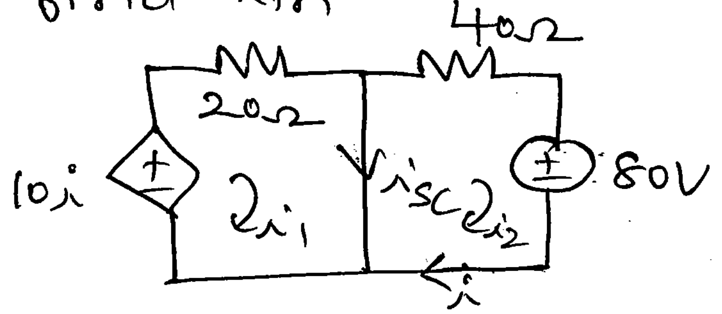
$$20i + 40i + 80 - 10i = 0$$

$$50i = -80$$

$$\therefore i = \frac{-80}{50} = -1.6\,A$$

$$V_{th} = V_{AB} = 40i + 80$$
$$= 40(-1.6) + 80$$
$$= 16 \text{ Volts}$$

to find R_{th}



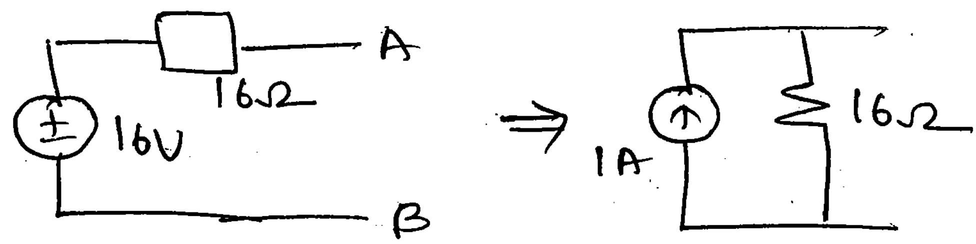
$$i_2 = -\frac{80}{40} = -2A$$

$$i_1 = \frac{10i}{20} = \frac{10(-2)}{20} = -1 \text{ amperes}$$

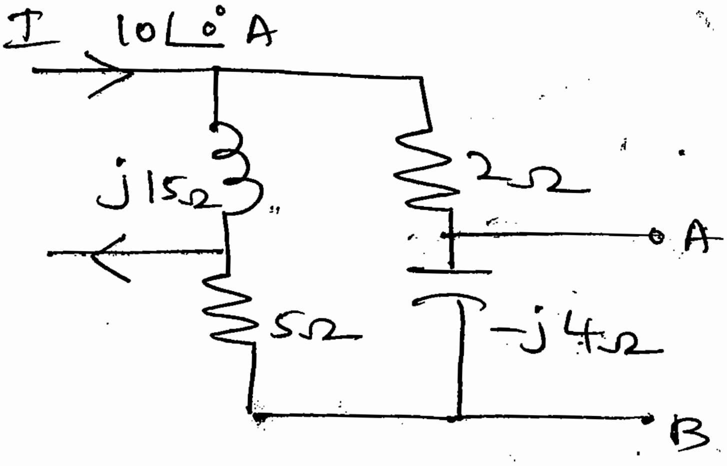
$$i_{sc} = i_1 - i_2$$

$$i_{sc} = -1 + 2 = 1 \text{ amperes}$$

$$\therefore R_{th} = \frac{V_{th}}{I_{sc}} = \frac{16}{1} = 16\Omega$$

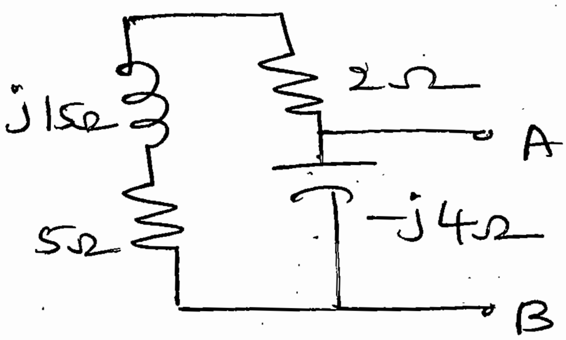


Thevenin's & Norton's equivalent circuit obtain Norton's & Thevenin's equivalent circuit & find the power delivered by $10\angle 60$ impedance



to find R_{th}

$10\angle 0$ current source is open circuited

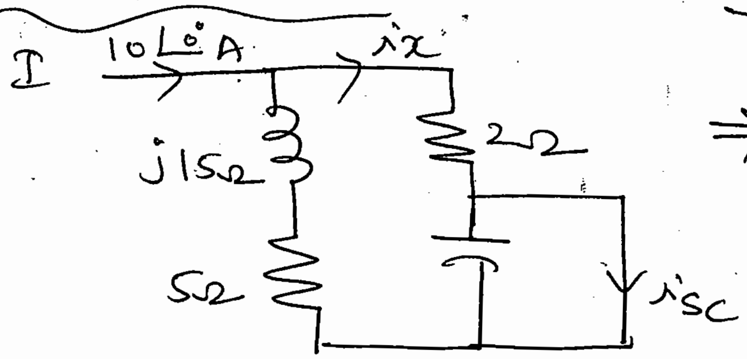


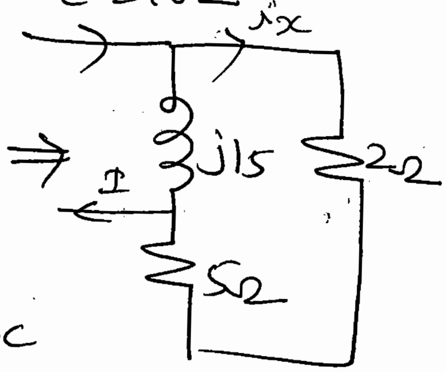
$$Z_{th} = \frac{(2+5+j5)(-j4)}{7+j15\cancel{+}j4}$$

$$Z_{th} = \frac{(7+j15)(-j4)}{7+j11}$$

$$Z_{th} = 5.08\angle -82.54\ \Omega$$

to find I





$$i_x = \frac{I\,(j15)}{j15+7}$$

$$= \frac{(10\angle 0^\circ)\;15\angle 90}{7+j15} = 9.062\angle 25.02\,A$$

$$i_x = i_{sc} = 9.062\angle 25.02\,A$$

$$I_L = i_{sc}\left[\frac{Z_{th}}{Z_{th}+Z_L}\right]$$

$$= \frac{(9.062\angle 25.02)\;(5.08\angle -82.54)}{5.08\angle -82.54 + 10\angle 60}$$

$$= \frac{46.034\angle -57.52}{0.66 - j5.03 + 5 + j8.66}$$

$$= \frac{46.034\angle -57.52}{6.724\angle 32.67} = 6.846\angle -90.19\,A$$

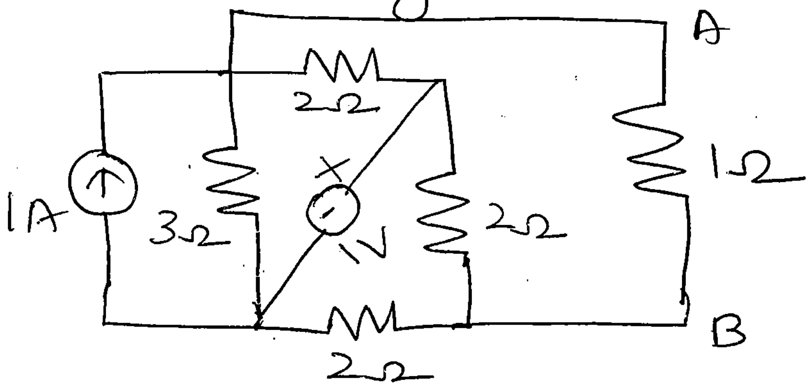
$\therefore$ power dissipated in $10\angle 60^\circ\,\Omega$

$$= I_L^2 \times 5$$

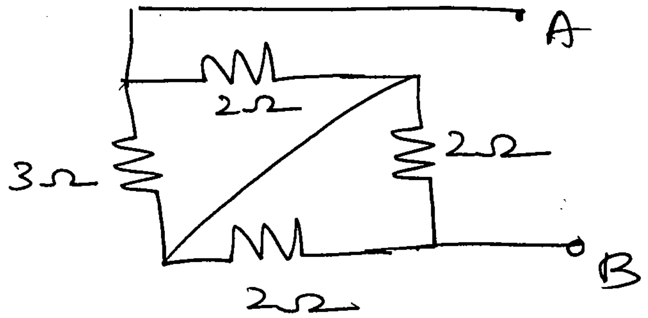
$$= (6.846)^2 \times 5$$

$$\boxed{P = 234.35 \text{ watts}}$$

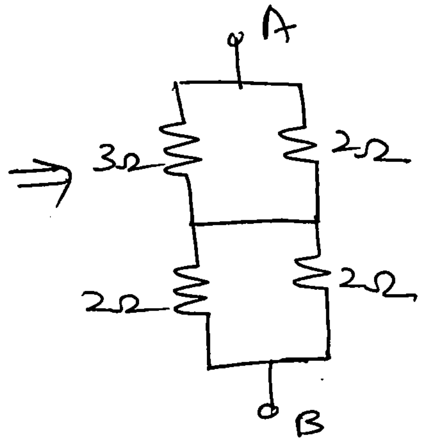
Find the current through 1Ω resistor using Norton's theorem



To find R_{th} | R_N



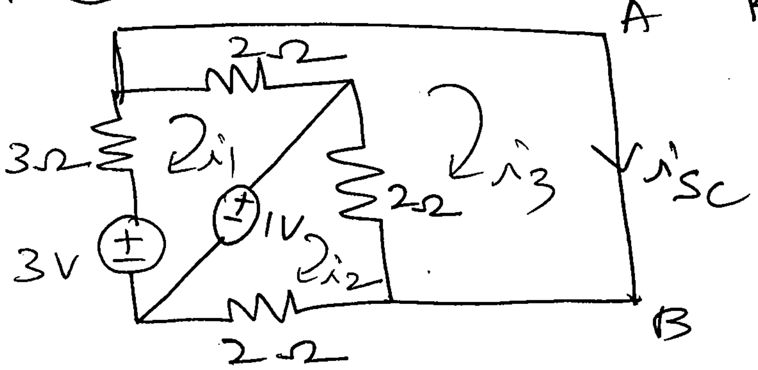
$\Rightarrow$



$$R_N = \frac{3\times 2}{3+2} + \frac{2\times 2}{2+2} = 2.2\Omega$$

To find V_{th}

KVL to 3 loops

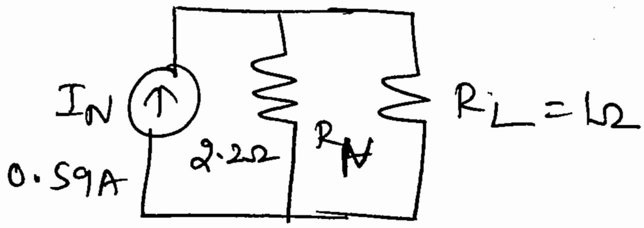


$5i_1 - 2i_3 = 2 \rightarrow (1)$ Loop (1)

Loop (2) $\quad 4i_2 - 2i_3 = 1 \Rightarrow (2)$

Loop (3) $\quad -2i_1 - 2i_2 + 4i_3 = 0 \rightarrow (3)$

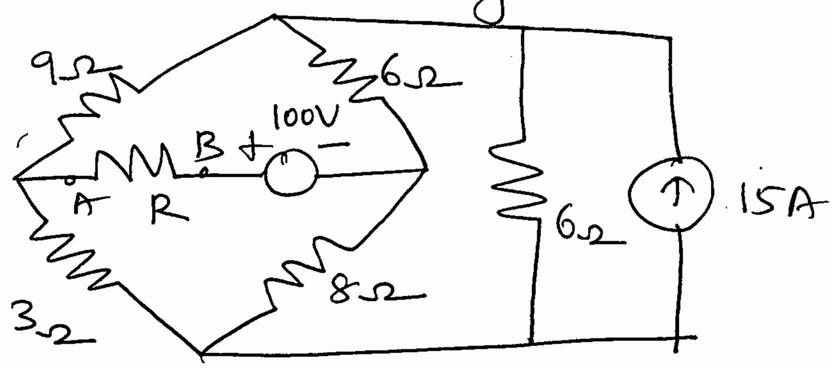
$i_3 = i_{sc} = 0.59$ amperes

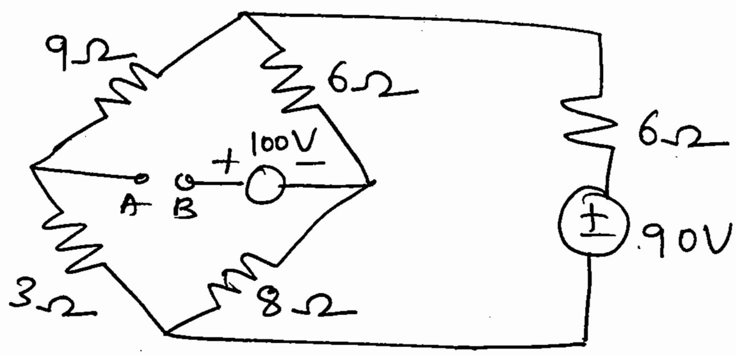


$$i_L = I_N\left(\frac{R_N}{R_N + R_L}\right) = \frac{0.59 \times 2.2}{3.2}$$

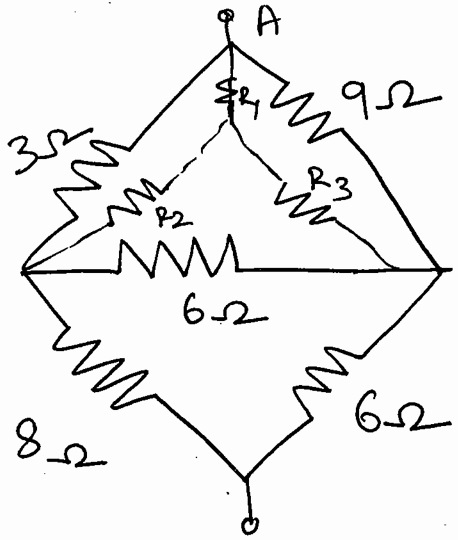
$$\boxed{i_L = 0.406 \text{ amperes}}$$

Find R_{Th} using Thevenin's theorem





both 100V & 90V voltagesources are set equal to zero

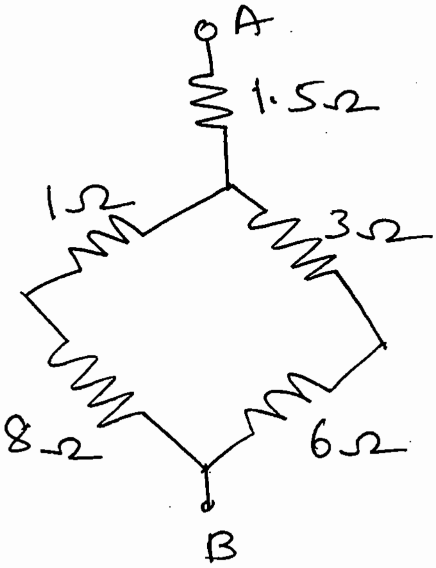


convert delta to star (3, 9, 6)

$$R_1 = \frac{3 \times 9}{3+9+6} = 1.5\,\Omega$$

$$R_2 = \frac{3 \times 6}{18} = 1\,\Omega$$

$$R_3 = \frac{9 \times 6}{18} = 3\,\Omega$$

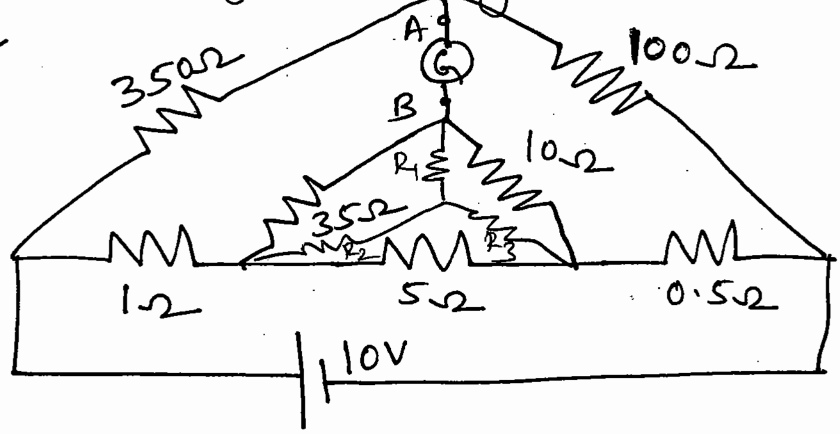


$$R_{th} = \frac{9 \times 9}{9+9} + 1.5$$

$$\boxed{R_{th} = 6\,\Omega}$$

Calculate the current through the galvanometer of 30Ω in Kelvin's double bridge using Thevenin's theorem

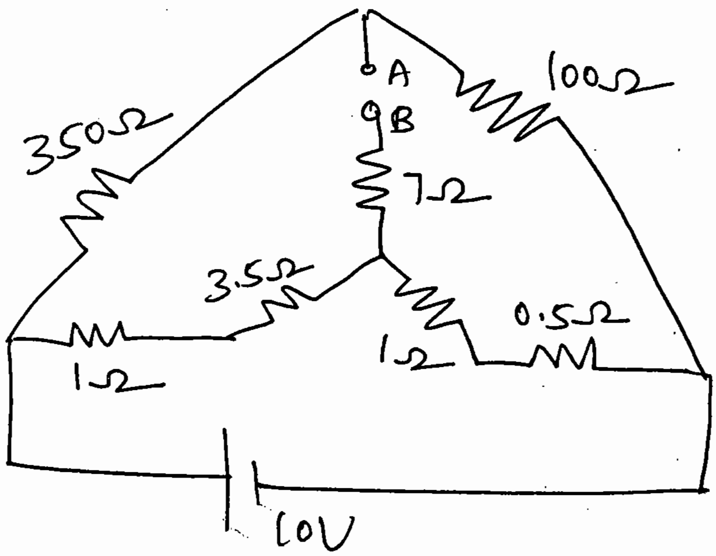
Imp



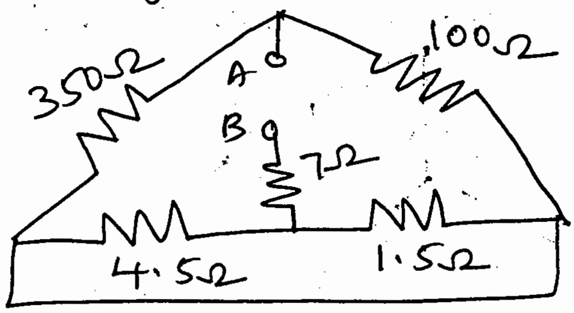
Remove the galvanometer to open circuit the terminals A & B & convert 35Ω, 10Ω, 5Ω resistor connected in delta into equivalent star

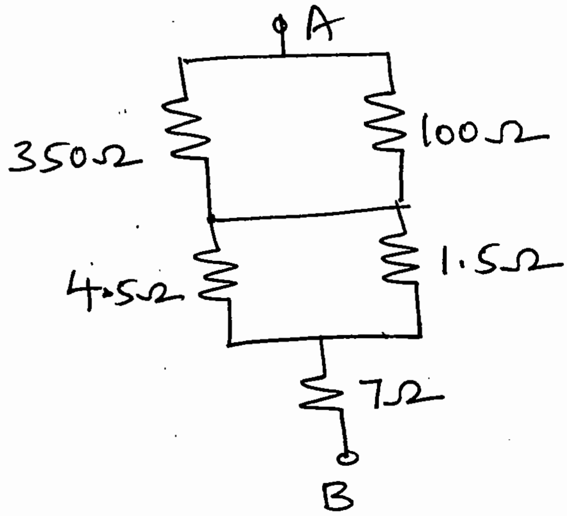
$$R_1 = \frac{35 \times 10}{35+5+10} = 7\Omega \; ; \; R_2 = \frac{35 \times 5}{50} = 3.5\Omega$$

$$R_3 = \frac{5 \times 10}{50} = 1\Omega$$



To find R_{th} replace voltage source by shortcircuit





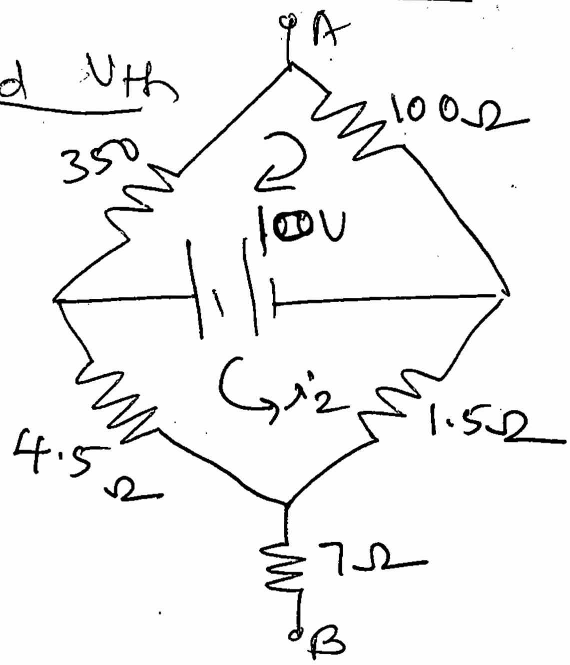
$$R_{th} = \frac{350 \times 100}{450} + \frac{4.5 \times 1.5}{6} + 7$$

$$R_{th} = (350 \parallel 100) + (4.5 \parallel 1.5) + 7$$

$$= 77.77 + 1.125 + 7$$

$$\boxed{R_{th} = 85.895\,\Omega}$$

To find V_{th}



$450 i_1 = 10 \qquad i_1 = \dfrac{10}{450} = 0.022A$

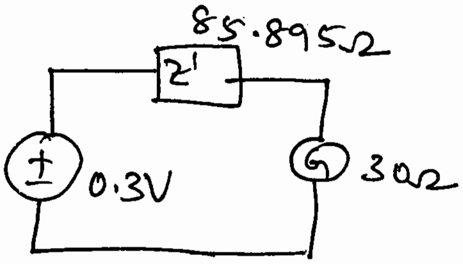
$6 i_2 = 10$

$i_2 = \dfrac{10}{6} = 1.666\ A$

$\therefore V_{Th} = V_{AB} = 100\, i_1 - 1.5\, i_2$

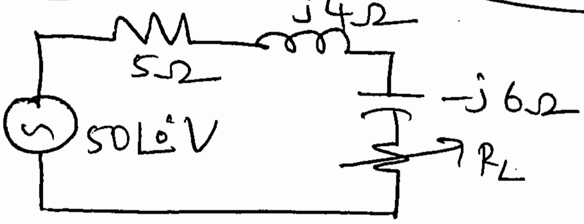
$= 100 \times 0.022 - 1.5 \times 1.66$

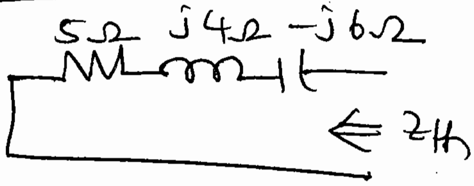
$\boxed{V_{Th} = 0.3 \text{ Volts}}$



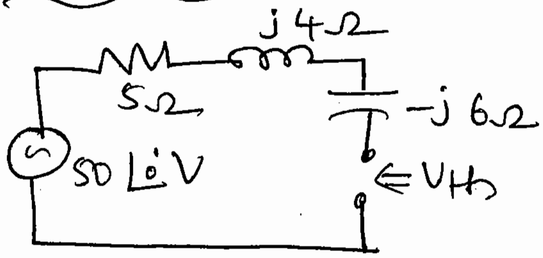
$i = \dfrac{V_{Th}}{R_{Th} + R_L} = \dfrac{0.3}{85.895 + 30} = 250mA$

Find the value of R_L & find the maximum power delivered





To find V_{Th}



$V_{Th} = 50\angle 0°\ V$

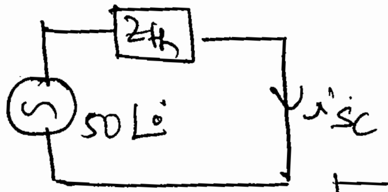
$R_{Th} = 5 + j4 - j6$

$R_{Th} = (5 - j2)\,\Omega$

$= 5.38\ \angle{-21.8°}\ \Omega$

$R_L = |R_{Th}| = 5.38\,\Omega$

to find i_{sc}



$$i_{sc} = \frac{50}{5-j2} = 8.62 + 3.45j$$

$$\boxed{i_{sc} = I_N = 9.284 \angle 21.8^\circ A}$$

$$I_L = \frac{V_{th}}{R_{th} + R_L} = 4.729 \angle -10.9^\circ A$$

$$P = I_L^2 \cdot R_L = (4.729)^2 \times 5.38$$

$$\boxed{P = 120.315 \text{ watts}}$$

module: 3

Transient Behavior and Initial conditions

Introduction ⇒ whenever a network containing energy storage elements such as inductor or capacitor is switched from one condition to another, either by change in applied source or change in network elements, the response current and voltage change from one state to the other state.

The time taken to change from an initial steady state to the final steady state is known as the Transient period. This response is known as transient response or transients.

The response of the network after it attains a final steadystate value is independent of time and is called the steady state response. The complete response of the network is determined with the help of a differential equation.

Initial condition provide information regarding how the elements behave during switching action. This knowledge is necessary in understanding nonlinear switching circuit

Initial condition is the condition that exist in a network at the instant of switching operation. The reference time is $t=0$, when one or more switches are operated. The condition just before switching operation will be represented as $i(0^-)$, $V(0^-)$ etc condition just after switching operation will be represented as $i(0^+)$, $V(0^+)$ -- etc

Final condition can be obtained by allowing time "t" to be infinity

In solving the differential equations in the network, we get some arbitary constant. Initial conditions are used to determine these arbitrary constants. It helps us to know the behaviour of elements at the instant of switching

In solving the problems for initial conditions in the network, we divide the time period in the following ways

i) Just before switching (from $t=-\infty$ to $t=0^-$)

ii) Just after switching (at $t=0^+$)

iii) After switching (for $t>0$)

Procedure for evaluating initial conditions

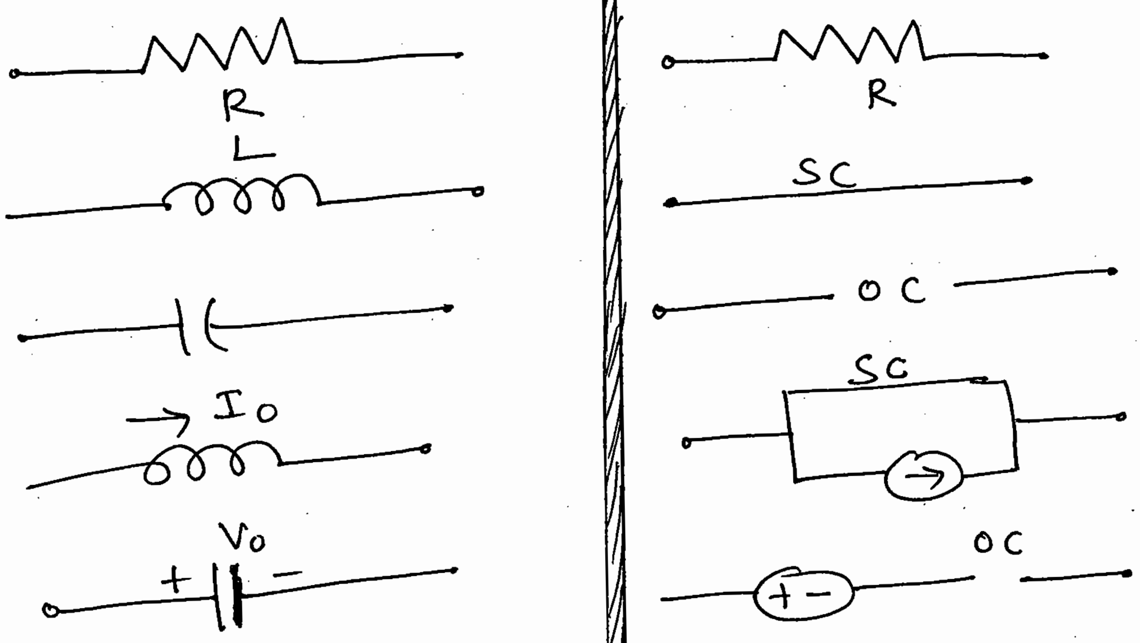
1) Draw the equivalent network at $t=0^-$ Before switching action takes place i.e, for $t=-\infty$ to $t=0^-$, the network is under steady state conditions, hence find the current flowing through the inductors $i_L(0^-)$ and Voltage across the capacitor $V_C(0^-)$

2) Draw the equivalent network at $t=0^+$ i.e, immediately after switching. Replace all the inductors with open circuits or with current sources $i_L(0^+)$ and replace all capacitors by short circuits or voltage sources $V_C(0^+)$. Resistors are kept as it is in the network

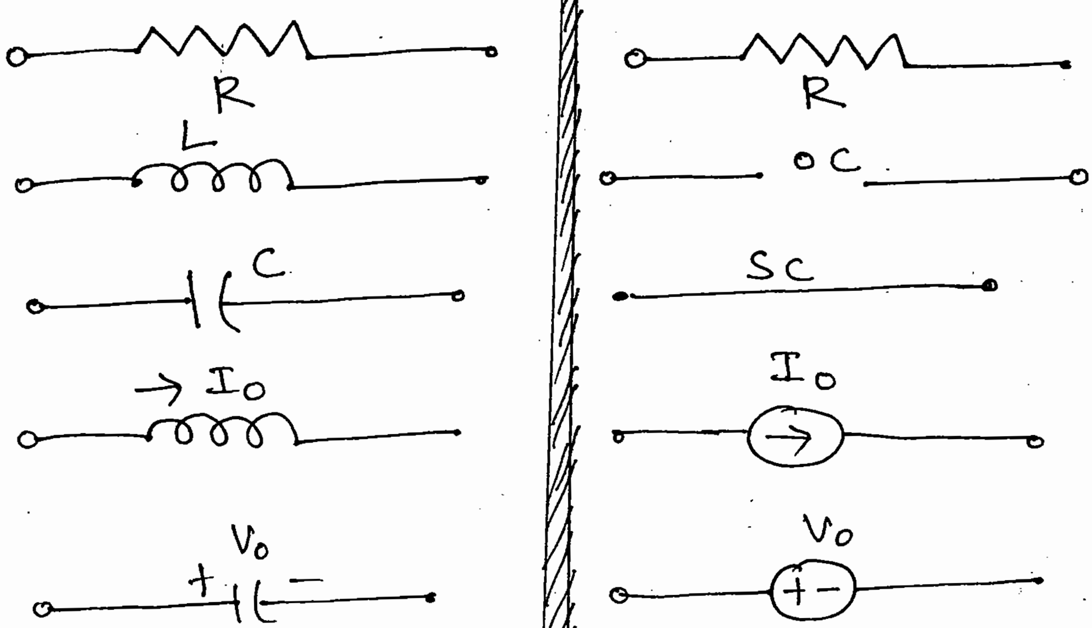
3) Intial voltages or currents in the network are determined from the equivalent network at $t=0^+$

4) Initial conditions i.e, $\frac{di}{dt}(0^+)$, $\frac{dV}{dt}(0^+)$ $\frac{d^2i}{dt^2}(0^+)$, $\frac{d^2V}{dt^2}(0^+)$ are determined by writing integro differential equations for the network for $t>0$, i.e, after the switching action by making use of initial condition

Element with initial conditions	Equivalent circuit at $t = \infty$



Element with Initial condition	Equivalent circuit at $t = 0^+$



Initial conditions for the resistor

For a resistor current and Voltage are related by $v(t) = R\,i(t)$. The current through a resistor will change instantaneously if the Voltage changes instantaneously, similarly the voltage will change instantaneously if the current changes instantaneously.

Initial conditions for the inductor

Voltage across the inductor is proportional to the rate of change of current. It is impossible to change the current through an inductor by an finite amount in zero time, this requires an infinite voltage across the inductor. An inductor does not allow an abrupt change in the current through it

If there is no current flowing through the inductor at $t=0^-$, the inductor will act as an open circuit at $t=0^+$

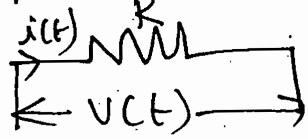
If a current of value I_0 flows through the inductor at $t=0^-$, the inductor can be regarded as a current source of I_0 ampere at $t=0^+$

Initial conditions for the capacitor

Current through a capacitor is proportional to the rate of change of voltage. It is impossible to change the voltage across a capacitor by a finite amount in zero time. This requires an infinite current

A capacitor does not allow an abrupt change in voltage across it. If there is no voltage across the capacitor at $t = 0^-$ the capacitor will act as a short circuit at $t = 0^+$. If the capacitor is charged to a voltage V_0 at $t = 0^-$, it can be regarded as a voltage source of V_0 Volt at $t = 0^+$

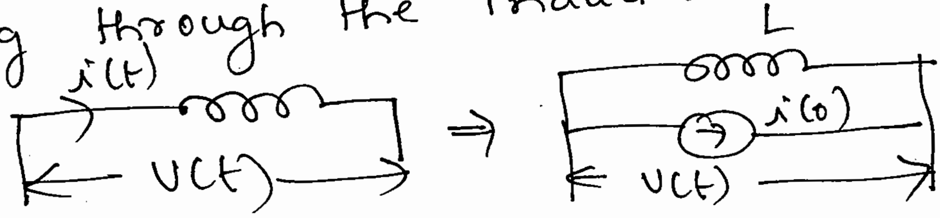
Resistor: The $v\,i$ relationship in time domain is $v(t) = R \cdot i(t)$



Inductor $\Rightarrow$ For the inductor the $v\,i$ relationship in time domain are

$$v(t) = L \cdot \frac{di}{dt}, \quad i(t) = \frac{1}{L}\int_0^t v(t)\,dt + i(0)$$

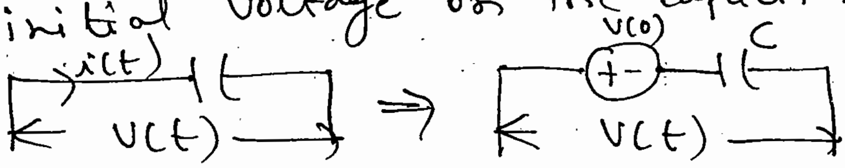
where $i(0)$ is the initial current passing through the inductor



Capacitor $\Rightarrow$ For the capacitor

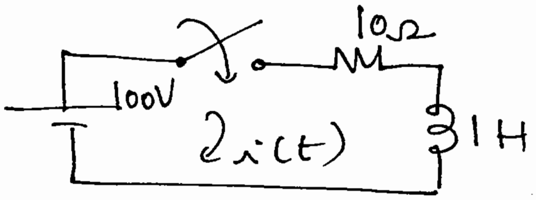
$$i(t) = C\frac{dv}{dt} \quad \text{and} \quad v(t) = \frac{1}{C}\int_0^t i(t)\,dt + v(0)$$

$v(0) \Rightarrow$ initial voltage on the capacitor



4

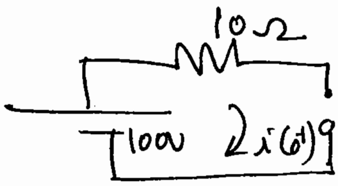
For the given network the switch is closed at $t=0$ with zero current in the inductor, find i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t=0^+$



at $t=0^-$ no current flows through the inductor $\therefore$ $i(0^-)=0$

at $t=0^+$, the inductor acts as an open circuit $i(0^+)=0$

writing KVL equation for $t>0$



$$100-10i-1\frac{di}{dt}=0$$

$$\frac{di}{dt}=100-10i \rightarrow (A)$$

at $t=0^+$ $\frac{di}{dt}(0^+)=100-10i(0^+)$

$$=100-10(0)=100\,A/s$$

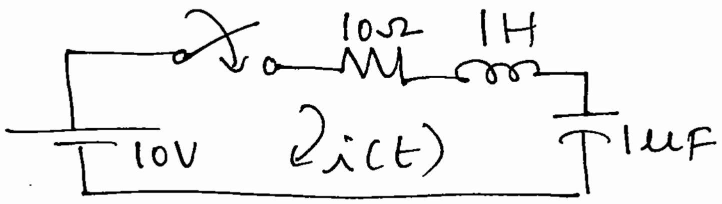
differentiating equation (A)

$$\frac{d^2i}{dt^2}=-10\frac{di}{dt} \quad \text{at } t=0^+$$

$$\frac{d^2i}{dt^2}(0^+)=-10\frac{di}{dt}(0^+)=-10(100)$$

$$=-1000\,A/s^2$$

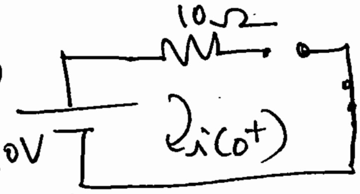
July 2017 QP For the network shown the switch is closed assuming all initial conditions zero find i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t=0^+$



At $t = 0^-$ $\quad i(0^-) = 0$

$V_c(0^-) = 0$

at $t = 0^+$ the inductor acts as an open circuit & the capacitor acts as a short circuit $\quad i(0^+) = 0, \; V_c(0^+) = 0$



writing KVL equation for $t > 0$

$$10 - 10i - 1\frac{di}{dt} - \frac{1}{10\times10^{-6}}\int_0^t i\,dt = 0$$

$$10 = 10i + \frac{di}{dt} + \frac{1}{10\times10^{-6}}\int_0^t i\,dt \Rightarrow (A)$$

at $t = 0^+$ $\quad 10 = 10\,i(0^+) + \frac{di}{dt}(0^+) + 0$

$$\therefore \frac{di}{dt}(0^+) = 10\,A/s$$

Differentiating (A) $\quad 0 = 10\frac{di}{dt} + \frac{d^2i}{dt^2} + \frac{1}{10\times10^{-6}}\,i$

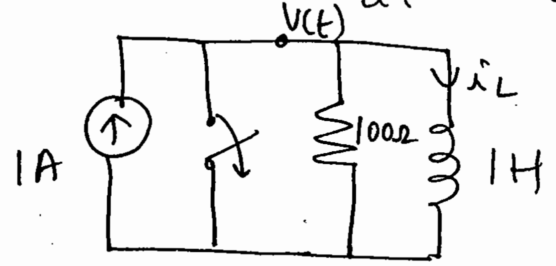
at $t = 0^+$

$$0 = 10\frac{di}{dt}(0^+) + \frac{d^2i(0^+)}{dt^2} + \frac{1}{10^{-5}}\,i(0^+)$$

$$\therefore \frac{d^2i(0^+)}{dt^2} = -10\times10 = -100\,A/s^2$$

(5)

July 2017 (8m) For the network shown in figure the switch is opened at $t=0$. Calculate V, $\frac{dV}{dt}$ & $\frac{d^2V}{dt^2}$ at $t=0^+$ (V, DV, DV²)
app



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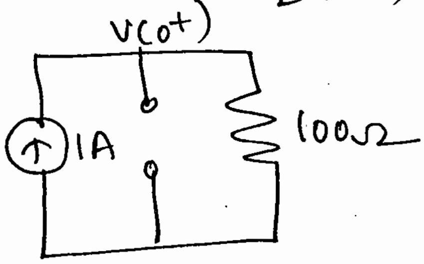
at $t=0^-$ the switch is closed hence no current flows through the inductor

$i_L(0^-)=0$

at $t=0^+$ the inductor acts as an open circuit, $i_L(0^+)=0$, $V(0^+)=100\times 1=100V$



writing KCL equation, for $t>0$

$$\frac{V}{100}+\frac{1}{1}\int_0^t V.dt=1 \rightarrow (a)$$

differentiating (a)

$$\frac{1}{100}\frac{dV}{dt}+V=0 \rightarrow (b)$$

at $t=0^+$

$$\frac{dV}{dt}(0^+) = -100V(0^+)$$

$$= -100\times 100 = -10000 V/s$$

differentiating equation (b)

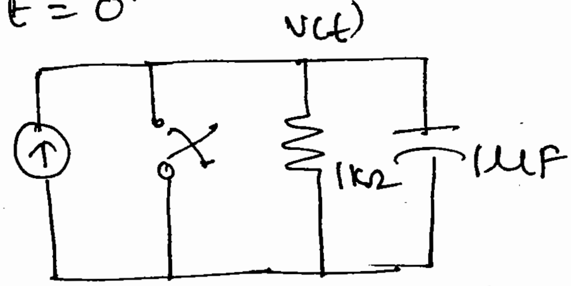
$$\frac{1}{100}\frac{d^2V}{dt^2}+\frac{dV}{dt}=0$$

at $t=0^+$

$$\frac{d^2V}{dt^2}(0^+) = -100\frac{dV}{dt}(0^+)$$

$$= -100\times(-10^4) = 10^6 \ V/s^2$$

For the given network the switch is opened at $t=0$ solve for v, $\frac{dv}{dt}$ & $\frac{d^2v}{dt^2}$ at $t=0^+$



at $t=0^-$ switch is closed hence the voltage across the capacitor is zero

$\therefore \quad v(0^-) = v_c(0^-) = 0$

at $t=0^+$ the capacitor acts as short circuit

$v(0^+) = 0$

write KCL equation for $t>0$

$$\frac{v}{1000} + 10^{-6}\frac{dv}{dt} = 10 \rightarrow (1)$$

at $t=0^+$ $\frac{v(0^+)}{1000} + 10^{-6}\frac{dv}{dt}(0^+) = 10$

$$\frac{dv}{dt}(0^+) = \frac{10}{10^{-6}} = 10\times10^{6} \text{ V/s}$$

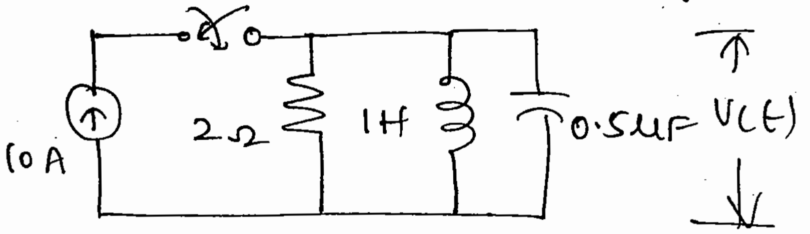
differentiating (1) $\frac{1}{1000}\frac{dv}{dt} + 10^{-6}\frac{d^2v}{dt^2} = 0$

at $t=0^+$

$$\frac{1}{1000}\frac{dv}{dt}(0^+) + 10^{-6}\frac{d^2v}{dt^2}(0^+) = 0$$

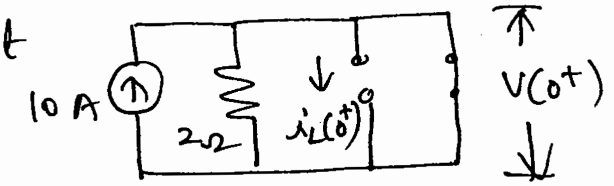
$$\frac{d^2v}{dt^2}(0^+) = -\frac{1}{1000\times10^{-6}}\times10\times10^{6} = -10\times10^{9} \text{ V/s}^2$$

For the network shown switch is closed at $t=0$, determine v, $\frac{dv}{dt}$ and $\frac{d^2v}{dt^2}$ at $t=0^+$



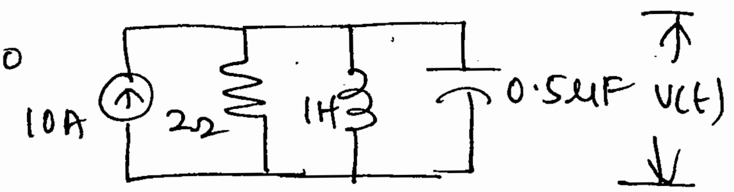
At $t=0^-$ no current flows through the inductor and there is no voltage across the capacitor $i_L(0^-)=0$, $v(0^-)=V_c(0^-)=0$

at $t=0^+$, the inductor acts as an open circuit and the capacitor acts as a short circuit



$i_L(0^+)=0$, $v(0^+)=V_c(0^+)=0$

for $t>0$



writing KCL equation for $t>0$

$$10=\frac{v}{2}+\frac{1}{1}\int_0^t v\,dt+0.5\times10^{-6}\frac{dv}{dt} \rightarrow (1)$$

at $t=0^+$ $\quad 10=\frac{v(0^+)}{2}+0+0.5\times10^{-6}\frac{dv}{dt}(0^+)$

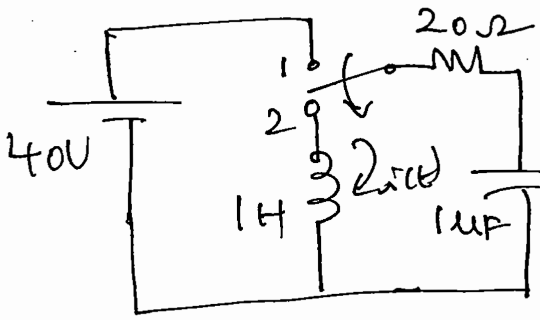
$$\frac{dv}{dt}(0^+)=20\times10^{6}\ \mathrm{V/s}$$

differentiating (1) $\quad 0=\frac{1}{2}\frac{dv}{dt}+v+0.5\times10^{-6}\frac{d^2v}{dt^2}$

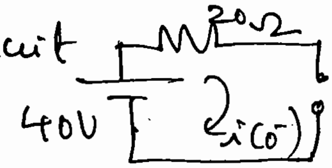
at $t=0^+$ $\quad 0=\frac{1}{2}\frac{dv}{dt}(0^+)+v(0^+)+0.5\times10^{-6}\frac{d^2v}{dt^2}(0^+)$

$$\frac{d^2v}{dt^2}(0^+)=-20\times10^{12}\ \mathrm{V/s^2}$$

For the network the switch is changed from the position 1 to the position 2 at $t=0$, steady condition having reached before switching. Find the values of i $\frac{di}{dt}$ & $\frac{d^2i}{dt^2}$ at $t=0^+$

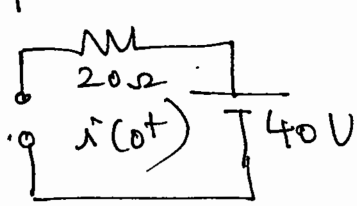


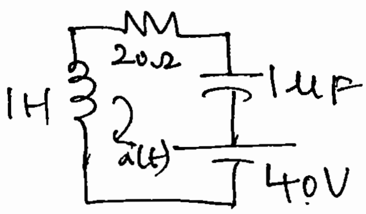
At $t=0^-$ the network attains steady state hence capacitor acts as an open circuit



$V_C(0^-)=40V$, $i(0^-)=0$

at $t=0^+$ the capacitor acts as a voltage source of 40V & the inductor acts as open circuit $V_C(0^+)=40V$, $i(0^+)=0$





writing KVL equation for $t>0$

$$-1\frac{di}{dt}-20i-\frac{1}{1\times10^{-6}}\int_0^t i\,dt-40=0 \rightarrow ① \text{ at } t=0^+$$

$$-\frac{di}{dt}(0^+)-20i(0^+)-0-40=0$$

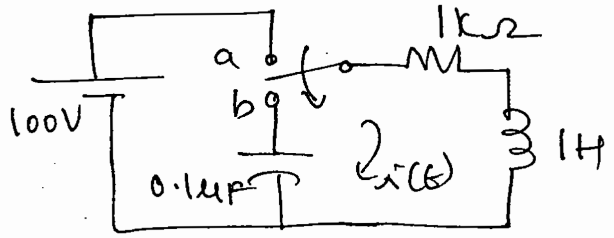
$$\frac{di}{dt}(0^+)=-40\ A/s$$

differentiating ① $\quad -\frac{d^2i}{dt^2}-20\frac{di}{dt}-10^6 i-0=0$

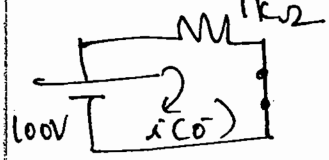
at $t=0^+$ $\quad -\frac{d^2i}{dt^2}(0^+)-20\frac{di}{dt}(0^+)-10^6 i(0^+)=0$

$$\frac{d^2i}{dt^2}(0^+)=800\ A/s^2$$

For the network shown the switch is changed from the position a to b at $t=0$ solve for i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t=0^+$

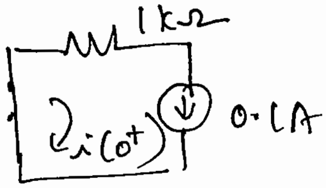


At $t=0^-$ the network attains steady condition hence the inductor acts as a short circuit



$$i(0^-) = \frac{100}{1000} = 0.1\,A$$

at $t=0^+$ the inductor acts as a current source of $0.1\,A$ and the capacitor acts as a short circuit $i(0^+) = 0.1\,A$



writing KVL equation for $t>0$

$$-\frac{1}{0.1\times10^{-6}}\int_0^t i\,dt - 1000i - 1\frac{di}{dt} = 0 \Rightarrow (1)$$

at $t=0^+$ $\quad -0 - 1000i(0^+) - \frac{di}{dt}(0^+) = 0$

$$\frac{di}{dt}(0^+) = -1000\,i(0^+) = -1000\times0.1 = -100\,A/s$$

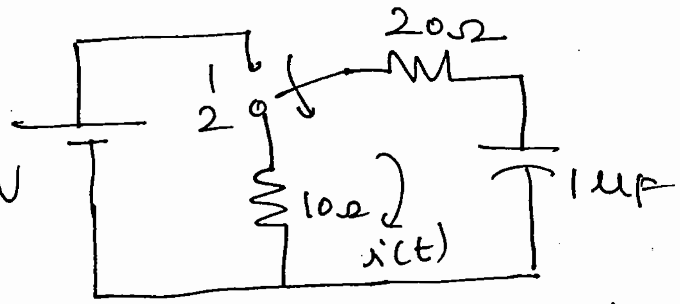
differentiating (1) $\quad -\frac{1}{10^{-7}}i - 1000\frac{di}{dt} - \frac{d^2i}{dt^2} = 0$

at $t=0^+$ $\quad -10^7 i(0^+) - 1000\frac{di}{dt}(0^+) - \frac{d^2i}{dt^2}(0^+) = 0$

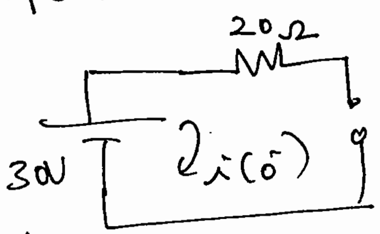
$$\frac{d^2i}{dt^2}(0^+) = -10^7(0.1) - 1000(-100)$$

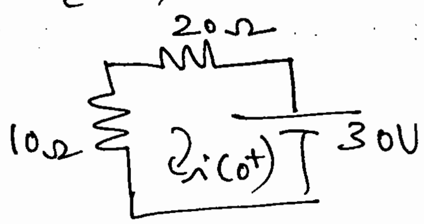
$$= -9\times10^5\,A/s^2$$

For the network shown the switch is changed from the position 1 to the position 2 at $t=0$, steady condition having reached before switching. Find the values of i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t=0^+$

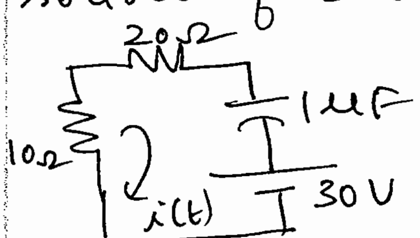


at $t=0^-$, the network attains steady state condition hence the capacitor acts as an open circuit $V_c(0^-)=30V$, $i(0^-)=0$





at $t=0^+$, the capacitor acts as a voltage source of 30V $\therefore\ i(0^+) = -\frac{30}{30} = -1A$



writing KVL equation for $t>0$

$$-10i - 20i - \frac{1}{1\times10^{-6}}\int_0^t i\,dt - 30 = 0 \rightarrow (1)$$

differentiating (1) $-30\frac{di}{dt} - 10^6 i = 0 \Rightarrow (2)$

at $t=0^+$ $-30\frac{di}{dt}(0^+) - 10^6 i(0^+) = 0$

$$\frac{di}{dt}(0^+) = \frac{-10^6(-1)}{30} = 0.33\times10^5\ A/s$$

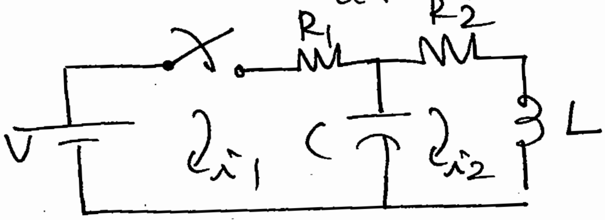
Differentiating (2)

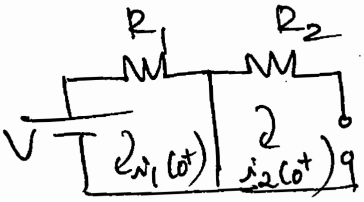
$$-30\frac{d^2 i}{dt^2} - 10^6 \frac{di}{dt} = 0$$

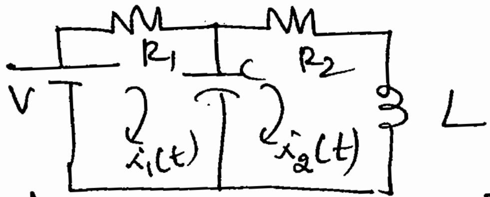
at $t = 0^+$ $\quad -30\frac{d^2 i}{dt^2}(0^+) - 10^6 \frac{di}{dt}(0^+) = 0$

$$\frac{d^2 i}{dt^2}(0^+) = \frac{-10^6 \times 0.33 \times 10^5}{30} = -1.1 \times 10^9 \text{ A/s}^2$$

For the circuit shown assuming all initial conditions as zero find $i_1(0^+)$, $i_2(0^+)$, $\frac{di_1}{dt}(0^+)$, $\frac{di_2}{dt}(0^+)$ $\frac{d^2 i_1}{dt^2}(0^+)$ & $\frac{d^2 i_2}{dt^2}(0^+)$







at $t = 0^-$, all initial conditions are zero

$V_c(0^-) = 0$, $i_1(0^-) = 0$, $i_2(0^-) = 0$

at $t = 0^+$ the inductor acts as open circuit & capacitor acts as a short circuit

$i_L(0^+) = \frac{V}{R_1}$ $\quad i_2(0^+) = \frac{V}{R_1}$, $i_2(0^+)\ 0$, $V_c(0^+) = 0$

writing KVL equations for 2 loops for $t > 0$

$$R_1 i_1 + \frac{1}{C}\int_0^t (i_1 - i_2)\,dt = V \rightarrow (a)$$

$$\& \quad R_2 i_2 + L\frac{di_2}{dt} + \frac{1}{C}\int_0^t (i_2 - i_1)\,dt = 0 \rightarrow (b)$$

from equation (b) at $t=0^+$

$$R_2 i_2(0^+) + L\frac{di_2}{dt}(0^+) + \frac{1}{C}\int_0^{0^+}(i_2 - i_1)\,dt = 0$$

$$\frac{di_2}{dt}(0^+) = 0$$

differentiating (a) $\quad R_1\frac{di_1}{dt} + \frac{1}{C}(i_1 - i_2) = 0 \rightarrow$ (c)

at $t=0^+$ $\quad R_1\frac{di_1}{dt}(0^+) + \frac{1}{C}i_1(0^+) - \frac{1}{C}i_2(0^+) = 0$

$$R_1\frac{di_1}{dt}(0^+) + \frac{1}{C}\frac{V}{R_1} = 0$$

$$\frac{di_1}{dt}(0^+) = \frac{-V}{R_1^2 C}$$

differentiating (c) $\quad R_1\frac{d^2i_1}{dt^2} + \frac{1}{C}\frac{di_1}{dt} - \frac{1}{C}\frac{di_2}{dt} = 0$

at $t=0^+$ $\quad R_1\frac{d^2i_1}{dt^2}(0^+) + \frac{1}{C}\frac{di_1}{dt}(0^+) - \frac{1}{C}\frac{di_2}{dt}(0^+) = 0$

$$\frac{d^2i_1}{dt^2}(0^+) = \frac{V}{R_1^3C^2}$$

differentiating (b)

$$R_2\frac{di_2}{dt} + L\frac{d^2i_2}{dt^2} + \frac{1}{C}(i_2 - i_1) = 0$$

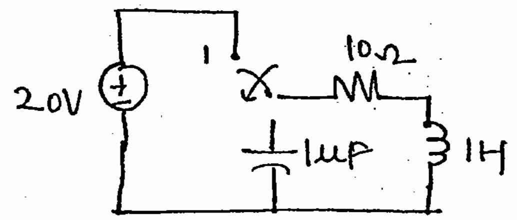
at $t=0^+$

$$\frac{d^2i_2}{dt^2}(0^+) = -\frac{R_2}{L}\frac{di_2}{dt}(0^+) - \frac{1}{LC}\left[i_2(0^+) - i_1(0^+)\right]$$

$$\boxed{\frac{d^2i_2}{dt^2}(0^+) = \frac{V}{R_1 LC}}$$

For the circuit shown the switch K is changed from position 1 to 2 at $t=0$ steady state condition having been reached at position - 1. Find the value of i, $\frac{di}{dt}$ & $\frac{d^2i}{dt^2}$ at $t=0^+$ (9)

Jan 2014 (10M)



Soln :- Before closing the switch $i(0^-)=2A$

at position 2 switch closed at $t=0$

$\therefore$ applying KV $Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0 \rightarrow (1)$

$V_C(0^-) = V_C(0^+) = 0$; $i(0^-) = i(0^+) = 2A$

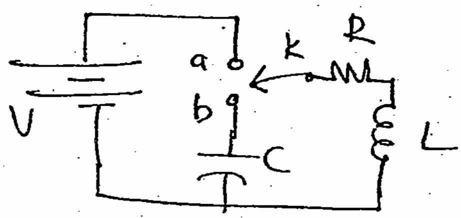
$\therefore \frac{di}{dt}(t>0^+) = -20\,A/sec$

differentiating equn (1) w.r.t "t"

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

$$(10)(-20) + 1\cdot\frac{d^2i}{dt^2} + \frac{2}{1\times10^{-6}} \quad \therefore \boxed{\frac{d^2i}{dt^2} = -2\times10^6\,A/sec^2}$$

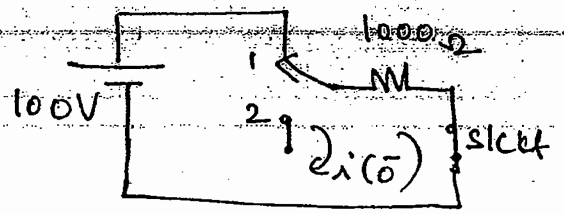
For the fig shown K is changed from position a to b at $t=0$. solve i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0^+$; $R=1000\Omega$ $L=1H$, $C=0.1\mu F$ & $V=100V$ assume capacitor is initially uncharged



ANS:-

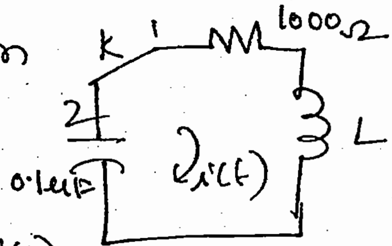
at $t=0^+$ switch K is at position 1, the network remains in steady state. hence L acts as short circuit

while C acts as open circuit



$$i(0^-) = \frac{100}{1000} = 0.1A = i(0^+)$$

$V_C(0^-) = 0 = V_C(0^+)$ because the current through L & voltage across "C" cannot change instantaneously for all $t \geq 0^+$ ∴ switch K is moved to position as shown



apply KUL

$$-1000\, i(t) - \frac{di(t)}{dt} - \frac{1}{0.1\times10^{-6}}\int i(t)\,dt = 0 \rightarrow (1)$$

$$\therefore 1000\, i(0^+) - \frac{di(0^+)}{dt} + \frac{1}{0.1\times10^{-6}}\int i(0^+)\,dt = 0$$

$$\frac{di}{dt}(0^+) = -1000\, i(0^+) = -1000\,(0.1) = -100\ A/sec$$

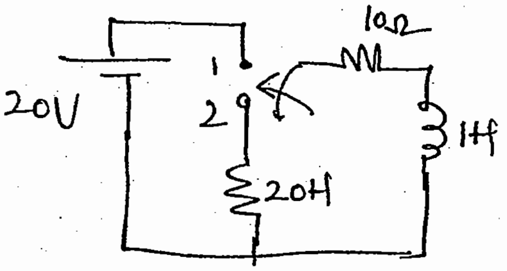
differentiating (1) $\quad 1000\frac{di(t)}{dt} + \frac{d^2 i(t)}{dt^2} + \frac{i(t)}{0.1\times10^{-6}} = 0$

at $t = 0^+$

$$\frac{d^2 i(0^+)}{dt^2} = -1000\frac{di(0^+)}{dt} - \frac{i(0^+)}{0.1\times10^{-6}} \quad -1000(-100) - \frac{0.1}{0.1\times10^{-1}}$$

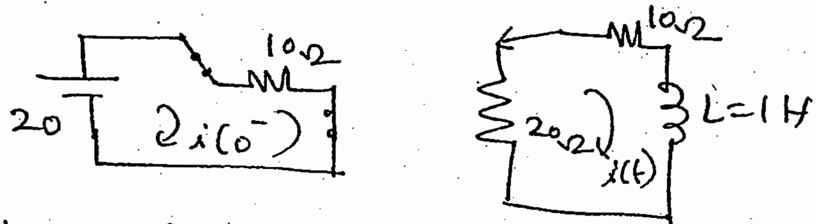
$$\therefore \frac{d^2 i(0^+)}{dt^2} = -9\times10^5\ A/sec^2$$

1. Determine $\frac{di}{dt}$ & $\frac{d^2 i}{dt^2}$ at $t=0^+$ when the switch K is moved from position 1 to 2 at $t=0$ for the fig shown. Steady state having reached before switching. 10



CB-C-DIVAKARA

Soln :- Inductor acts as short circuit under steady state condition ∴



$$10\,i(t) + 20\,i(t) + 1\cdot\frac{di(t)}{dt} = 0$$

$$i(0^-) = i(0^+) = \frac{20}{10} = 2A$$

$$30\,i(t) + \frac{di(t)}{dt} = 0$$

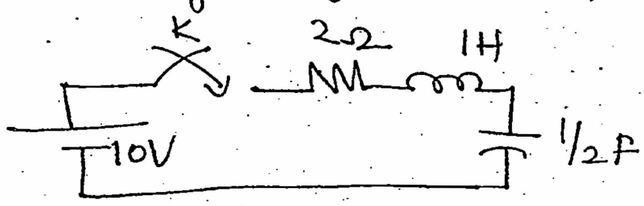
$$\frac{di(t)}{dt} = -30\,i(t) \rightarrow (1)$$

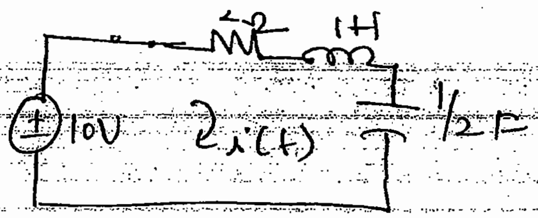
at $t=0^+$ $\frac{di(0^+)}{dt} = -30\,i(0^+) = -30\times 2 = -60\ A/sec$

differentiate eqn (1) $\Rightarrow$ $\frac{d^2 i(t)}{dt^2} = -30\frac{di(t)}{dt}$

$$\frac{d^2 i(t)}{dt^2} = -30\times(-60) = 1800\ A/sec^2$$

For the n/w shown switch K is closed at $t=0$ with capacitor uncharged. Find $i(0^+)$, $\frac{di(t)}{dt}$ at 0^+ & $\frac{d^2 i(0^+)}{dt^2}$





apply KVL

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i(t)\,dt = 10 \rightarrow (B)$$

$$\Rightarrow \quad Ri + L\frac{di}{dt} + V_c(t) = 10 \longrightarrow (A)$$

at $t = 0^+$ $\quad R(i(0^+)) + L\frac{di(0^+)}{dt} + V_c(0^+) = 10$

$$R \times 0 + L\frac{di(0^+)}{dt} + 0 = 10$$

$$\frac{di(0^+)}{dt} = 10 \text{ A/sec}$$

differentiate equ^n (B) $\quad R\frac{di(t)}{dt} + L\frac{d^2i(t)}{dt^2} + \frac{i(t)}{C} = 0$

Put $\quad t = 0^+ \quad R \times 10 + L\frac{d^2 i(0^+)}{dt^2} + \frac{0}{C} = 0$

$$2 \times 10 + 1 \cdot \frac{d^2 i(0^+)}{dt^2} + \frac{0}{C} = 0$$

$$20 + \frac{d^2 (i(0^+))}{dt^2} = 0$$

$$\boxed{\frac{d^2 i(0^+)}{dt^2} = -20 \text{ A/sec}^2}$$

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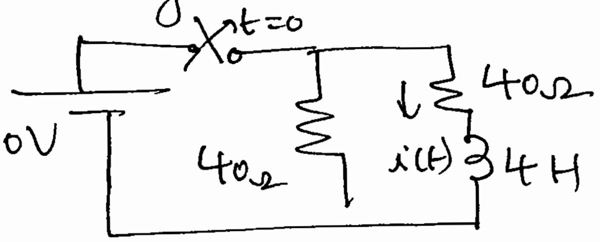
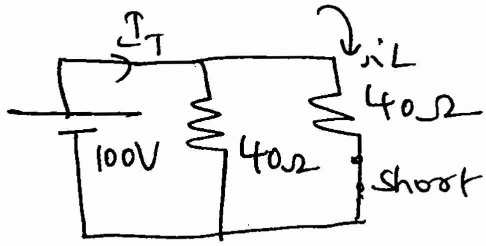
Find the current $i(t)$ when switch K is opened at $t=0$ with the circuit having reached steady state before the switching find current at $t=0.5\,sec$



When the switch is closed and steady state is reached hence inductor behaves as short circuit



$$I_T = \frac{100}{40 \parallel 40} = \frac{100}{20} = 5A$$

$$\therefore i_L = 5 \times \frac{40}{40+40} = 2.5A$$

$$i_L(0^-) = 2.5\,A$$

When the switch is opened

$$-40\,i(t) - 4\frac{di(t)}{dt} - 40\,i(t) = 0$$

$$\therefore 4\frac{di(t)}{dt} + 80\,i(t) = 0$$

taking Laplace transform

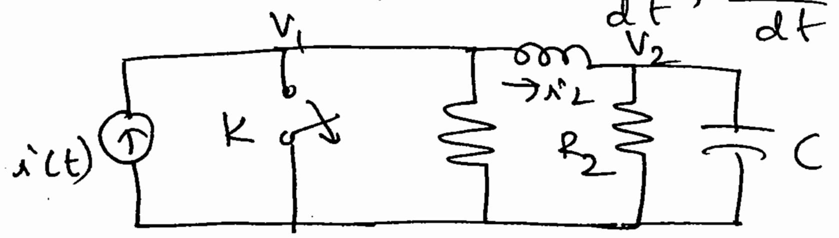
$$4[s\,I(s) - I_L(0^-)] + 80\,I(s) = 0$$

$$I(s)[4s+80] = 4 \times 2.5$$

$$I(s) = \frac{10}{4s+80} = \frac{2.5}{s+20}$$

$$\therefore i(t) = L^{-1}[I(s)] = 2.5\,e^{-20t}\,A$$

The network shown in fig has 2 independent node pairs & the switch K is opened at $t=0$, find the following at $t=0^+$, V_1, V_2, $\frac{dV_1}{dt}$, $\frac{dV_2}{dt}$ & $\frac{di_L}{dt}$



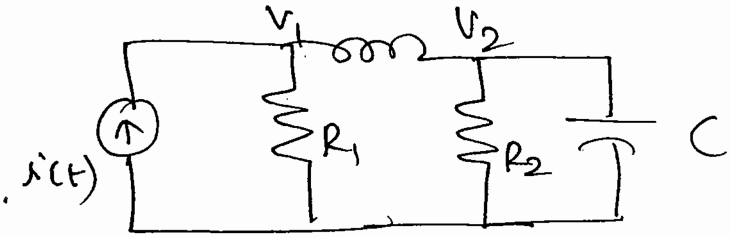
at $t=0^-$ switch K is closed

$$i_L(0^-) = 0 = i_L(0^+) \rightarrow ①$$

$$V_c(0^-) = V_2(0^-) = V_c(0^+) = V_2(0^+) = 0 \rightarrow ②$$

because voltage across C and current through L cannot change instantaneously

for all $t \geq 0^+$ switch K is opened



KCL @ node 1

$$i(t) = \frac{V_1(t)}{R_1} + \frac{1}{L}\int_{-\infty}^{t} [V_1(t) - V_2(t)\, dt]$$

$$i(t) = \frac{V_1(t)}{R_1} \left\{ \frac{1}{L}\int_{-\infty}^{0} [V_1(t) - V_2(t)]\, dt \right.$$

$$\left. + \frac{1}{L}\int_{0^-}^{t} [V_1(t) - V_2(t)]\, dt \right\} = 0$$

the first integral term indicates $i_L(0^-)$ which has zero value from ①

$$i(t) = \frac{V_1(t)}{R_1} + \frac{1}{L}\int_{0^-}^{t} [V_1(t) - V_2(t)]\, dt \rightarrow ③$$

at $t = 0^+$ ③ $\Rightarrow i(t) = \frac{V_1(0^+)}{R_1} + \frac{1}{L}\int_{0^-}^{0^+}[V_1(0^+) - V_2(0^+)]\,dt$

now the value of integral term is zero as limit is zero

$\boxed{V_1(0^+) = R_1 i(t)}$ & $V_2(0^+) = 0$

differentiating ③ w.r.t to t

$$\frac{di(t)}{dt} = \frac{1}{R_1}\frac{dV_1(t)}{dt} + \frac{1}{L}\left[V_1(t) - V_2(t)\right] \Rightarrow ④$$

at $t = 0^+$ ④ $\Rightarrow \frac{di(t)}{dt} = \frac{1}{R_1}\frac{dV_1}{dt}(0^+) + \frac{1}{L}\left(V_1(0^+) - V_2(0^+)\right)$

$$\frac{di}{dt} = \frac{1}{R_1}\frac{dV_1}{dt}(0^+) + \frac{1}{L}\left[R_1 i(t) - 0\right]$$

$$\therefore \frac{1}{R_1}\frac{dV_1}{dt}(0^+) = \frac{di}{dt} - \frac{R_1 i(t)}{L_1}$$

$$\therefore \frac{dV_1}{dt}(0^+) = R_1\frac{di}{dt} - \frac{R_1^2 i(t)}{L_1} \quad V/sec$$

apply KCL @ node 2

$$\frac{1}{L}\int_{0^-}^{t}[V_1(t) - V_2(t)]\,dt = \frac{V_2(t)}{R_2} + C\frac{dV_2(t)}{dt} \rightarrow ⑤$$

at $t = 0^+$

$$\frac{1}{L}\int_{0^-}^{0^+}[V_1(0^+) - V_2(0^+)]\,dt = \frac{V_2(0^+)}{R_2} + C\frac{dV_2(0^+)}{dt}$$

$$0 = 0 + C\frac{dV_2(0^+)}{dt}$$

$$\therefore \frac{dv_2}{dt}(0^+) = 0$$

$$i_L(t) = \frac{1}{L}\int_0^t \left[v_1(t) - v_2(t) \right] dt$$

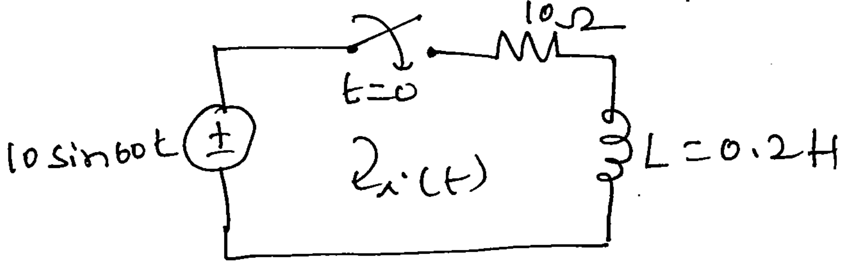
$$\frac{di_L(t)}{dt} = \frac{v_1(t) - v_2(t)}{L}$$

at $t = 0^+$

$$\frac{di_L}{dt}(0^+) = \frac{v_1(0^+) - v_2(0^+)}{L}$$

$$= \frac{R\,i(t)}{L} \text{ A/sec}$$

A series RL circuit is as shown if the switch k in the circuit is closed at $t=0$ find an expression for $i(t)$



For a series RL circuit

$$i(t) = \frac{V_m}{Z}\left[\sin(\omega t+\theta-\alpha) - \sin(\theta-\alpha)e^{-\frac{R}{L}t}\right]$$

with $V_s(t) = V_m \sin(\omega t+\theta)$

$\theta = 0$, $V_m = 10V$, $\omega = 60$ rad/sec

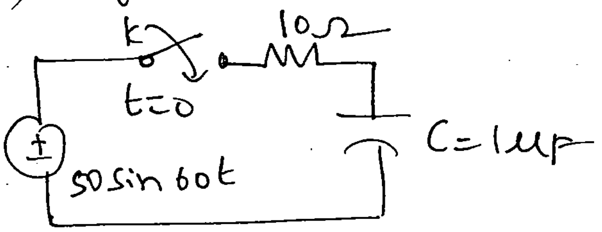
$$Z = \sqrt{R^2+\omega^2L^2} = 15.62\Omega$$

$$\alpha = \tan^{-1}\left(\frac{\omega L}{R}\right) = 50.19^\circ$$

$$i(t) = \frac{10}{15.62}\left[\sin(60t-50.19) - \sin(-50.19)e^{\frac{-10}{0.2}t}\right]$$

$$i(t) = 0.49e^{-50t} + 0.64\sin(\omega t - 50.19^\circ)$$

Find the expression for the current $i(t)$ if the switch is closed at $t=0$



For a series RC circuit

$$i(t) = \left[\frac{V_m}{R}\sin\theta - \frac{V_m}{z}\sin(\theta+\alpha)\right]e^{-t/RC} + \frac{V_m}{z}\sin(\omega t+\theta+\alpha)$$

$$V_s(t) = V_m \sin(\omega t+\theta)$$

$\theta = 0$, $V_m = 50V$, $W = 60$ rad/sec

$$z = \sqrt{\left(R^2 + \frac{1}{WC}\right)^2} = \sqrt{10^2 + \left(\frac{1}{60\times10^{-6}}\right)^2}$$

$$z = 16.67\,k\Omega$$

$$\alpha = \tan^{-1}\left[\frac{1}{WCR}\right] = 89.97^\circ$$

$$i(t) = -3e^{-10^5 t} + 3\sin(60t + 89.97^\circ)\,mA$$

Laplace transform

The Laplace transform method is very much suitable for obtaining solution of higher order differential equations. The method transforms the time domain equations into frequency domain using a complex frequency variable method denoted by $s = \sigma + j\omega$

Advantages of Laplace transform

(1) The convolution in time domain gets converted to multiplication in s-domain

(2) The Laplace transform can analyse discontinuous inputs

(3) It is applicable for the analysis of unstable systems

(4) Initial or boundary conditions are automatically incorporated in Laplace transform

(5) Simplifies the calculations

(6) Differential equations in time domain can be converted into simple algebraic equations in s-domain

Disadvantages of Laplace transform

1. Laplace transform is unsuitable for data processing in random vibrations
2. The conversion $s = j\omega$ is only for sinusoidal steady state analysis
3. The frequency response of the system cannot be estimated. only pole zelo plot can be drawn
4. Laplace transform does not exists for few probability distribution functions

Definition of Laplace transform

The Laplace transform of function $f(t)$ is defined as $F(s) = L\{f(t)\}$

$$= \int_{0^-}^{\infty} f(t)\, e^{-st} \cdot dt \rightarrow (A)$$

s is the complex frequency valiable given by $s = \sigma + j\omega$. The real part σ is attenuation constant (or) damping factor & ω is the angular frequency

Inverse Laplace transform $L^{-1}\{F(s)\}$

$$f(t) = L^{-1}[F(s)] = \frac{1}{2\pi j} \int_{\sigma_1 - j\infty}^{\sigma_1 + j\infty} F(s)\, e^{st} \cdot ds$$

Initial Value theorem:-

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The Initial Value of the time function $f(t)$ can be obtained from its laplace transform $F(s)$ by first multiplying the transform by s & then putting s approach Infinity

i.e, $\underset{t \to 0}{lt} f(t) = \underset{s \to \infty}{lt} [s F(s)]$

Proof:- consider $L\{\frac{d}{dt} f(t)\} = s F(s) - f(0^+)$

$$\therefore L\left[\frac{d}{dt} f(t)\right] = \int_0^\infty \left[\frac{d}{dt} f(t)\right] e^{-st} dt$$

now let s approach Infinity

$$\underset{s \to \infty}{lt} \{s F(s) - f(0^+)\} = \underset{s \to \infty}{lt} \int_0^\infty \left[\frac{d}{dt} f(t)\right] e^{-st} dt$$

$$= \underset{s \to \infty}{lt} e^{-st} f(t)$$

$$-f(0^+) + \underset{s \to \infty}{lt} [s F(s)] = 0$$

$$\therefore f(0^+) = \underset{s \to \infty}{lt} s \cdot F(s)$$

$$f(0^+) = \underset{s \to \infty}{lt} f(t) = \underset{s \to \infty}{lt} [s F(s)]$$

In order to apply Initial Value theorem the condition is

$$F(s) = \frac{\text{Numerator}}{\text{Denominator}}$$

The degree of the Numerator must be less than that of the Denominator

<u>Final Value theorem:-</u>

To apply the final value theorem the condition is the Denominator of F(s) must not have roots on the RHS plane and on the imaginary axis except a simple pole at the origin

The theorem states that "t" tends to ∞ can be determined from its laplace transform by first multiplying the transform by S & then letting S approach zero

i.e, $$\lim_{t \to \infty} f(t) = \lim_{s \to 0} [S F(s)]$$

Proof:- consider the laplace transform of derivative of a function f(t)

i.e, $L\left[\frac{d}{dt}f(t)\right] = \int_0^\infty \left[\frac{d}{dt}f(t)\right] e^{-st}\,dt$

$= \left[s\,F(s) - f(0^+)\right] = \int_0^\infty \left[\frac{d}{dt}f(t)\right] e^{-st}\,dt$

since "s" is not a function of t

Now let s approach zero

$\lim_{s\to 0}\left[s\,F(s) - f(0^+)\right] = \lim_{s\to 0}\int_0^\infty \frac{d}{dt}f(t)\,e^{-st}\,dt$

$= \int_0^\infty \frac{d}{dt}f(t)\,dt \quad = \lim_{t\to\infty}\int_0^t \left[\frac{d}{dt}f(t)\right] dt$

$\lim_{s\to 0}\left[s\,F(s) - F(0^+)\right] = \lim_{t\to\infty}\left[f(t) - f(0^+)\right]$

$f(0^+)$ is not a function of 's' & may be removed from the limit

$-f(0^+) + \lim_{s\to 0}\left[s\,F(s)\right] = \lim_{t\to\infty} f(t) - f(0^+)$

$\therefore \lim_{t\to\infty} f(t) = \lim_{s\to 0}\left[s\,F(s)\right]$

using Initial value theorem find $i(0^+)$ whose laplace transform $I(s)$ is $\dfrac{4s+5}{(s+1)(s+3)}$

Soln:- $I(s) = \dfrac{4s+5}{s^2+4s+3}$

according to Initial value theorem

$$f(0^+) = \lim_{s\to\infty} [s\,F(s)]$$

$$i(0^+) = \lim_{s\to\infty} [s\,I(s)]$$

$$= \lim_{s\to\infty} s\left[\frac{4s+5}{s^2+4s+3}\right]$$

$$= \lim_{s\to\infty}\left[\frac{s}{s^2}\left[\frac{4s+5}{1+\frac{4}{s}+\frac{3}{s^2}}\right]\right] = \lim_{s\to\infty}\left[\frac{4+\frac{5}{s}}{\left(1+\frac{4}{s}+\frac{3}{s^2}\right)}\right]$$

$$i(0^+) = \frac{4}{1} = 4$$

Find the laplace transform of

ⓐ t ⓑ $s(t) = 1$ for $t = 0$

$s(t) = 0$ for $t \neq 0$

ⓐ $$L\{f(t)\} = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$= \int_0^\infty t\, e^{-st}\, dt$$

$$= \int_0^\infty \frac{d}{ds}\left(e^{-st}\right) dt = -\frac{d}{ds} \int_0^\infty (1)\, e^{-st}\, dt$$

$$= \frac{d}{ds}\left(\frac{1}{s}\right) = \frac{1}{s^2}$$

ⓑ $$s(t) = \frac{d\, u(t)}{dt}$$

taking laplace transform on both sides

$$L\{s(t)\} = L\left\{\frac{d u(t)}{dt}\right\}$$

$$L\left\{\frac{d\, f(t)}{dt}\right\} = S\,F(s) - f(0^-)$$

$$= L\{s(t)\} = S \cdot L\{u(t)\} - u(t)\Big|_{t=0^-}$$

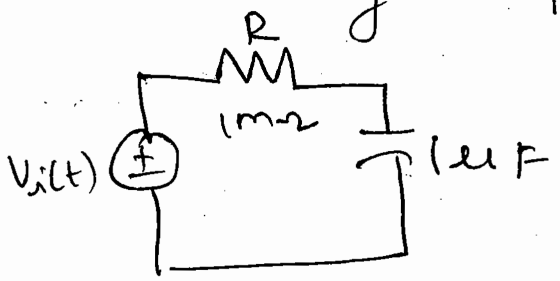
but $u(t)\Big|_{t=0} = 0$

$$\therefore L\{u(t)\} = \frac{1}{S}$$

$$\therefore L\{u(t)\} = S \cdot \frac{1}{S} - 0$$

$$L\{s(t)\} = 1$$

for the critically related network obtain the expression for the current using Laplace transform



$$V_i(t) = \delta(t) \quad \therefore \quad V_i(s) = 1$$

applying KVL to the loop

$$Ri + \frac{1}{C}\int i\,dt = V_i(t)$$

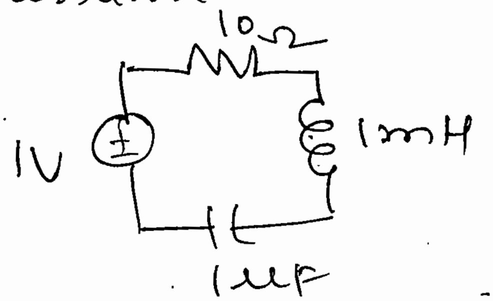
taking Laplace transform

$$R\,I(s) + \frac{1}{C}\int I(s)\,dt = V_i(s)$$

$$I(s)\left[\frac{RCS+1}{CS}\right] = 1$$

$$I(s) = 10^{-6}\,\frac{s}{s+1} = 10^{-6}\left[\delta(t) - e^{-t}\right]$$

using Laplace transform obtain an expression for the current $i(t)$ assume zero Initial conditions



Soln:- $v_i(t) = 1 \quad \therefore \quad V_i(s) = \frac{1}{s}$

applying KVL to the loop

$$10i + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = v_i(t)$$

taking laplace transform of both sides

$$10\,I(s) + L\,I(s) + \frac{1}{C}\frac{1}{s} = \frac{1}{s}$$

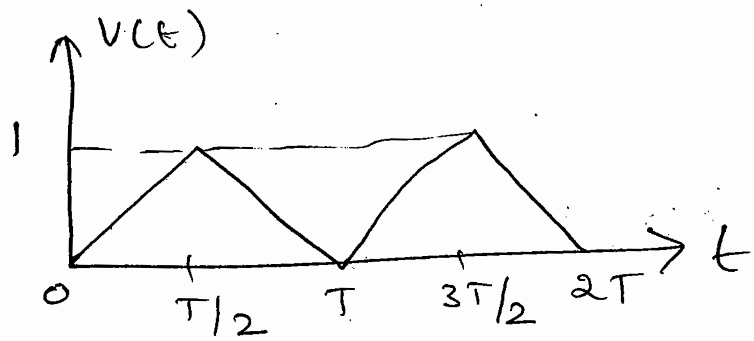
$$10\,I(s) + 10^{-3}\,s\,I(s) + \frac{1}{10^{-6}}\frac{1}{s} = \frac{1}{s}$$

$$I(s)\left[10 + 10^{-3} + \frac{10^{6}}{s}\right] = \frac{1}{s}$$

$$I(s)\,\mathcal{L}\left\{\frac{1}{(s+5000)^2 + (31250)^2}\right\}$$

$$= 3.2\times10^{-5}\,\frac{31250}{(s+5000)^2 + (31250)^2}$$

Find the laplace transform of the periodic wave form



The slope is $\left[\frac{1-0}{T/2-0}\right] = \frac{2}{T}$

Slope = $\tan\theta = \frac{y}{x} = \frac{\text{vertical}}{\text{horizontal}}$

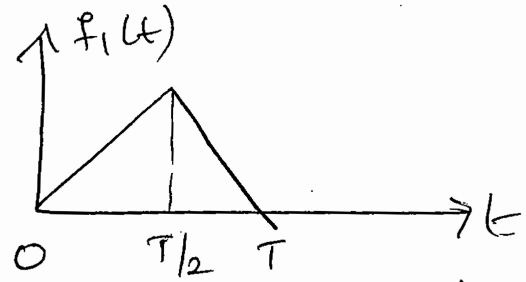
vertical distance $= 1-0$

horizontal distance $= T/2 - 0$ (from right to left)

Note:-

Rising	$T/2 - 0$
Falling	$T/2 - T$

$\therefore \quad f_a(t) = \frac{2}{T} t.u(t)$



$f_b(t)$ is to stop the wave form @ $t = \frac{T}{2}$

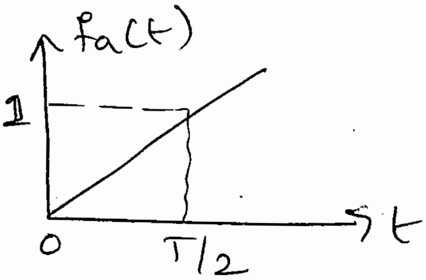
& $f_c(t) \Rightarrow$ bring the waveform back to zero.

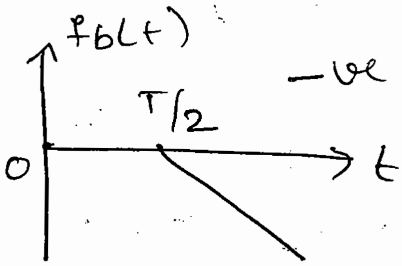
(1a) 5.

The ramp $f_a(t)$ is stopped @ $t = \frac{T}{2}$

Thus there starts a ramp @ $t = T/2$ with slope $-\frac{2}{T}$.

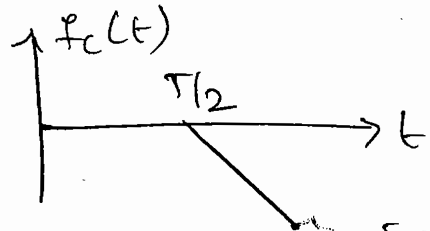
$\therefore \; f_b(t) = -\frac{2}{T}\left(t - \frac{T}{2}\right) u(t - T/2)$





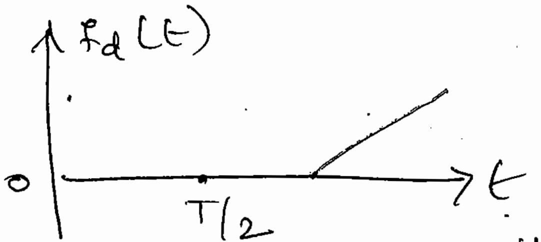
−ve indicates negative slope

but @ same time $t = \frac{T}{2}$ another ramp is starting @ slope $-\frac{2}{T}$



$\therefore \; f_c(t) = -\frac{2}{T}\left(t - \frac{T}{2}\right) u\left(t - \frac{T}{2}\right)$

this ramp is stopped @ $t = T$ hence there exists a ramp with slope $\frac{2}{T}$ @ $t = T$



$\therefore \; f_d(t) = \frac{2}{T}(t - T)\, u(t - T)$

$f_d \Rightarrow$ stop f_c @ $t = T$ otherwise f_c will continue Infinitely

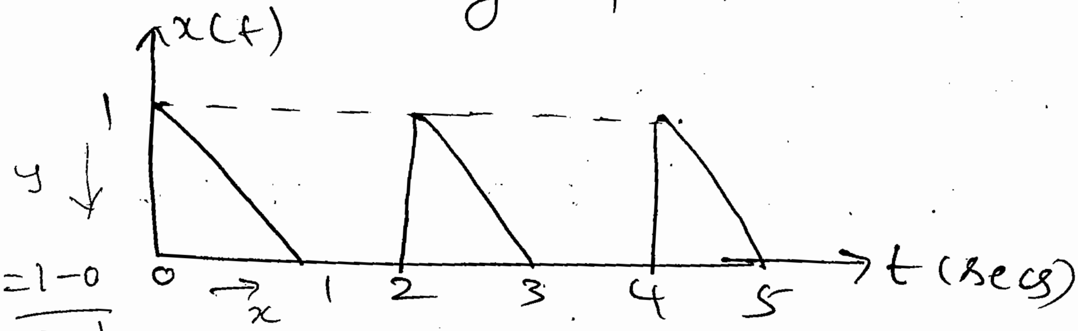
$\therefore \; f_1(t) = f_a(t) + f_b(t) + f_c(t) + f_d(t)$

$= \frac{2}{T} t\, u(t) - 2\left\{\frac{2}{T}\left(t - \frac{T}{2}\right) u\left(t - \frac{T}{2}\right)\right\} + \frac{2}{T}(t - T)\, u(t - T)$

$F_1(s) = \frac{2}{T} \cdot \frac{1}{s^2} - \frac{4}{T} \cdot \frac{1}{s^2} e^{-T/2 s} + \frac{2}{T} \frac{1}{s^2} e^{-Ts}$

$\therefore \; F(s) = \frac{F_1(s)}{1 - e^{-Ts}} = \frac{\frac{2}{T} \frac{1}{s^2}\left(1 - 2e^{-T/2 s} + e^{-Ts}\right)}{\left(1 - e^{-Ts}\right)}$

obtain the laplace transform of the periodic signal



$\frac{y}{x} = \frac{1-0}{0-1}$

as the signal is periodic

1) step of 1 @ $t = 0$ $\therefore f_a(t) = u(t)$

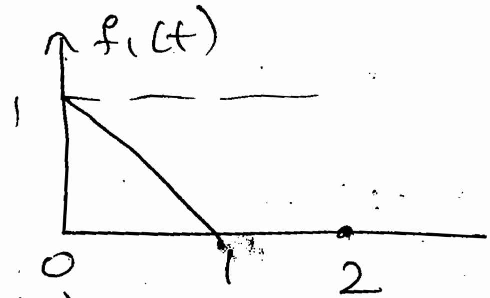
2) Ramp @ $t = 0$ with slope $\frac{0-1}{1-0} = -1$

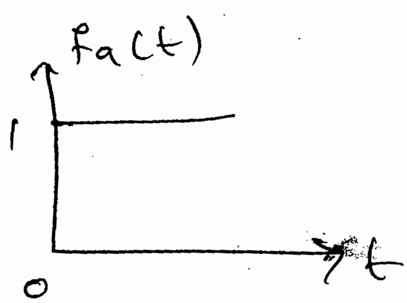
$\therefore$ $f_b(t) = -t\,u(t)$

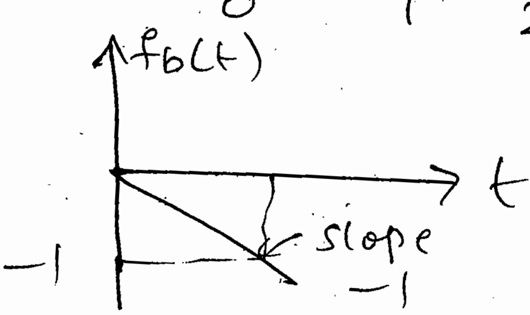
3) Ramp @ $t=0$ is stopped & converted to constant valued function @ $t=1$; so there exists a ramp @ $t=1$ of same slope as that of $f_b(t)$ but opposite sign

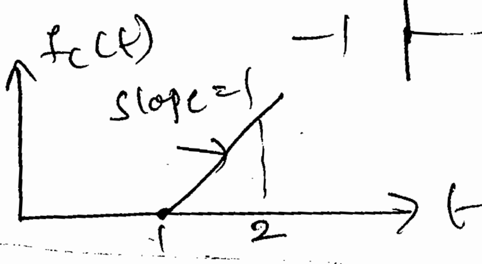
$f_c(t) = (t-1)\,u(t-1)$

consider first cycle









$$f_1(t) = f_a(t) + f_b(t) + f_c(t)$$

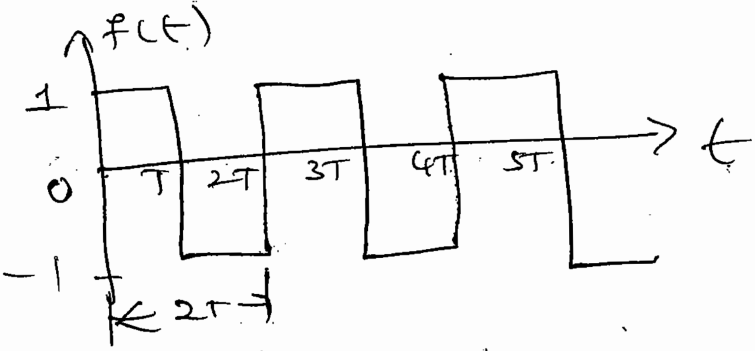
$$= u(t) - t\,u(t) + (t-1)\,u(t-1)$$

$$F_1(s) = \frac{1}{s} - \frac{1}{s^2} + \frac{1}{s^2} e^{-s}$$

$$\therefore \quad F(s) = \frac{F_1(s)}{1 - e^{-Ts}}$$

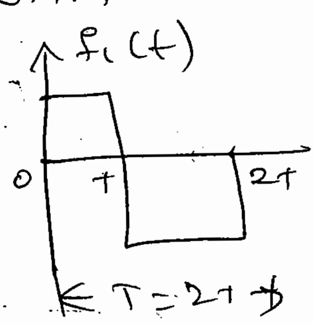
$$= \frac{\frac{1}{s} - \frac{1}{s^2} + \frac{1}{s^2} e^{-s}}{1 - e^{-2s}}$$

(3) obtain the laplace transform of the square wave train



$f_1(t)$ starts with a step of 1 at $t = 0$

$f_a(t) = u(t)$



$u(t) - 2\,u(t-T)$

already $u(t)$ has $+1$ to bring it to -1 add -2

@ $t = T$, there is instantaneous change in $f_1(t)$ from 1 to -1 so there is step of -2 @ $t = 2T$

$f_b(t) = -2u(t-T)$

@ $t = 2T$ there is increase in $f_1(t)$ from -1 to 0 so there is step of 1 @ $t = 2T$

$f_c(t) = u(t-2T)$

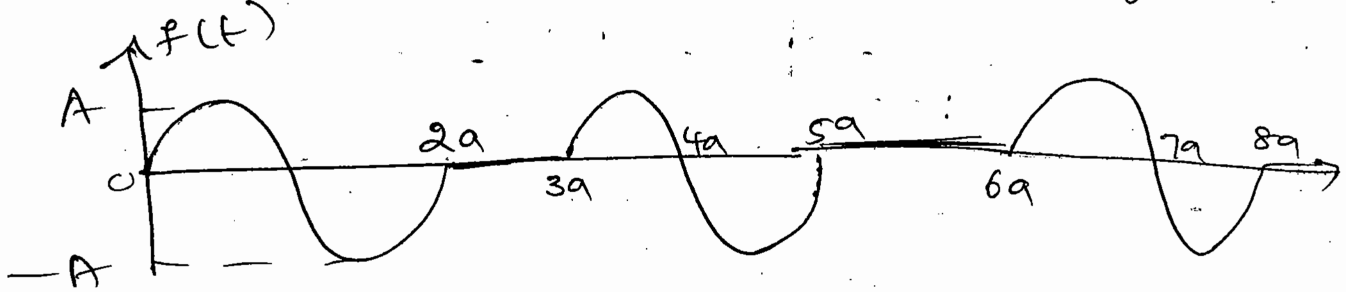
$\therefore f_1(t) = f_a(t) + f_b(t) + f_c(t)$

$= u(t) - 2u(t-T) + u(t-2T)$

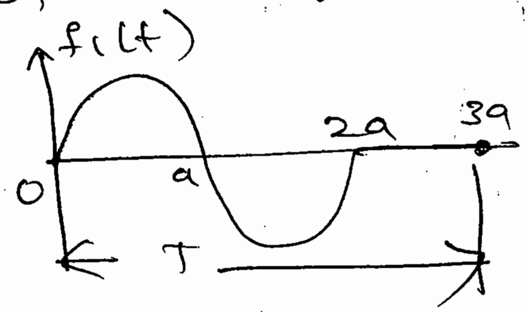
$F_1(s) = \frac{1}{s} - \frac{2}{s} e^{-Ts} + \frac{1}{s} e^{-2Ts}$

$\therefore F(s) = \frac{F_1(s)}{1-e^{-Ts}} = \frac{\frac{1}{s}\left[1-2e^{-Ts}+e^{-2Ts}\right]}{1-e^{-2Ts}}$

* Find the laplace transform of



consider the first cycle



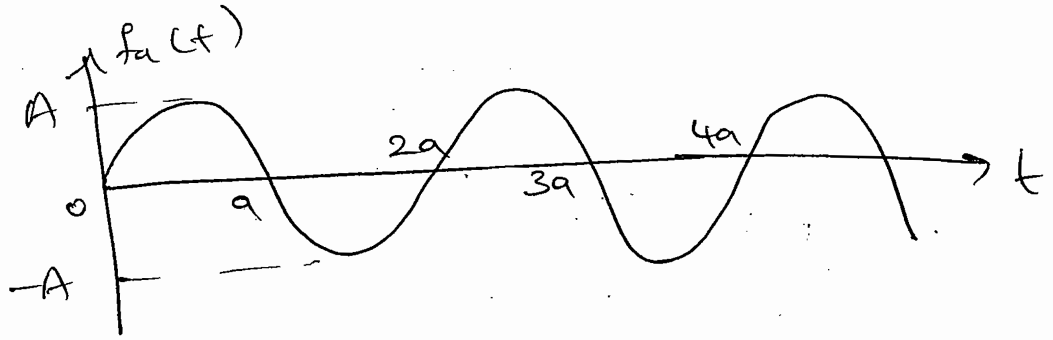
Given signal is periodic. (4)

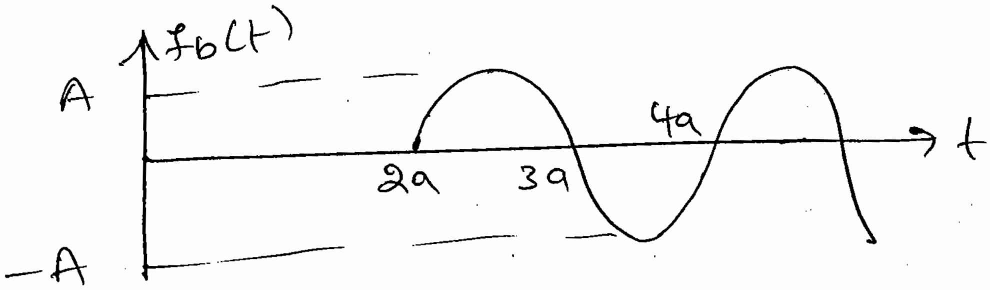
& this cycle is made up of 2 sine waves $f_a(t) = A \sin \omega t\, u(t)$.

we want to cancel the sinewave for $t > 2a$

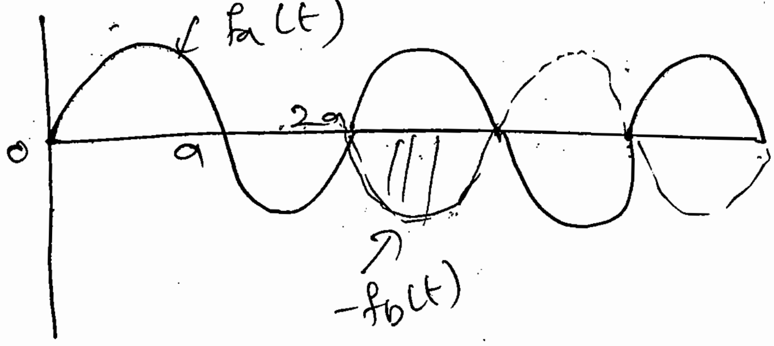
$\therefore$ consider a sinewave which is shifted @ $t = 2a$

$\therefore \quad f_b(t) = A \sin \omega(t-2a)\, u(t-2a)$





this $f_b(t)$ is to be subtracted from $f_a(t)$ so that each half cycle of $f_a(t)$ will get cancelled by $f_b(t)$ to get required first cycle of the given $f(t)$.



$$f_1(t) = f_a(t) - f_b(t)$$

with $T = 3a$

$$f_1(t) = A \sin \omega t \, u(t) - A \sin[\omega(t-2a)] \, u(t-2a)$$

$$\therefore F_1(s) = A \cdot \frac{\omega}{s^2+\omega^2} - A \frac{\omega}{s^2+\omega^2} e^{-2as}$$

$$\therefore F(s) = \frac{F_1(s)}{1-e^{-Ts}}$$

where $T = 3a$ of $f_1(t)$

$$F(s) = \frac{A\omega(1-e^{-2as})}{s^2+\omega^2(1-e^{-3as})}$$

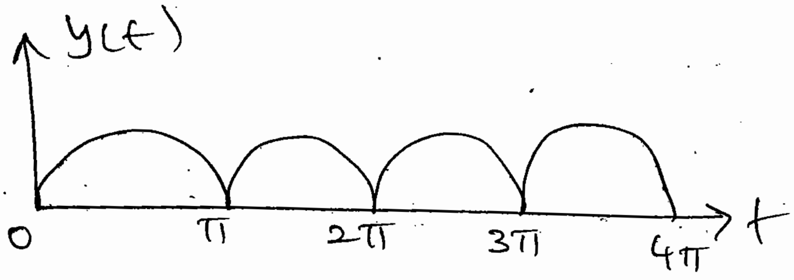
ω for $f_a(t)$ & $f_b(t)$ is related to T by the relation

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2a} = \frac{\pi}{a}$$

this is because time period T of $f_a(t)$ & $f_b(t)$ is $2a$ while the time period of given waveform is $3a$ $\therefore$ T in the expression

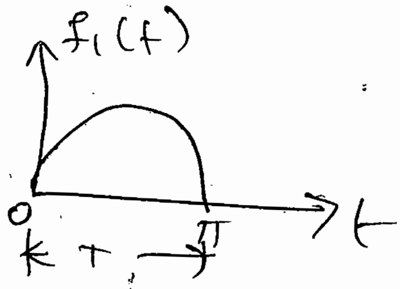
$$\frac{F_1(s)}{1-e^{-Ts}}$$

Find the laplace transform



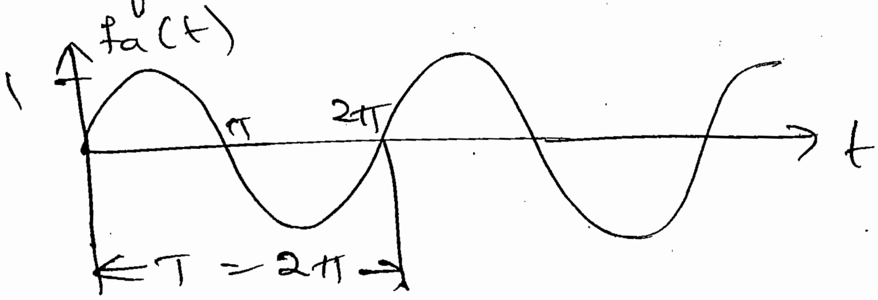
consider the first cycle

time period of this cycle is:



$T = \pi$

$f_1(t)$ is made up of 2 sinusoidal waveforms. consider a sinusoidal waveform with time period $T = 2\pi$



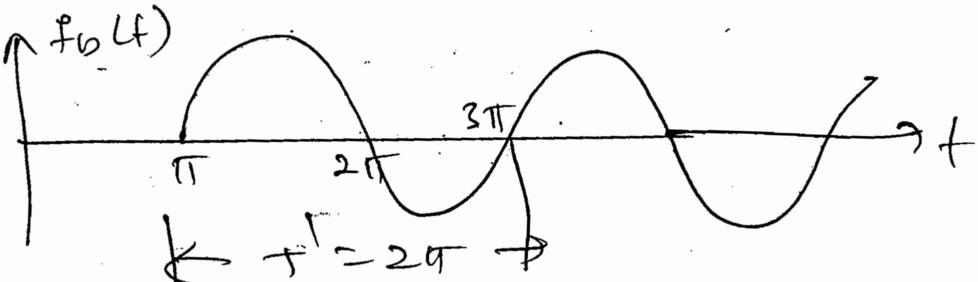
CB-C-DIVAKARA

$fa(t) = 1 \sin w'(t)$

$$w' = \frac{2\pi}{T'} = \frac{2\pi}{2\pi} = 1$$

$fa(t) = \sin(t)\, u(t)$

Now we want to cancel the half cycles after $t = \pi$, so consider shifted sine wave as show with period $T' = 2\pi$

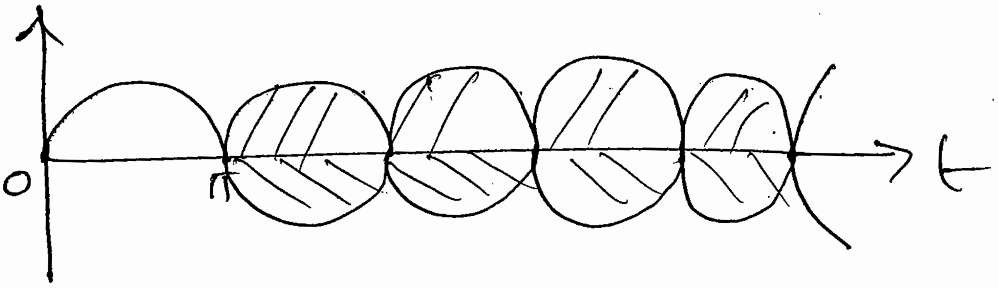


$$f_b(t) = 1 \sin\left[\omega'(t-\pi)\right] u(t-\pi)$$

$$\omega' = \frac{2\pi}{T'} = 1$$

$$f_b(t) = \sin(t-\pi)\, u(t-\pi)$$

The addition of $f_a(t)$ & $f_b(t)$ gives $f_1(t)$



$$f_1(t) = f_a(t) + f_b(t)$$

$$= \sin(t)\, u(t) + \sin(t-\pi)\, u(t-\pi)$$

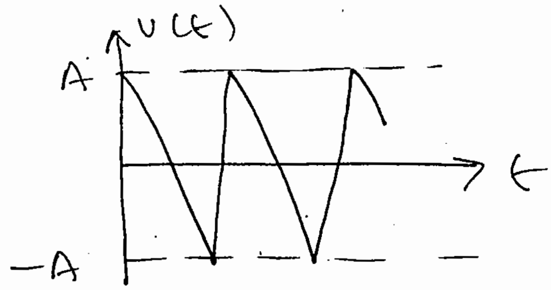
$$F_1(s) = \frac{1}{s^2+1} + \frac{1}{s^2+1} e^{-\pi s}$$

$$Y(s) = \frac{F_1(s)}{1-e^{-Ts}}$$

$$= \frac{\frac{1}{s^2+1}\left[1+e^{-\pi s}\right]}{\left[1-e^{-\pi s}\right]}$$

$$Y(s) = \frac{1}{s^2+1}\left[\frac{1+e^{-\pi s}}{1-e^{-\pi s}}\right] = \frac{1}{s^2+1} \coth\left(\frac{\pi s}{2}\right)$$

Determine the laplace transform of the periodic sawtooth waveform. (23)



It starts with step of A @ $t=0$

$$f_a(t) = A\,u(t)$$

then there is a ramp of slope $\frac{-2A}{T}$.

$$\therefore\ f_b(t) = \frac{-2A}{T}(t-T)u(t-T)$$

and a step of A units is again added at $t=T$ to make the final value zero

$$f_d(t) = A\,u(t-T)$$

$$f_1(t) = f_a(t) + f_b(t) + f_c(t) + f_d(t)$$

$$= A\,u(t) - \frac{2A}{t}\,t \cdot u(t) + \frac{2A}{\cancel{T}}(t-T)\,u(t-T)$$
$$+ A\,u(t-T)$$

$$F_1(s) = \frac{A}{s} - \frac{2A}{Ts^2} + \frac{2A}{Ts^2}e^{-Ts} + \frac{Ae^{-Ts}}{s}$$

$$F(s) = \frac{F_1(s)}{1-e^{-Ts}}$$

$$F(s) = \frac{\frac{A}{s} - \frac{2A}{Ts^2} + \frac{2A}{Ts^2}e^{-Ts} + \frac{Ae^{-Ts}}{s}}{1-e^{-Ts}}$$

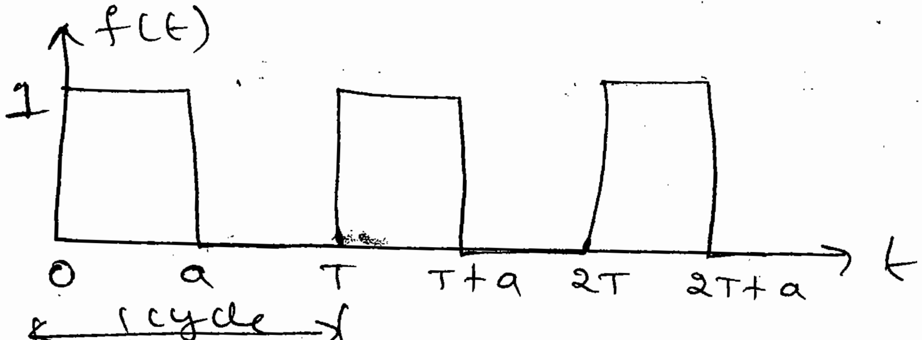
Laplace transform of a periodic function

Consider a periodic function of time period T satisfying the condition

$f(t+nT) = f(t)$ where "n" is positive or negative integer

LT of such periodic function $F(s) = \frac{1}{1-e^{-sT}} F_1(s)$

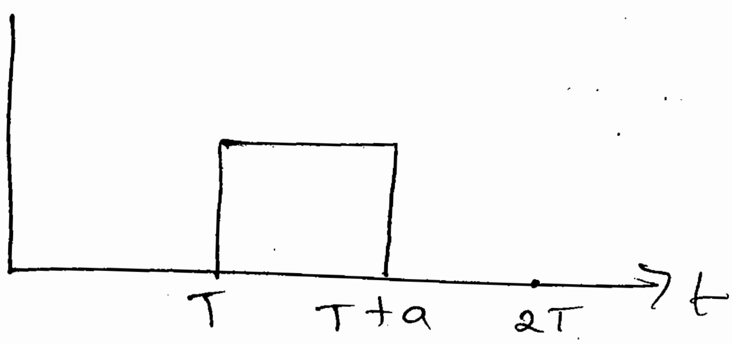
$F_1(s)$ is the laplace transform of the first cycle of the periodic function



Let $f_1(t)$ be the first cycle of a periodic function $f(t)$

$f_1(t) = u(t) - u(t-a)$

$f_2(t) = u(t-T) - u(t-(T+a))$

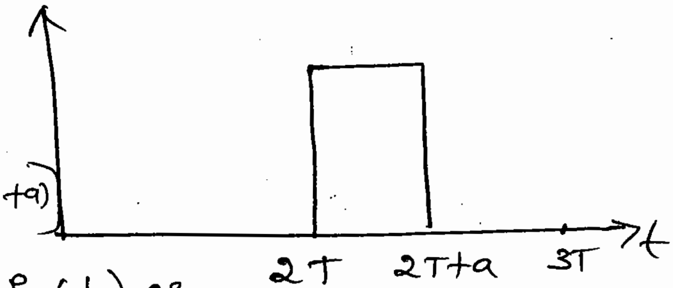


from $f_1(t)$

$f_2(t) = f_1(t-T)\,u(t-T)$

multiplication by $u(t-T)$ confirms that $f_2(t)$ i.e, second cycle is zero for $t < T$

consider 3rd cycle



$f_3(t) = u(t-2T) - u[t-(2T+a)]$

it can be seen from $f_1(t)$ as

$f_3(t) = f_1(t-2T)\,u(t-2T)$

multiplication $u(t-2T)$ confirms that $f_3(t)$ i.e, second cycle is zero for $t < 2T$

$\therefore \quad f_4(t) = f_1(t-3T)\,u(t-3T)$

|||ly for $f_n(t) = f_1(t-(n-1)T)\,u(t-(n-1)T)$

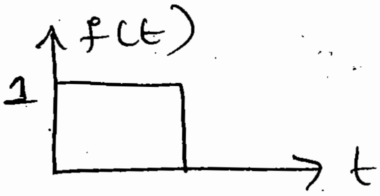
$\therefore \quad L\{f_1(t)\} = F_1(s)$

& $\quad L\{f(t-T)\} = e^{-Ts}F(s)$

$$\frac{1}{1-e^{-Ts}} = 1 + e^{-Ts} + e^{-2Ts} + e^{-3Ts} + \cdots$$

$$\therefore \quad F(s) = F_1(s) \cdot \frac{1}{1-e^{-Ts}}$$

Obtain the laplace transform of the function



Sol:-

The given function consists of unit step function $u(t)$ & a delayed unit step function is subtracted from $u(t)$

$$\therefore \quad f(t) = u(t) - u(t-T)$$

taking laplace transform of both sides

$$F(s) = L\{u(t)\} - L\{u(t-T)\}$$

Imp

Laplace transforms of standard functions

I. unit impulse function

$$\delta(t) = 1 \quad \text{for } t=0$$
$$= 0 \quad \text{for } t \neq 0$$

$$L\{\delta(t)\} = 1$$

II. Ramp function :-

$$r(t) = t \quad ; \; t \geq 0$$
$$= 0 \quad ; \; t < 0$$

$$L\{r(t)\} = \frac{1}{s^2}$$

& $$L\{t.u(t)\} = \frac{1}{s^2}$$

III Step function

$$u(t) = 1 \quad ; t \geq 0$$
$$= 0 \quad ; t < 0$$

$$L\{u(t)\} = \frac{1}{s}$$

$$L\{A\,u(t)\} = \frac{A}{s} \quad ; \text{ A is constant}$$

$$L\{u(t-T)\} = e^{-Ts} \cdot \frac{1}{s} = \frac{e^{-Ts}}{s}$$

use Initial & final value theorem where they apply to find f(0) & f(∞) for

(a) $$F(s) = \frac{e^{2s}(s+2)}{s^2+5}$$

by Initial value theorem

$$f(0) = f(0^+) = \lim_{s\to\infty} z.\, s\,F(s) = \lim_{s\to\infty} s\left[\frac{(s+2)e^{2s}}{s^2+5}\right]$$

$$= \lim_{s\to\infty} \frac{s^2\left(1+\frac{2}{s}\right)e^{-2s}}{s^2\left(1+\frac{5}{s^2}\right)} = 0$$

by final value theorem

$$f(\infty) = \lim_{s\to 0} s.F(s) = \lim_{s\to 0} s\left[\frac{(s+2)e^{2s}}{(s^2+5)}\right] = 0.$$

(b) $$F(s) = \frac{s^3+7s^2+5}{s(s^3+3s^2+4s+2)}$$

by Initial Value theorem

$$f(0) = f(0^+) = \lim_{s\to\infty} s.F(s)$$

$$= \lim_{s\to\infty} s\left[\frac{s^3+7s^2+5}{s(s^3+3s^2+4s+2)}\right]$$

$$= \lim_{s\to\infty} \frac{s^3\left[1+\frac{7}{s}+\frac{5}{s^3}\right]}{s^3\left(1+\frac{3}{s}+\frac{4}{s^2}+\frac{2}{s^3}\right)} = 1$$

By Final value theorem

$$f(\infty) = \lim_{s\to\infty} s.F(s) = \lim_{s\to 0} s\left[\frac{s^3+7s^2+5}{s(s^3+3s^2+4s+2)}\right]$$

$$= \lim_{s\to 0}\left[\frac{s^3+7s^2+5}{s^3+3s^2+4s+2}\right] = \frac{5}{2}$$

Find the laplace transform of

$f(t) = 5 + 4e^{-2t}$

$F(s) = L\,f(t) = L\left[5+4e^{-2t}\right]$

$= L\{5\} + 4\,L\{e^{-2t}\}$

$= \frac{5}{s} + 4\left[\frac{1}{s+2}\right]$

$= \frac{5(s+2)+4s}{s(s+2)} = \frac{9s+10}{s(s+2)}$

properties of LT

Scaling theorem:- {multiplication by constant K}

If K is a constant then the LT of K. f(t) is given as K times the laplace transform of f(t)

$L\{K\,f(t)\} = K.\,F(s)$

Complex translation:-

$F(s-a) = L\{e^{at}f(t)\}$

a is the complex number

$L\{e^{-at}f(t)\} = F(s+a)$

Real translation {Shifting theorem}.

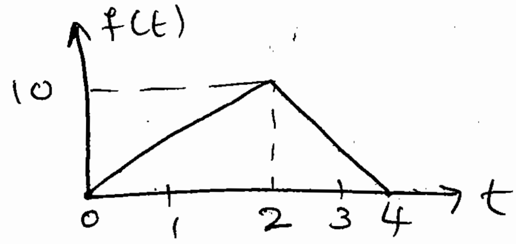
This theorem is used to obtain the laplace transform of the shifted (or) delayed function of time

$$L\{f(t-T)\} = e^{-Ts} F(s)$$

Real Integration

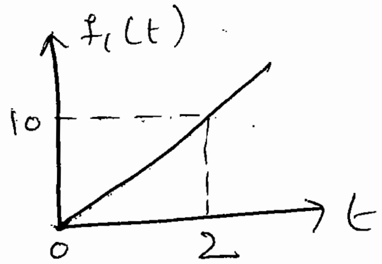
$$L\left\{ \int_0^t f(t)\cdot dt \right\} = \frac{F(s)}{s}$$

Obtain the laplace transform of the function shown



23

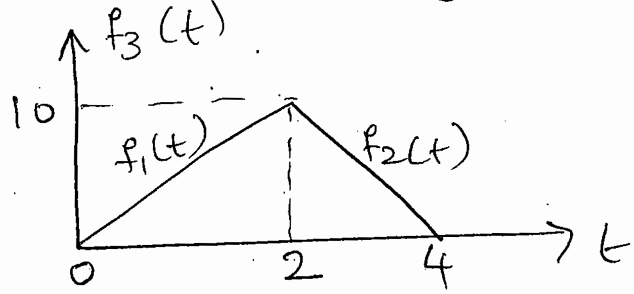
Soln:- The function f(t) starts with a ramp which passes through origin with 2 points as (0, 0) & (2, 10)



$\therefore$ slope $= \dfrac{10-0}{2-0} = 5$

$\therefore f_1(t) = 5 \cdot t \, u(t)$

this ramp is to be stopped @ $t=2$ so we must add a ramp of equal but opposite sign slope at $t=2$.



$f_2(t) = -5(t-2)\,u(t-2)$

slope $= \dfrac{0-10}{4-2} = -5$

addition of $f_1(t)$ & $f_2(t)$ gives a constant valued function $f_3(t)$.

but from original waveform it is clear that there is another ramp

2 points on this ramp are

$(2, 10)$ & $(4, 0)$.

$f_3(t) = 5(t-4)\, u(t-4)$

since the ramp continue to ∞, we add a ramp of equal but opposite sign slope @ $t=4$, so the overall function stops @ $t=4$

$\therefore\ f_4(t) = 5(t-4)\, u(t-4)$

$\therefore\ f(t) = f_1(t) + f_2(t) + f_3(t) + f_4(t)$

$= 5\, t\, u(t) - 5(t-2)\, u(t-2) - 5(t-2)\, u(t-2)$

$+ 5(t-4)\, u(t-4)$

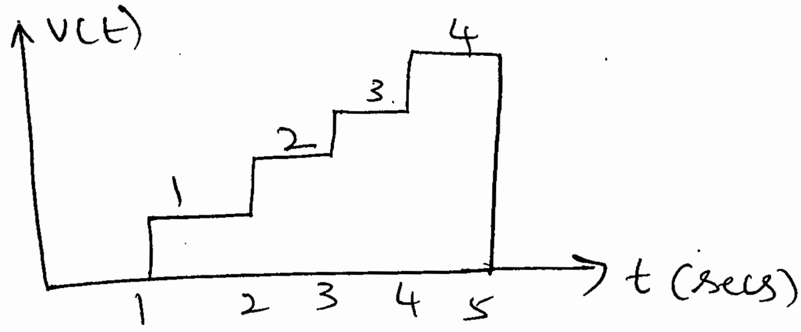
$= 5\left\{t\, u(t) - 2(t-2)\, u(t-2) + (t-4)\, u(t-4)\right\}$

$\therefore\ F(s) = 5\left\{\frac{1}{s^2} - \frac{2}{s^2} e^{-2s} + \frac{1}{s^2} e^{-4s}\right]$

$= \frac{5}{s^2}\left[1 - 2e^{-2s} + e^{-4s}\right]$

22/11/2016

* Assuming that the stair case voltage waveform is not repeated find its laplace transform.



Sol^n :- $v(t) = u(t-1) + u(t-2) + u(t-3) + u(t-4) - 4u(t-5)$

$$\therefore V(s) = \frac{1}{s}\left[e^{-s} + e^{-2s} + e^{-3s} + e^{-4s} - 4e^{-5s}\right]$$

Convolution theorem :-

If $F_1(s)$ & $F_2(s)$ are the laplace transforms of $f_1(t)$ & $f_2(t)$ respectively

$$L\int_0^t f_1(\tau)\, f_2(t-\tau)\, d\tau$$

$$= L\int_0^t f_1(t-\tau)\, f_2(\tau)\, d\tau = L\{f_1(t) * f_2(t)\}$$

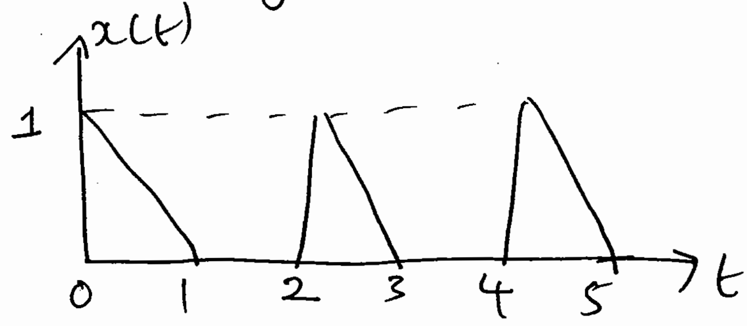
τ is a dummy variable for t

$$= L\{f_2(t) * f_1(t)\} = F_1(s) \cdot F_2(s)$$

obtain the Laplace transform of the periodic signal shown (29)

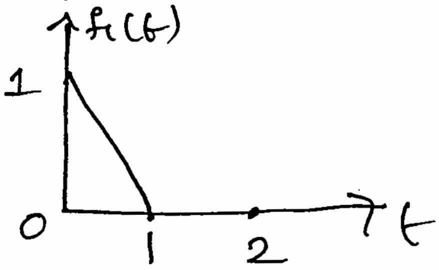
July 2017

OSM



B.C. DIVAKARA

as the signal is periodic consider its first cycle



it is made up of

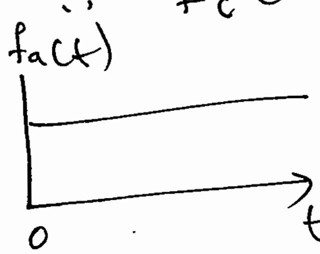
i) Step of 1 at $t=0$ i.e, $f_a(t) = u(t)$

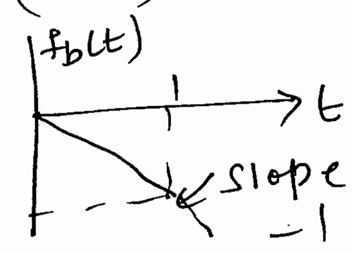
ii) Ramp at $t=0$ with slope $\frac{0-1}{1-0} = -1$

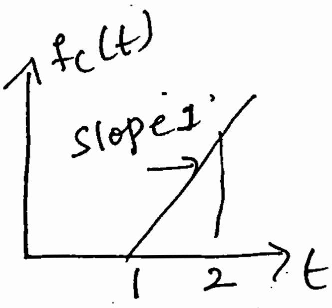
i.e, $f_B(t) = -t\,u(t)$

iii) The ramp at $t=0$ is stopped and converted to constant valued function at $t=1$. so there exists a ramp at $t=1$ of same slope as that of $f_b(t)$ but opposite sign

$\therefore \quad f_c(t) = (t-1)\,u(t-1)$







$$f_1(t) = f_a(t) + f_b(t) + f_c(t)$$
$$= u(t) - t\,u(t) + (t-1)\,u(t-1)$$

$$f_1(s) = \frac{1}{s} - \frac{1}{s^2} + \frac{1}{s^2}e^{-s}$$

$$F(s) = \frac{F_1(s)}{1-e^{-Ts}}$$

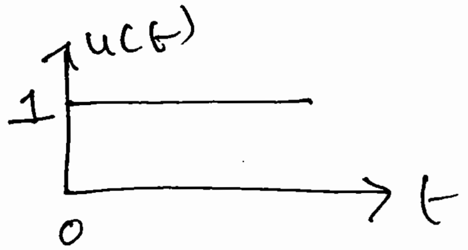
$$= \frac{\frac{1}{s} - \frac{1}{s^2} + \frac{1}{s^2}e^{-s}}{1-e^{-2s}}$$

<u>Singularity functions</u>

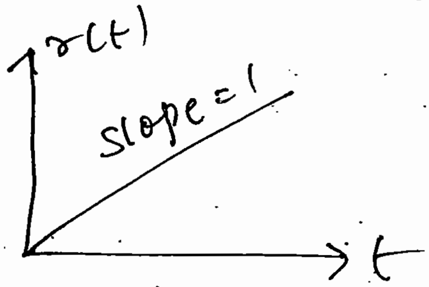
1. unit step function $u(t)$
2. Delta function $\delta(t)$
3. Ramp function $r(t)$

They are called singularity functions because they are either not finite or they do not possess finite derivatives everywhere

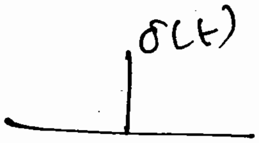
<u>Unit step function</u> $u(t) = \begin{cases} 0, & t \leq 0 \\ 1, & t \geq 0 \end{cases}$



<u>Ramp function</u> $r(t) = \begin{cases} 0, & t \leq 0 \\ t & t \geq 0 \end{cases}$



Delta function $\delta(t) = \frac{d}{dt} u(t) = \begin{cases} 0, & t<0 \\ 0, & t>0 \end{cases}$



July 2017 (8m) Given the signal $x(t) = \begin{cases} 3, & t<0 \\ -2, & 0<t<1 \\ 2t-4, & t>1 \end{cases}$

express $x(t)$ in terms of singularity functions, also find Laplace transform of $x(t)$

Soln ⇒ i) The signal $x(t)$ may be regarded as $3u(-t)$

ii) in the intervals, $t<0$, $x(t)$ may be regarded as $-2[u(+t) - u(t-1)]$ and

iii) for $t>1$, $x(t)$ may be viewed as $(2t-4)u(t-1)$

Thus $x(t) = 3u(-t) - 2[u(t) - u(t-1)]$
$+ (2t-4)u(t-1)$

$\therefore x(t) = 3[1-u(t)] - 2u(t) + 2u(t-1)]$
$+ 2t\,u(t-1) - 4u(t-1)$

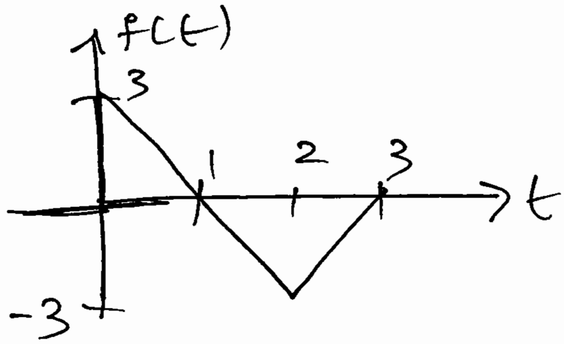
$= 3 - 5u(t) - 2u(t-1) + 2(t-1+1)u(t-1)$

$= 3 - 5u(t) - 2u(t-1) + 2(t-1)u(t-1)$
$+ 2u(t-1)$

$= 3 - 5u(t) + 2r(t-1)$

$L\{x(t)\}$ cannot be found because $x(t)$ contains a constant 3 for $-\infty < t < 0$ { a non causal signal }.

Express f(t) in terms of singularity function and find F(S)

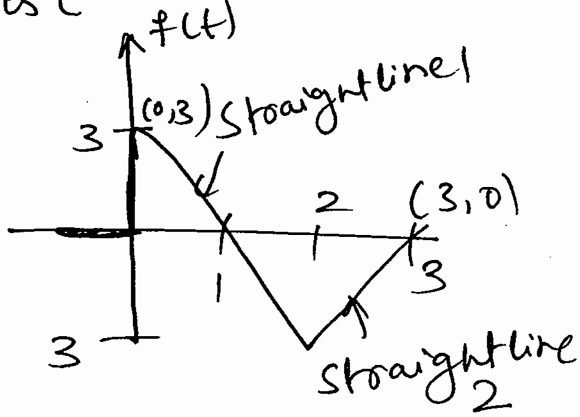


to find f(t) for $0<t<2$

Equation of the straight line 1 is

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

here y is f(t) and x is t



$$\therefore \frac{f(t)-3}{t-0} = \frac{-3-3}{2-0}$$

$$\Rightarrow 2f(t)-6 = -6t$$

$$f(t) = 3-3t$$

to find f(t) for $2<t<3$

$$\frac{f(t)+3}{t-2} = \frac{0+3}{3-2}$$

$$\Rightarrow f(t)+3 = 3t-6$$

$$f(t) = 3t-9$$

$$f(t) = \begin{cases} 3-3t & , 0<t<2 \\ 3t-9 & , 2<t<3 \\ 0 & , \text{otherwise} \end{cases}$$

$\therefore$

$f(t) = [3-3t][u(t) - u(t-2)] + [3t-9][u(t-2) - u(t-3)]$

$= 3u(t) - 3u(t-2) - 3tu(t) + 3tu(t-2)$
$+ 3t\,u(t-2) - 3tu(t-3) - 9u(t-2) + 9u(t-3)$

$f(t) = 3u(t) - 12u(t-2) - 3t\,u(t) + 6t\,u(t-2)$
$-3t\,u(t-3) + 9u(t-3)$

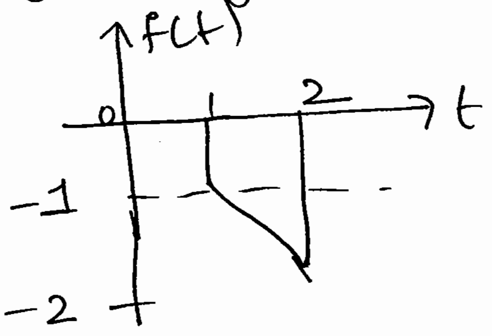
$= 3u(t) - 12u(t-2) - 3t\,u(t) + 6(t-2)u(t-2)$
$+ 12u(t-2) - 3(t-3)u(t-3) - 9u(t-3) +$
$9u(t-3)$

$\therefore f(t) = 3u(t) - 3tu(t) + 6(t-2)u(t-2)$
$- 3(t-3)u(t-3)$

$F(s) = L\{f(t)\}$

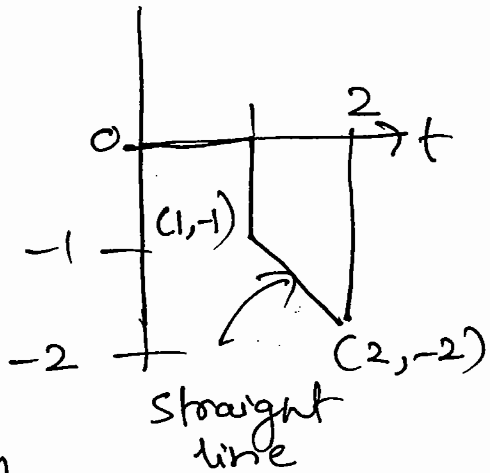
$$F(s) = \frac{3}{s} + \frac{6}{s^2}e^{-2s} - \frac{3}{s^2} - \frac{3}{s^2}e^{-3s}$$

Express the function $f(t)$ using singularity functions and find $F(s)$



Equation of the straight line

$$\frac{f_1(t)+1}{t-1} = \frac{-2+1}{2-1}$$

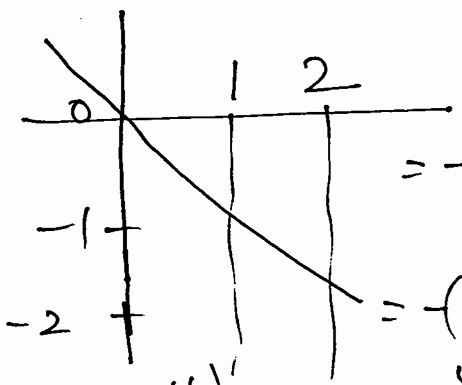


$\therefore f_1(t)+1 = -t+1$

$f_1(t) = -t$

above equation is for the values t lying between 1 & 2

$\therefore f(t) = f_1(t) \cdot g(t)$

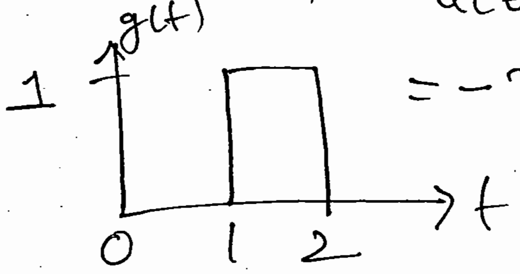


$$f(t) = -t[u(t-1) - u(t-2)]$$

$$= -(t-1+1)u(t-1) + (t-2+2)u(t-2)$$

$$= -(t-1)u(t-1) - u(t-1) + (t-2)u(t-2) + 2u(t-2)$$

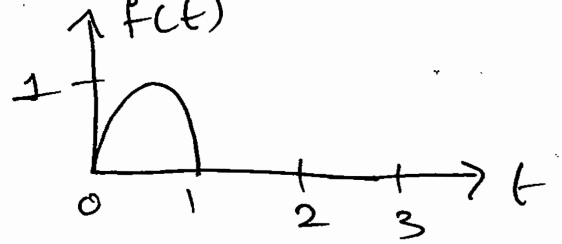
$$= -r(t-1) - u(t-1) + r(t-2) + 2u(t-2)$$

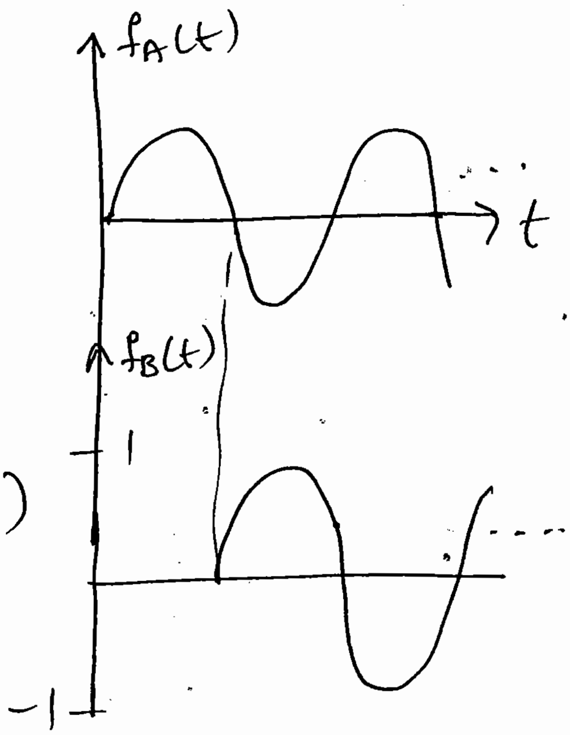


$$\therefore F(s) = \mathcal{L}\{f(t)\}$$

$$= -\frac{1}{s^2}e^{-s} - \frac{1}{s}e^{-s} + \frac{1}{s^2}e^{-2s} + \frac{2}{s}e^{-2s}$$

Find the Laplace transform of the function $f(t)$





$$f(t) = f_A(t) + f_B(t)$$

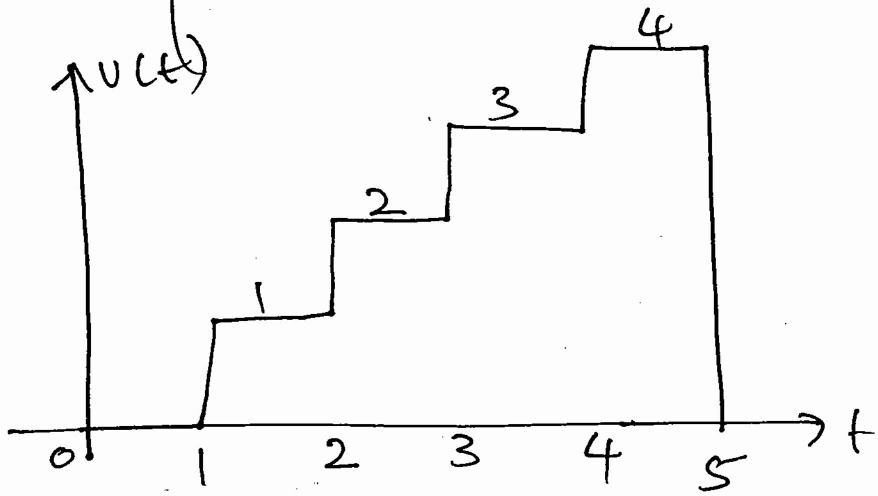
$$= \sin \pi t\, u(t) + \sin \pi (t-1)\, u(t-1)$$

$$F(s) = \mathcal{L}\{f(t)\} = \frac{\pi}{s^2+\pi^2} + \frac{\pi}{s^2+\pi^2}e^{-s}$$

$$= \frac{\pi}{s^2+\pi^2}\left[1+e^{-s}\right]$$

Find the laplace transform

$$v(t) = \begin{cases} 1, & 1<t<2 \\ 2, & 2<t<3 \\ 3, & 3<t<4 \\ 4, & 4<t<5 \\ 0, & \text{elsewhere} \end{cases}$$

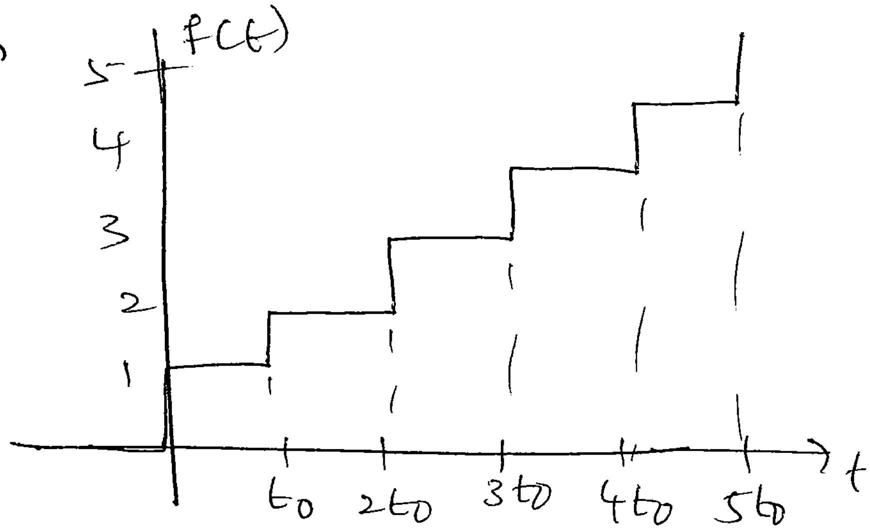


$$v(t) = [u(t-1)-u(t-2)] + 2[u(t-2)-u(t-3)]$$
$$+ 3[u(t-3)-u(t-4)] + 4[u(t-4)-u(t-5)]$$
$$= u(t-1) + u(t-2) + u(t-3) + u(t-4) - 4u(t-5)$$

taking Laplace transform

$$V(s) = \frac{1}{s}\left[e^{-s} + e^{-2s} + e^{-3s} + e^{-4s} - 4e^{-5s}\right]$$

Consider a staircase waveform which extends to infinity and $t = n t_0$ jumps to the value $n+1$ being a superposition of unit step functions determine the Laplace transform



$$f(t) = u(t) + u(t-t_0) + u(t-2t_0) + \cdots$$

$$F(s) = \mathcal{L}\{f(t)\} = \frac{1}{s} + \frac{1}{s} e^{-t_0 s} + \frac{1}{s} e^{-2t_0 s} + \cdots$$

$$= \frac{1}{s}\left[1 + e^{-t_0 s} + e^{-2t_0 s} + \cdots\right]$$

let $e^{-t_0 s} = x$

$$\therefore F(s) = \frac{1}{s}\left[1 + x + x^2 + \cdots\right]$$

from binomial theorem

$$(1-x)^{-1} = 1 + x + x^2 + \cdots$$

$$\therefore F(s) = \frac{1}{s(1-x)} = \frac{1}{s(1-e^{-t_0 s})}$$

$R_{eq} = (10) \parallel (10+20) = \frac{(10)(30)}{40} = 7.5\Omega$

$\therefore I_T = \frac{5}{7.5} = 0.666A$

using current divider rule

$I = I_T \left[\frac{10}{10 + (10+20)} \right] = 0.666\left(\frac{10}{40}\right)$

$I = 0.166 A$

node voltage V_a is the voltage across 20Ω resistor due to current I

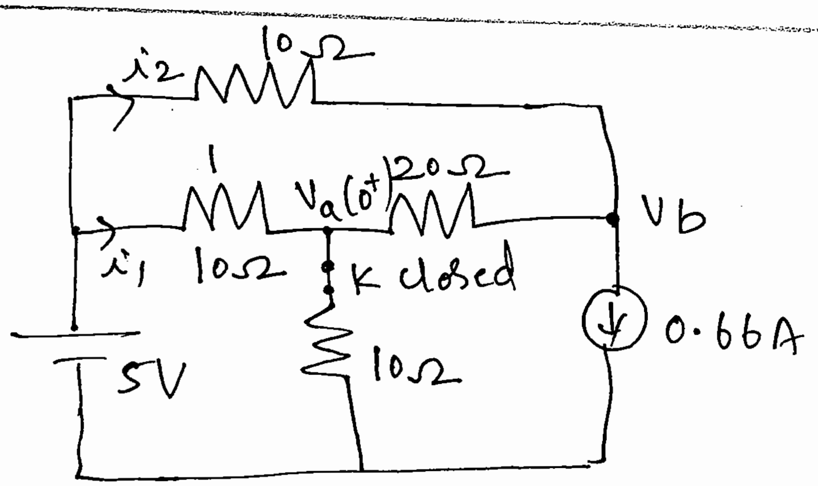
$V_a(0^-) = I \times 20 = (0.1666)(20)$

$\boxed{V_a(0^-) = 3.33V}$

$\therefore i_L(0^-) = 0.666 A = i_L(0^+)$

since the total current I_T is also flowing through short circuit branch i.e. inductor L

because the current through the inductor cannot change instantaneously at $t=0^+$ switch k is closed.



It is clear that at instant $t=0^+$ only inductor current is same as initial current, hence it acts as a constant current source 0.666 A

$$\frac{5-V_a}{10} = \frac{V_a}{10} + \frac{V_a - V_b}{20}$$ apply KCL @ node a

$$10 - 2V_a = 2V_a + V_a - V_b$$

$$\therefore 5V_a - V_b = 10 \rightarrow (1)$$

KCL @ node b

$$\frac{V_a - V_b}{20} + \frac{5 - V_b}{10} = 0.666$$

$$\therefore V_a - V_b + 10 - 2V_b = 13.332$$

$$\therefore V_a - 3V_b = 3.33 \rightarrow (2)$$

$V_a = 1.904\,V$

Thus at $t = 0^+$ $V_a(0^+) = 1.904\,V$

B.C. DIVAKARA

module : 4

RESONANT CIRCUITS

Introduction ⇒ Resonance is a phenomenon that occurs in ac circuits. Ac circuits comprising of resistors, inductors and capacitors are said to be in Resonance when the total current and the applied voltage are in phase.

Under the condition of resonance the circuit will behave like a pure resistance.

Resonance condition is achieved by varying frequency and keeping circuit elements constant (or) by varying circuit elements and keeping the frequency constant.

In a resonant circuit equal amounts of energy are interchanged periodically between L & C. The power drawn from the source is the only to provide for the energy dissipation in the resistance. Hence a circuit in Resonance stores a constant amount of energy.

Resonance circuits are classified as (1) series Resonance (2) parallel resonance

series Resonance circuit ⇒

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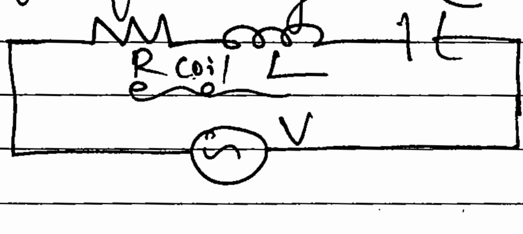
A series resonance circuit consists of a coil and a capacitance connected in series across an alternating voltage of variable frequency



Impedance of the circuit
$Z = R + j(X_L - X_C)$ by Varying the supply frequency the inductive reactance X_L is made equal to capacitive reactance X_C then the circuit is said to be Resonance.

At resonance the supply voltage and current are in phase the impedance of the circuit is resistive. The powerfactor of the circuit is unity.

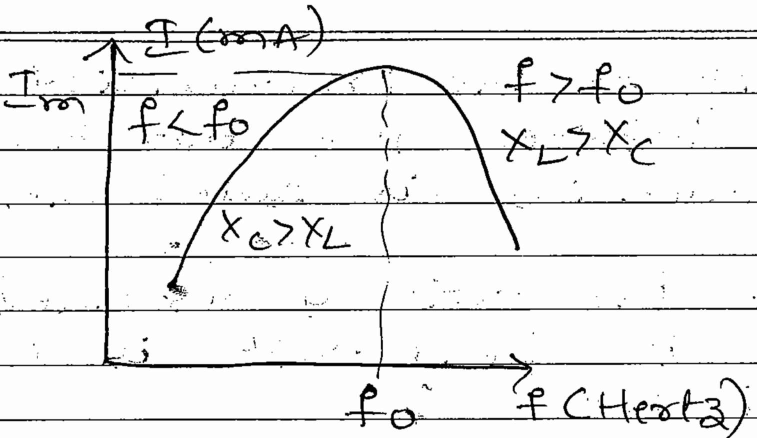
To obtain the resonant frequency equate imaginary part of the total impedance zero

i.e, $X_L - X_C = 0$

(or) $X_L = X_C$

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$\therefore \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$



when the frequency less than the resonant frequency f_o, capacitive reactance X_C is greater than inductive reactance X_L for frequencies greater than resonant frequency f_o, inductive reactance X_L will be greater than the capacitive reactance X_C, at resonance the current is maximum and $I_m = \frac{V}{R}$

as the current is maximum at resonance it is called as Acceptor circuit

properties of series Resonance

1. current is maximum
2. Impedance of circuit is resistive & minimum
3. Voltage across inductance is equal to the Voltage across the capacitor
4. power factor of the circuit is unity

Bandwidth ⇛ The frequencies separation between the two half power frequencies is called Bandwidth

Q-factor ⇛ The ratio of resonant frequency to bandwidth is called Quality factor

circuits having a higher quality factor have a narrower bandwidth (or) a sharper frequency response curve. such circuits are said to have greater frequency selectivity.

properties of parallel Resonance

1. At resonance the Imaginary part of admittance is zero
2. Impedance at resonance is resistive and is maximum
3. Current at resonance is minimum
4. power factor of the circuit is unity
5. Impedance at resonance is Known as dynamic resistance

The most common application of resonance is Tuning

The use of resonance is to establish a condition of stable frequency in circuits, designed to produce ac signals

Bandwidth ⇒ The band of frequency over which power of the circuit is half of its maximum value

Q-factor $Q = \frac{V_L}{V}$ or $\frac{V_C}{V}$

$$Q = \frac{\cancel{I_0} \cdot X_L}{\cancel{I_0} \cdot R} = \frac{WL}{R} \quad \therefore \quad Q = \frac{W_0 L}{R}$$

also $Q = \frac{V_C}{V} = \frac{\cancel{I_0} \cdot X_C}{\cancel{I_0} \cdot R} = \frac{1}{W_0 CR}$

$$\boxed{Q = \frac{1}{W_0 CR}}$$

but $Q = \frac{W_0 L}{R}$ & $W_0 = \frac{1}{\sqrt{LC}}$

$$Q = \frac{1}{\sqrt{LC}} \frac{L}{R} = \frac{1}{\sqrt{LC}} \left(\frac{L}{R}\right) = \frac{\sqrt{L}}{R\sqrt{C}}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

& also Voltage across the inductor $V_L = QV$
& $V_C = QV$

Selectivity ⇒ It is defined as the ratio of resonant frequency to the band width

$$\boxed{\text{Selectivity} = \frac{f_0}{f_2 - f_1}}$$

It is the ability of the circuit to distinguish between desired and undesired frequencies

3M Relation between f_0, f_1 & f_2 or
prove that $f_0 = \sqrt{f_1 f_2}$ or
show that resonant frequency is the geometric mean of half power frequency

At half power frequencies the current is $\frac{1}{\sqrt{2}}$ times I_0 which means that magnitude of the impedance remains same at that point

$\therefore$ when $f_1 < f_0$ $X_C > X_L$

$$|Z_1| = \sqrt{R^2 + (XC_1 - XL_1)^2}$$

similarly impedance Z_2 at f_2 when $f_2 > f_0$; $X_L > X_C$

$$|Z_2| = \sqrt{R^2 + (XL_2 - XC_2)^2}$$

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since $Z_1 = Z_2$

$$\sqrt{R^2 + (XC_1 - XL_1)^2} = \sqrt{R^2 + (XL_2 - XC_2)^2}$$

$$R^2 + (XC_1 - XL_1)^2 = R^2 + (XL_2 - XC_2)^2$$

$$\therefore (XC_1 - XL_1)^2 = (XL_2 - XC_2)^2$$

$$\therefore XC_1 - XL_1 = XL_2 - XC_2$$

i.e, $XC_1 + XC_2 - XL_1 - XL_2 = 0$

$$XC_1 + XC_2 = XL_1 + XL_2$$

$$\frac{1}{W_1 C} + \frac{1}{W_2 C} = W_1 L + W_2 L$$

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$$\frac{1}{C}\left[\frac{W_2+W_1}{W_1W_2}\right] = L\left[W_1+W_2\right]$$

$$W_1W_2 = \frac{1}{LC} \rightarrow (1)$$

but for series resonance

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad ; \quad W_0 = \frac{1}{\sqrt{LC}} \rightarrow (2)$$

$$\therefore \quad W_0^2 = \frac{1}{LC} \rightarrow (3)$$

equating (1) & (3)

$$W_0^2 = W_1W_2$$

$$\therefore \quad \boxed{f_0 = \sqrt{f_1 f_2}}$$ hence proved

Expression for Half power frequencies

Let f_0 be the resonant frequency and f_1 and f_2 be the lower and upper half power frequencies, at resonance, current is maximum

$$I_m = \frac{V}{R}$$

at half power frequencies the equivalent reactance of the circuit is equal to the resistance $X_L - X_C = R$

at lower half power frequency f_1, $X_C > X_L$

$$\therefore X_{C_1} - X_{L_1} = R \rightarrow (1)$$

at upper half power frequency f_2, $X_L > X_C$

$$\therefore X_{L_2} - X_{C_2} = R \rightarrow (2)$$

from (1)

$$X_{C_1} - X_{L_1} = R$$

$$\frac{1}{W_1 C} - W_1 L = R$$

$$1 - W_1^2 LC = W_1 RC$$

$$W_1^2 LC + W_1 RC - 1 = 0 \qquad \div LC$$

or

$$W_1^2 + \frac{R}{L} W_1 - \frac{1}{LC} = 0$$

This is in the form of $ax^2 + bx + c = 0$

$$a = 1, \quad b = \frac{-R}{L}, \quad c = \frac{-1}{LC}$$

$$\boxed{W_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}$$

negative sign not taken as W_1 cannot be negative classmate Date Page

$$\therefore \quad f_1 = \frac{1}{2\pi}\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right.$$

or $$\boxed{f_1 = -\frac{R}{4\pi L} + \sqrt{\left(\frac{R}{4\pi L}\right)^2 + \frac{1}{4\pi^2 LC}}} \Rightarrow (A)$$

from (2) $\quad X_{L_2} - X_{C_2} = R$

$$W_2 L - \frac{1}{W_2 C} = R$$

$$W_2^2 LC - W_2 RC - 1 = 0 \quad \div LC$$

$$W_2^2 - \frac{R}{L} W_2 - \frac{1}{LC} = 0$$

solving for W_2

$$W_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

(or) $$f_2 = \frac{1}{2\pi} + \left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right.$$

$$\therefore \quad f_2 = \frac{R}{4\pi L} + \sqrt{\left(\frac{R}{4\pi L}\right)^2 + \frac{1}{4\pi^2 LC}} \Rightarrow (B)$$

Band Width $= f_2 - f_1 \quad \therefore$ from (A) & (B)

$$= \frac{R}{4\pi L} - \left(-\frac{R}{4\pi L}\right)$$

$$\boxed{BW = \frac{R}{2\pi L}}$$

also $\quad BW = W_2 - W_1 = \frac{R}{L}$

but to prove $Q = \frac{W_0 L}{R} = \frac{f_0}{f_2 - f_1} = \frac{f_0}{R/2\pi L}$

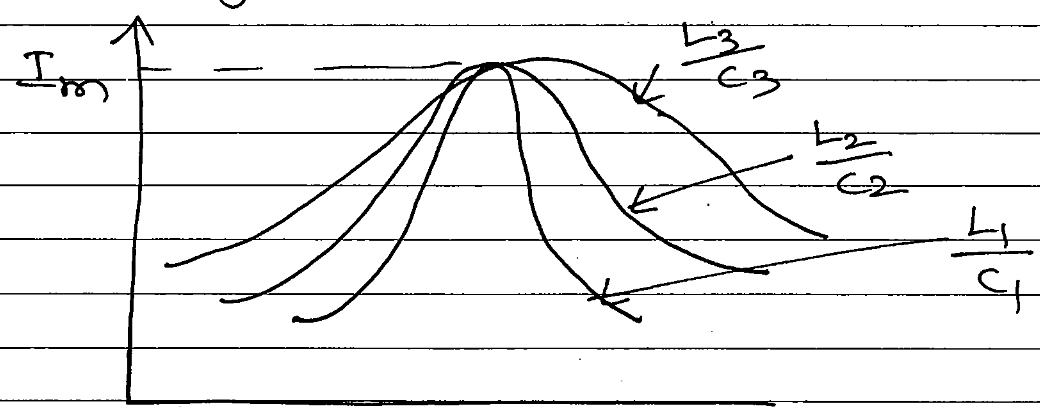
$$= \frac{2\pi f_0 L}{R} \quad \therefore \boxed{Q = \frac{W_0 L}{R}}$$

Voltage magnification in series resonance

$Q = \frac{V_L}{V}$ or $Q = \frac{V_C}{V}$

$\therefore \boxed{V_L = V \cdot Q}$ & $\boxed{V_C = Q \cdot V}$

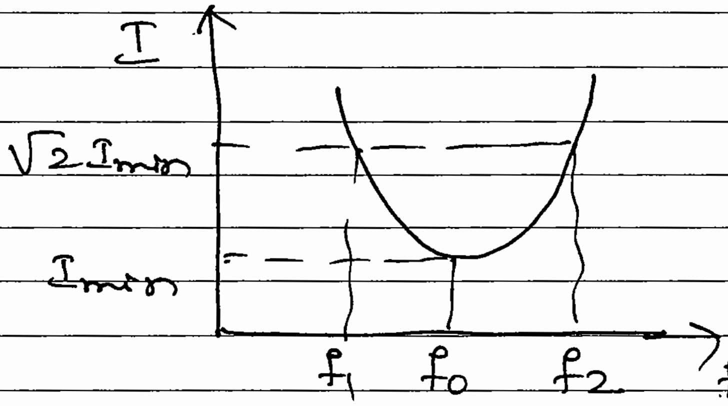
Note ⇒ As the ratio of L and C is increased the current varies more abruptly in the region of resonant frequency. Therefore for large values of $\frac{L}{C}$ the bandwidth decreases and selectivity increases



Effect of Variation of $\frac{L}{C}$ on frequency response curve

$$\frac{L_3}{C_3} < \frac{L_2}{C_2} < \frac{L_1}{C_1}$$

Frequency response of parallel circuit



It is seen from the graph that current is minimum at resonance. This is because the current in RL & RC circuit are complex and at resonance the imaginary part of the total current is zero, leaving only the in-phase component of current. This results in the impedance of the circuit to be maximum at resonance. As the current is minimum at resonance it is called Rejector | Antiresonance circuit.

* A RLC series circuit is composed of the components having the values $R = 0.2\Omega$, $L = 100mH$, $C = 50\mu F$, determine the resonant frequency and current at 24V

Solⁿ:- $R = 0.2\Omega$, $L = 100mH$, $C = 50\mu F$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100\times10^{-3}\times 50\times10^{-6}}} = 71.17\ Hz$$

$$i = \frac{V}{R} = \frac{24}{0.2} = 120\ A$$

It is required that a series RLC circuit should resonate at 1 MHz determine the values of R, L & C if bandwidth of the circuit is 5kHz and its impedance is 50Ω at resonance

Ans Given $Z = 50\Omega$

at resonance $Z = R \therefore R = 50\Omega$

$BW = 5kHz$, $f_0 = 1\times10^6$ Hz

$$BW = \frac{R}{2\pi L} \Rightarrow \frac{5k \times 2\pi}{R} = \frac{1}{L}$$

$$\therefore L = \frac{50}{5\times10^3 \times 2\pi} = 1.59\,mH$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow C = \frac{1}{4\pi^2 f_0^2 L}$$

$$C = 15.9\ pF$$

Imp

A 220V, 100Hz ac supply supplies a series RLC circuit with a capacitor and a coil. If the coil has 50mΩ resistance and 5mH inductance. Find at a resonant frequency of 100Hz what is the value of capacitor also calculate the Q-factor and half power frequencies

Given $V = 220V$, $f = 100Hz$, $L = 5\cancel{\times}10^{-3}$ H

$R = 50\times10^{-3}\,\Omega$

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$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{5\times10^{-3}\times C}}$$

$$C = \frac{1}{4\pi^2(5\times10^{-3})(100)^2}$$

$$C = 506.605\,\mu F$$

$$Q = \frac{W_0 L}{R} = \frac{2\pi(100)(5\times10^{-3})}{50\times10^{-3}}$$

$$Q = 62.83$$

$$Q = \frac{f_0}{BW} \Rightarrow 62.83 = \frac{100}{BW}$$

Bandwidth = 1.591 Hertz

$$* \quad \boxed{Q = \frac{W_0 L}{R}} = \boxed{\frac{2\pi f_0 L}{R}} = \frac{2\pi f_0}{R/L} = \frac{2\pi f_0}{2\pi(f_2 - f_1)}$$

$$\boxed{Q = \frac{f_0}{f_2 - f_1} = \frac{f_0}{BW}}$$

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Dec 2012 app

A series RLC circuit has R = 10Ω, L = 0.01H, C = 0.01μF & it is connected across 10mV supply calculate f_0, Q, BW, f_1 & f_2 & I_0

a) $$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01\times0.01\times10^{-6}}} = 15.915\,KHz$$

b) $$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 100$$

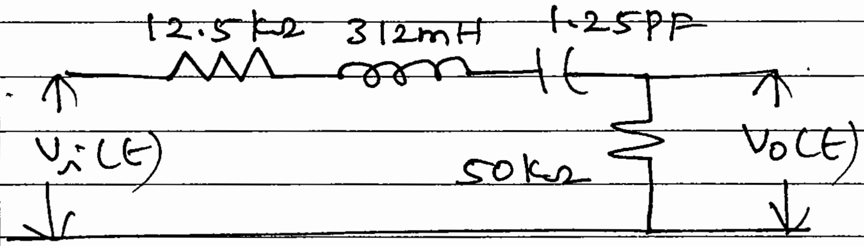
$$BW = \frac{R}{2\pi L} = \frac{10}{2\pi(0.01)} = \underline{159.15\ Hz}$$

$$f_1 = f_0 - \frac{BW}{2} = \underline{15.835\ kHz}$$

$$f_2 = f_0 + \frac{BW}{2} = \underline{15.994\ kHz}$$

$$I_0 = \frac{V}{R} = \frac{10\times10^{-3}}{10} = \underline{1\ mA}$$

Q) For the network shown in figure find f_0, Q, f_1, f_2 & Bandwidth



$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \underline{254.85\ kHz}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{62.5\times10^{3}}\sqrt{\frac{312\times10^{-3}}{1.25\times10^{-12}}}$$

$$\boxed{Q = 7.99}$$

$$BW = \frac{R}{2\pi L} = \underline{31.88\ kHz}$$

$$f_1 = f_0 - \frac{BW}{2} = \underline{238.91\ kHz}$$

$$f_2 = f_0 + \frac{BW}{2} = \underline{270.79\ kHz}$$

A series Resonant circuit has $Z = 500\Omega$ at resonant frequency, cutoff frequencies are 10 KHz and 100 Hz, find f_0. Values of R, L & C, Q-factor & Bandwidth

$f_0 = \sqrt{f_1 f_2} = 1\,KHz$

$R = Z = 500\Omega$, $BW = f_2 - f_1 = 9.9\,KHz$

$$Q\text{-factor} = \frac{f_0}{BW} = \frac{1\times10^3}{9.9\times10^3} = 0.101$$

$$BW = \frac{R}{2\pi L} \Rightarrow L = \frac{R}{2\pi(BW)} = \frac{500}{2\pi(9.9k)}$$

$L = 8.038\,mH$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow C = \frac{1}{4\pi^2 f_0^2 L} = 3.1512\,\mu F$$

Obtain the value of RLC in a series resonance circuit that resonates at 1.5 KHz and consumes 50 watts from a 50 V source operating at resonant frequency for the Bandwidth 0.75 KHz

$$P = \frac{V^2}{R} \quad \therefore \quad R = \frac{V^2}{P} = \frac{50^2}{50} = 50\Omega$$

$$BW = \frac{R}{2\pi L} \Rightarrow L = \frac{R}{2\pi(BW)} = \frac{50}{2\pi(0.75\times10^3)}$$

$L = 10.61\,mH$

$$C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2 (1.5\times10^3)^2 (10.61\times10^{-3})}$$

$$C = 1.06\,\mu F$$

Calculate the half power frequencies of series Resonant circuit where bandwidth is 75 KHz and $f_0 = 150$ KHz

$$\boxed{f_1 = f_0 - \frac{BW}{2} = 112.5\text{ KHz}}$$

$$\boxed{f_2 = f_0 + \frac{BW}{2} = 187.5\text{ KHz}}$$

A RLC series circuit is designed to have bandwidth of 300 rad/sec. The inductance is 0.1 H. If it has to resonate at 3000 rad/sec determine R, C if the applied voltage is 100V and also determine the voltage across inductor and capacitor

$$BW = 300\text{ rad/sec} = \frac{300}{2\pi} = 47.74\text{ Hz}$$

$$BW = \frac{R}{2\pi L} \quad ; \quad V_C = I.X_C = \frac{V}{R} X_C = \frac{100}{30}\left(\frac{1}{3000(1.11\times10^{-6})}\right)$$

$$(47.74)\,2\pi\,(0.1) = R \qquad \therefore R = 30\,\Omega$$

$$\therefore V_C = 1000\text{ V}$$

$$\boxed{W = 3000}$$

$$\therefore f_0 = \frac{3000}{2\pi} = 477.46\text{ Hz}$$

$$W_0 = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{W_0^2 L} = 1.11\,\mu F$$

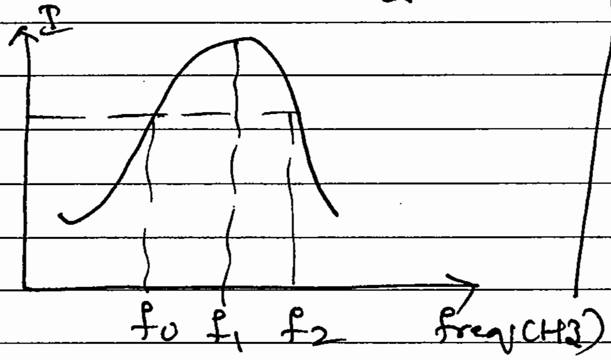
$$V_L = I.X_L = \frac{V}{R}\cdot X_L = \frac{100}{30}(3000\times0.1)$$

$$\boxed{V_L = 1000\text{ V}}$$

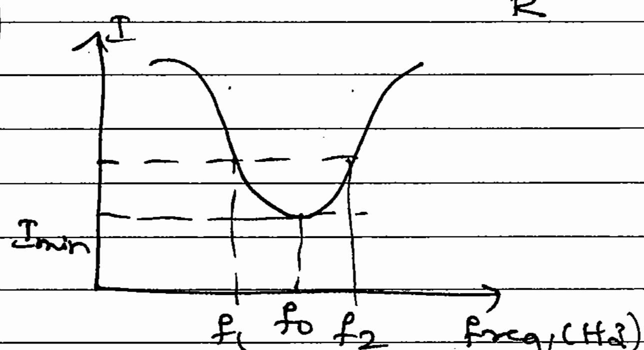
Comparison between Series and parallel resonance

Series Resonance	parallel resonance
a) $f_0 = \frac{1}{2\pi\sqrt{LC}}$	a) $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
b) current is maximum	b) current is minimum
c) Acceptor circuit	c) Rejector circuit
d) Exhibits minimum impedance when on tune	d) Exhibits maximum impedance on tune
e) Energy stored is $V = \frac{1}{2} L \lvert I \rvert^2$	e) Energy stored $V = \frac{1}{2} \lvert v^2 \rvert C$
f) power loss $P = \frac{1}{2} \lvert I \rvert^2 R$	f) power loss $= \frac{1}{2} \frac{\lvert V \rvert^2}{R}$

g)



frequency response



frequency response

A coil is connected in series with a variable capacitor across $v(t) = 10 \cos 1000t$, the capacitance is varied and the current is maximum when $C = 10\,\mu F$ & when the value of capacitor is $12.5\,\mu F$ the current is 0.707 times the maximum value find the values of R, L & Q-factor of the coil.

$$f_0 = \frac{W_0}{2\pi} = \frac{1000}{2\pi} = 159.155\,Hz$$

when the capacitor is $12.5\mu F$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{1000\,(12.5\times10^{-6})} = 80$$

when $C = 10\mu F$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi f\,(10\times10^{-6})} = 100\Omega$$

at resonance $X_L = X_C$

$$\therefore\ L = \frac{X_C}{2\pi f} = \frac{100}{1000} = 0.1\,H$$

@ resonance $I = \frac{V}{Z}$ but $I = \frac{I_0}{\sqrt{2}}$

@ half power frequencies

$$I = \frac{V}{\sqrt{2}R} \qquad \therefore\ Z = \sqrt{2}R$$

w.k.t $\quad Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\sqrt{2}R = \sqrt{R^2 + (X_L - X_C)^2}$$

Squaring on both sides

$$2R^2 = R^2 + (X_L - X_C)^2$$

$$R^2 = (X_L - X_C)^2$$

$$\boxed{R = X_L - X_C} \longrightarrow (1)$$

$$R = 100 - 80 = 20\Omega$$

$$\boxed{Q = \frac{W_0 L}{R} = \frac{(1000)(0.1)}{20} = 5}$$

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A constant voltage at a frequency of 1 MHz is applied to an inductor in series with a variable capacitor. When the capacitor is set to 500pF the current has its maximum value while it is reduced to one half when the capacitance is 600pF find R, L & Q-factor

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow L = \frac{1}{4\pi^2 f_0^2 C} = 0.05066 mH$$

when the capacitor is 600pF

$$I = \frac{1}{2} I_0$$

$$\frac{V}{Z} = \frac{1}{2}\frac{V}{R} \quad \therefore \quad Z = 2R$$

$$X_L = 2\pi f L = 318.3\Omega$$

$$X_C = \frac{1}{2\pi f C} = 265.253\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z^2 = R^2 + (X_L - X_C)^2$$

$$3R^2 = (X_L - X_C)^2$$

$$R^2 = 937.809 \Rightarrow R = 30.62\Omega$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 10.39$$

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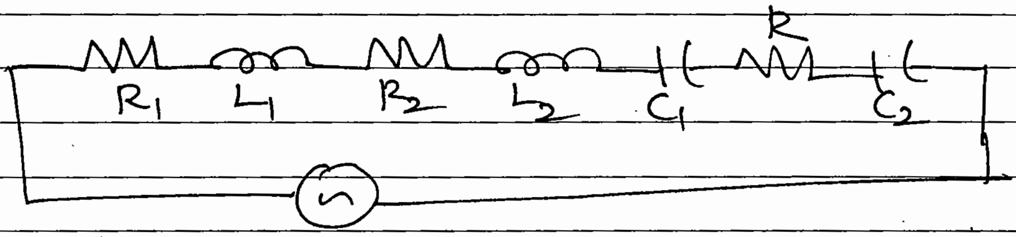
Two coils, one of $R_1 = 0.51\Omega$, $L_1 = 32\,mH$ and other coil of $R_2 = 1.3\Omega$, $L_2 = 15\,mH$ are in series and these two are in series with a capacitor of $25\,\mu F$ & $62\,\mu F$ and a series of Resistor of resistance $0.24\,\Omega$ find Resonant frequency, Q-factor, power dissipated in the circuit for the applied voltage 10V



$R_{eff} = R_1 + R_2 + R = 0.51 + 1.3 + 0.24$
$= 2.05\,\Omega$

$L_{eff} = L_1 + L_2 = 32 + 15 = 47\,mH$

$C_{eff} = \dfrac{C_1 C_2}{C_1 + C_2} = \dfrac{25 \times 62 \times 10^{-12}}{(25+62)\times 10^{-6}} = 17.81\,\mu F$

$f_0 = \dfrac{1}{2\pi\sqrt{LC}} = 173.92\,Hz$

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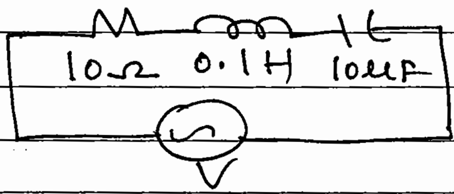
$Q = \dfrac{\omega_0 L}{R} = 25.054$

$P = \dfrac{V^2}{R} = \dfrac{(10)^2}{2.05} = 48.78$ watts

For the circuit shown find the impedance at 10 Hz above and below the resonance



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$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 159.15\,Hz$$

a) $f > f_0$ by $10\,Hz$

$f = 10 + 159.15 = 169.15\,Hz$

$X_L = W_0 L = 106.2831\,\Omega$

$$X_C = \frac{1}{WC} = 94.086\,\Omega$$

$X_{L_2} > X_{C_2}$

$Z_1 = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{10^2 + (106.2831 - 94.086)^2}$

$\boxed{Z_1 = 15.77\,\Omega}$

b) when $f < f_0$

$f = 159.15 - 10 = 149.15\,Hz$

$X_L = W_0 L = 2\pi f L = 93.7137\,\Omega$

$$X_C = \frac{1}{WC} = \frac{1}{2\pi f C} = 106.708\,\Omega$$

$Z_2 = \sqrt{R^2 + (X_C - X_L)^2}$

$= \sqrt{10^2 + (106.708 - 93.7)^2}$

$\boxed{Z_2 = 16.396\,\Omega}$

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A series RLC circuit has a resistance of 10Ω, an inductance of 0.3H and a capacitance of 100µF, the applied voltage is 230V find f_0, Q, f_1 & f_2 Bandwidth Lower and upper cut off freq's current at resonance, voltage across inductance at resonance

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.3 \times 100 \times 10^{-6}}} = 29.1\,Hz$$

$$Q = \frac{2\pi f_0 L}{R} = \frac{2\pi \times 29.1 \times 0.3}{10} = 5.48$$

Half power frequency ω_1 in rad/sec

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

$$= -\frac{10}{2\times 0.3} + \sqrt{\left(\frac{10}{2\times 0.3}\right)^2 + \left(\frac{1}{0.3\times 100\times 10^{-6}}\right)}$$

$$= -16.67 + 183.33 = 166.66 \text{ rad/sec}$$

half power frequency $f_1 = \frac{166.66}{2\pi} = 26.54\,Hz$

w.k.t $f_0^2 = f_1 f_2$

$$\therefore f_2 = \frac{f_0^2}{f_1} = \frac{(29.1)^2}{26.54} = 31.9\,Hz$$

$$\text{Bandwidth} = f_2 - f_1$$
$$= 31.9 - 26.54$$
$$= 5.36\,Hz$$

$I = \frac{V}{R} = \frac{230}{10} = 23\ A$

current at half power frequencies

$\frac{I}{\sqrt{2}} = \frac{23}{\sqrt{2}} = 16.265$ amperes

$X_L = 2\pi f_0 L$
$= 2\pi \times 29.1 \times 0.3$
$= 54.82\ \Omega$

Voltage across inductance

$V_L = I \cdot X_L = 23 \times 54.82$
$= 1260.9$ watts

A voltage $V(t) = 10 \sin \omega t$ is applied to a series RLC circuit, at the resonant frequency of the circuit the maximum voltage across the capacitor is found to be 500V. bandwidth of the circuit is 400 rad/sec and the impedance at resonance is $100\ \Omega$. Find the values of L & C & the resonant frequency

$V(t) = 10 \sin \omega t$

rms value of supply voltage

$V = \frac{10}{\sqrt{2}} = 7.07$ volts

$\boxed{V_C = 500V}$ $\therefore$ $\boxed{Q = \frac{V_C}{V} = \frac{500}{7.07}}$

$Q = 70.7$

but $Q = \frac{\omega_0}{\omega_2 - \omega_1}$

$70.7 = \frac{\omega_0}{400}$

$\therefore \quad W_0 = 2828 \text{ rad/sec}$

$\therefore \quad f_0 = 450.1 \text{ Hz}$

@ Resonance $\quad Z = R = 100\,\Omega$

$BW = W_2 - W_1 = \dfrac{R}{L}$

$\therefore \quad 400 = \dfrac{100}{L}$

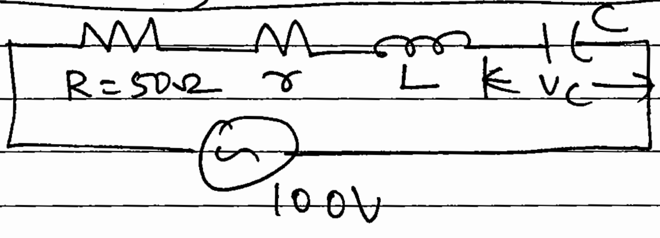
$\therefore \quad L = 0.25\,H$

as $f_0 = \dfrac{1}{2\pi\sqrt{LC}} \quad \therefore \quad C = \dfrac{1}{(2\pi f_0)^2 L}$

$C = \dfrac{1}{(2828)^2 \times 0.25}$

$C = 0.5\,\mu F$

A $50\,\Omega$ resistor is connected in series with an inductor having internal resistance, a capacitor & 100 volt variable frequency supply as shown in figure at a frequency of 200 Hz, the maximum current of 0.7 A flows through the circuit and voltage across the capacitor is 200 volts, determine the circuit constants.



Voltage across capacitor at resonance

$V_C = I_{max} X_C$

$200 = 0.7\, X_C$

$\therefore \quad \boxed{X_C = 285.71\,\Omega}$

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$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 200 \times 285.71}$$

$$C = 2.79\,\mu F$$

at resonance $X_L = X_C$

$= 285.71\,\Omega$

$$\therefore L = \frac{285.71}{2\pi f} = \frac{285.71}{2\pi \times 200}$$

$$L = 0.227\,H$$

Impedance at resonance

$$Z = \frac{V}{I} = \frac{100}{0.7} = 142.86\,\Omega$$

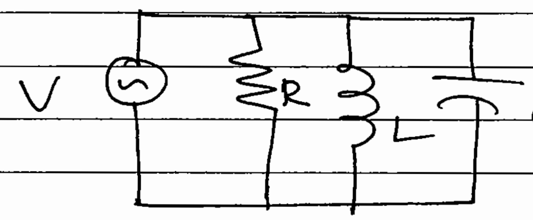
but $Z = R + r$ at resonance

$\therefore R + r = 142.86$

$\therefore 50 + r = 142.86$

$$r = 92.86\,\Omega$$

parallel Resonance circuit



admittance for the parallel circuit

$$Y = \frac{1}{Z} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$= \frac{1}{R} - \frac{j}{X_L} + \frac{j}{X_C}$$

$$\frac{1}{Z} = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

for resonance equate imaginary part to zero

$$\frac{1}{X_C} - \frac{1}{X_L} = 0$$

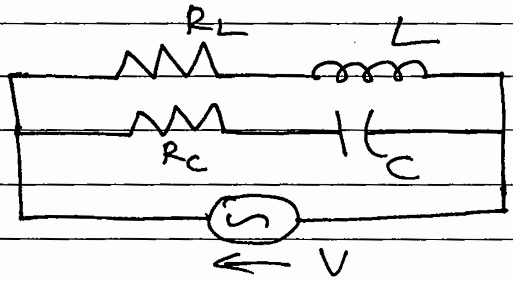
$$X_L = X_C$$

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

DIVAKAR B C

parallel Resonance two branch circuit



Admittance for the parallel circuit

$$Y = \frac{1}{Z} = \frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_C}$$

Rationalising the denominator

$$\frac{1}{Z} = \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{R_C + jX_C}{R_C^2 + X_C^2}$$

$$\frac{1}{Z} = \frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} + j\left[\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2}\right]$$

for resonance equate the imaginary part to zero

$$\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2} = 0$$

$$\frac{X_C}{R_C^2 + X_C^2} = \frac{X_L}{R_L^2 + X_L^2}$$

$$X_C(R_L^2 + X_L^2) = X_L(R_C^2 + X_C^2) \Rightarrow (1)$$

There are five variables R_L, R_C, L, C & frequency by varying which resonance condition can be obtained

case<i> Variable R_L

From (1) $R_L^2 + X_L^2 = \frac{X_L}{X_C}(R_C^2 + X_C^2)$

$$R_L^2 = \frac{X_L}{X_C}(R_C^2 + X_C^2) - X_L^2$$

$$\therefore R_L = \sqrt{\frac{X_L}{X_C}(R_C^2 + X_C^2) - X_L^2}$$

case<ii> Variable R_C

From (1) $R_C^2 + X_C^2 = \frac{X_C}{X_L}(R_L^2 + X_L^2)$

$$R_C^2 = \frac{X_C}{X_L}(R_L^2 + X_L^2) - X_C^2$$

$$\therefore \boxed{R_C = \sqrt{\frac{X_C}{X_L}(R_L^2 + X_L^2 - X_C^2}}$$

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Variable frequency (f)

Case 3 ⇒ Variable frequency (f)

from ①

$$\omega_0 L\left(R_C^2 + \frac{1}{\omega_0^2 C^2}\right) = \frac{1}{\omega_0 C}\left(R_L^2 + \omega_0^2 L^2\right)$$

$$\omega_0^2 LC\left(R_C^2 + \frac{1}{\omega_0^2 C^2}\right) = R_L^2 + \omega_0^2 L^2$$

$$\omega_0^2 LC R_C^2 + \frac{L}{C} = R_L^2 + \omega_0^2 L^2$$

$$\omega_0^2\left[LC R_C^2 - L^2\right] = R_L^2 - \frac{L}{C}$$

$$\omega_0^2 LC\left[R_C^2 - \frac{L}{C}\right] = R_L^2 - \frac{L}{C}$$

$$\omega_0^2 LC = \frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\left(\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}\right)}$$

the circuit will resonate only if
$R_L^2 < \frac{L}{C}$ and $R_C^2 > \frac{L}{C}$

or if $R_L^2 < \frac{L}{C}$ & $R_C^2 < \frac{L}{C}$

when $R_L^2 = R_C^2 = \frac{L}{C}$ the circuit will resonate at all frequencies

magnification in parallel resonance circuit

In parallel circuit the current at resonance is minimum

$\therefore \quad I_{min} = \frac{V}{R}$ or $V = I_{min} \cdot R$

current through capacitor at resonance

$$I_C = \frac{V}{X_C} = \frac{V}{\frac{1}{\omega_0 C}} \quad \frac{I_{min} \cdot R}{\frac{1}{\omega_0 C}}$$

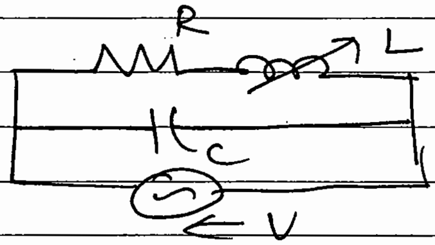
$$I_C = \omega_0 R C \cdot I_{min}$$

$$I_C = I_{min} \cdot Q$$

the ratio of current through either capacitor or Inductor to the total current in the circuit at resonance is defined as current magnification

$$Q = \frac{I_C}{I_{min}} = \text{current magnification}$$

A fixed condenser is placed in parallel with fixed resistance and variable inductance show that at resonance $X_L = \frac{X_C}{2} \pm \sqrt{\frac{X_C^2}{4} - R^2}$



admittance of the circuit

$$Y = \frac{1}{Z} = \frac{1}{-jX_C} + \frac{1}{R+jX_L}$$

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Rationalising the denominator

$$\frac{1}{Z} = \frac{j}{X_C} + \frac{R - jX_L}{R^2 + X_L^2}$$

$$= \frac{R}{R^2+X_L^2} + j\left[\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right]$$

at resonance imaginary part is zero

$$\therefore \frac{1}{X_C} - \frac{X_L}{R^2+X_L^2} = 0$$

$$\frac{1}{X_C} = \frac{X_L}{R^2+X_L^2}$$

$$R^2 + X_L^2 = X_L X_C$$

$$X_L^2 - X_L X_C + R^2 = 0$$

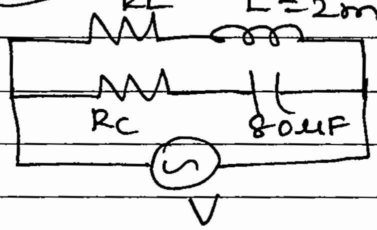
$$\therefore X_L = \frac{X_C \pm \sqrt{X_C^2 - 4R^2}}{2}$$

$$X_L = \frac{X_C}{2} \pm \sqrt{\left(\frac{X_C}{2}\right)^2 - R^2}$$

$$X_L = \frac{X_C}{2} \pm \sqrt{\frac{X_C^2}{4} - R^2}$$

DIVAKAR.B.C

Find R_L & R_C which causes the circuit given to be resonant at all frequencies



for the circuit to be resonant at all frequencies

$$R_L^2 = R_C^2 = \frac{L}{C}$$

$$R_L = R_C = \sqrt{\frac{L}{C}} = \sqrt{\frac{2\times10^{-3}}{80\times10^{-6}}} = 5\,\Omega$$

A coil of resistance of 40Ω and inductance 0.75H forms a part of series circuit for which the resonant frequency is 55Hz. If the supply is 250V, 50Hz find the line current, powerfactor the voltage across the coil

@ resonance $\frac{1}{W_r C} = W_r L$

$= 2 \times 3.14 \times 55 \times 0.75 = 259.05\Omega = X_C$ at 55 Hz

X_L at 50 Hz $= 2 \times 3.14 \times 50 \times 0.75$
$= 235.5\Omega$

X_C at 50 Hz $= 259.05 \times \frac{55}{50} = 284.96\Omega$

Z @ 50 Hz $= 40 + j(X_L - X_C) = 40 + j(235.5 - 284.96)$

$= 40 - j49.46$
$= 63.61 \angle -51.04^\circ \ \Omega$ //

current $= \frac{250}{63.61} = 3.93\text{A}$

powerfactor $= \cos\phi = \cos\phi = \cos 51.04$
$pf = 0.629$ leading

The voltage across the coil $= I Z_{coil}$

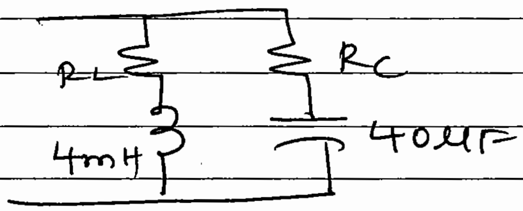
$= I\sqrt{R^2 + X_L^2} = 3.93\sqrt{40^2 + (235.5)^2}$
$V_{coil} = 938.77V$

DIVA (CAPA: B-C)

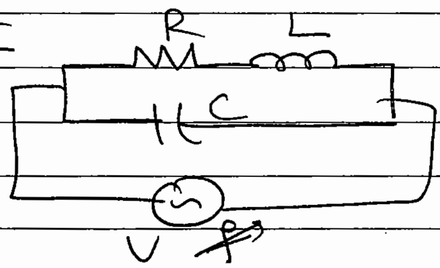
Find the value of R_L & R_C for the circuit shown in figure resonates at all frequencies

The circuit can resonate at any frequency if $R_L^2 = R_C^2 = \frac{L}{C}$

$$\therefore R_L = R_C = \sqrt{\frac{L}{C}} = \sqrt{\frac{4\times10^{-3}}{40\times10^{-6}}} = 10\,\Omega$$



Expression for dynamic impedance of a parallel resonance circuit & derive the expression of a resonance frequency



$$Z_1 = R + jX_L, \quad Z_2 = \frac{-j}{W_C}$$

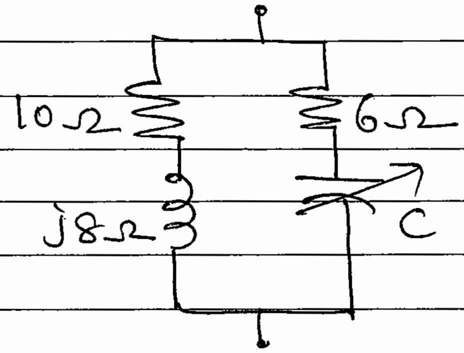
$$Y = \frac{1}{Z_1} = \frac{1}{R+jWL} \times \frac{R-jWL}{R-jWL}$$

$$Y_2 = \frac{1}{Z_2} = jWC$$

Total admittance $Y = Y_1 + Y_2$

$$Y = \frac{R-jWL}{R^2+W^2L^2} + jW_C$$

Find the value of the capacitance C for which the circuit given in figure resonates at 750 Hz



$Z_L = 10 + j8, \quad Z_C = 6 - jX_C$

$$Y_T = \frac{1}{Z_L} + \frac{1}{Z_C} \Rightarrow Y_T = Y_L + Y_C$$

$$Y_L = \frac{10 - j8}{100 + 64} = \frac{10 - j8}{164}$$

$$Y_C = \frac{1}{6 - jX_C} \times \frac{6 + jX_C}{6 + jX_C} = \frac{6 + jX_C}{36 + X_C^2}$$

$$\therefore Y_T = Y_L + Y_C$$

$$= \frac{10 - j8}{164} + \frac{6 + jX_C}{36 + X_C^2}$$

$$Y_T = \frac{6}{36} + \frac{10}{164} + j\left[\frac{X_C}{36 + X_C^2} - \frac{8}{164}\right]$$

equating imaginary parts to zero

$$\frac{X_C}{36 + X_C^2} = \frac{8}{164}$$

$$164 X_C = 288 + 8X_C^2$$

$$8X_C^2 - 164 X_C + 288 = 0$$

$$\boxed{X_{C_1} = 18.56\,\Omega}, \quad \boxed{X_{C_2} = 1.9396\,\Omega}$$

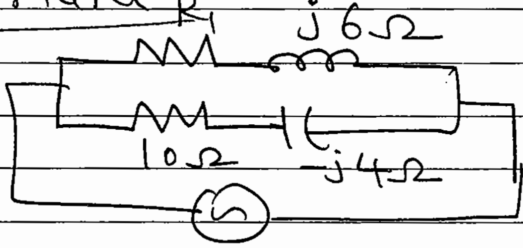
$\therefore C_1 = \frac{1}{\omega X_{C_1}} = \frac{1}{18.56 (2\pi \times 750)}$

$\boxed{C = 1.143 \times 10^{-5} \text{ F}}$

$C_2 = \frac{1}{X_{C_2}\omega} = \frac{1}{1.93 \times 2\pi \times 750}$

$\boxed{C_2 = 109.95 \mu F}$

Jan 2014 Determine the value of R_1 such that the circuit given in figure resonant.



$Z_1 = R_1 + j6\Omega$

$Z_2 = 10 - j4\Omega$

$Y_T = \frac{1}{R_1 + j6} \times \frac{R_1 - j6}{R_1 - j6} + \frac{1}{10 - j4} \times \frac{10 + j4}{10 + j4}$

$= \frac{R_1 - j6}{R_1^2 + 36} + \frac{10 + j4}{100 + 16}$

$Y_T = \frac{R_1}{R_1^2 + 36} + \frac{10}{116} + j\left[\frac{4}{116} - \frac{6}{R_1^2 + 36}\right]$

equating imaginary parts to zero

$\frac{4}{116} - \frac{6}{R_1^2 + 36} = 0$

$\therefore \frac{4}{116} = \frac{6}{R_1^2 + 36} \Rightarrow 4R_1^2 + 144 = 696$

$R_1^2 = 138$

$\boxed{R_1 = 11.747 \Omega}$

A practical parallel resonant circuit consists of a coil of 0.1H, $R = 10\Omega$, $C = 10\mu F$ find dynamic impedance & Resonant frequency

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.1 \times 10 \times 10^{-6}} - \frac{100}{0.01}}$$

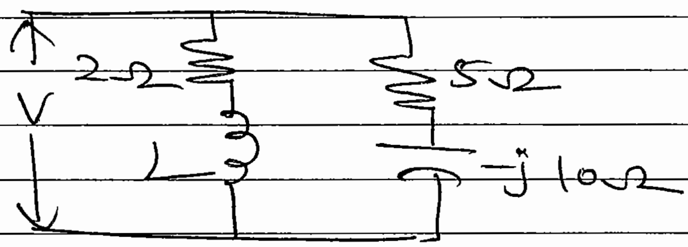
$$f_r = 158.357\ Hz$$

Dynamic impedance $Z_0 = \frac{L}{CR}$

$$Z_0 = \frac{0.1}{10 \times 10^{-6} \times 10} = 1k\Omega$$

July 2017 (07m)

Find the value of L at which circuit resonates at $\omega = 500$ rad/sec



$$Y_T = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{2+jX_L} \times \frac{2-jX_L}{2-jX_L} + \frac{1}{5-j10} \times \frac{5+j10}{5+j10}$$

$$= \frac{2-jX_L}{4+X_L^2} + \frac{5+j10}{25+100}$$

equating Imaginary parts to zero

$$\frac{X_L}{4+X_L^2} = \frac{10}{125}$$

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$$125 X_L = 40 + X_L^2$$

$$10 X_L^2 - 125 X_L + 40 = 0$$

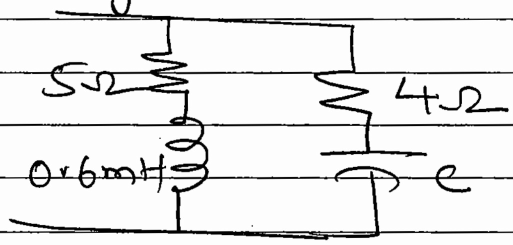
$$X_L = \frac{25 + \sqrt{561}}{4}$$

$$\boxed{X_{L_1} = 12.17\,\Omega} \qquad \boxed{X_{L_2} = 0.328\,\Omega}$$

$$\therefore L_1 = \frac{X_L}{w} = 24.34\,mH$$

$$\boxed{L_2 = \frac{X_{L_2}}{w} = 0.657\,\mu H}$$

For the circuit shown find the 2 values of C at resonance when the frequency of driving voltage is 5000 rad/sec



$$X_L = wL$$
$$= 5000 \times 0.6 \times 10^{-3}$$
$$jX_L = j3\,\Omega$$

$$\therefore Y_T = \frac{1}{5+j3} + \frac{1}{4-jX_C}$$
$$= \frac{5-3j}{25+9} + \frac{4+jX_C}{16+X_C^2}$$

equating imaginary parts to zero

$$\frac{3}{25+9} = \frac{X_C}{16+X_C^2}$$

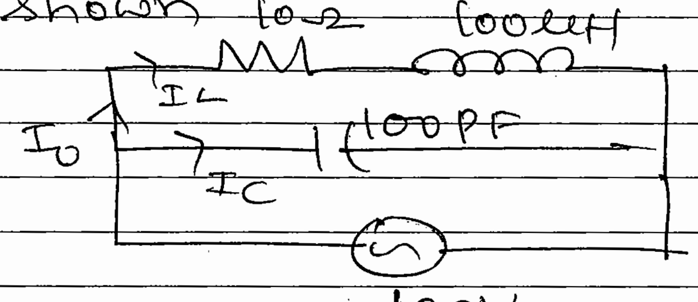
$$3X_C^2 - 34X_C + 48 = 0$$

$$X_{C_1} = 9.68\ \Omega \ / \ X_{C_2} = 1.652\ \Omega$$

$$C_1 = \frac{1}{X_{C_1} \omega} = \frac{1}{9.68 \times 5000} = 20.66\ \mu F$$

$$C_2 = \frac{1}{X_{C_2} \omega} = \frac{1}{1.652 \times 5000} = 121.06\ \mu F$$

For the parallel resonant circuit shown find I_0, I_L, I_C, f_0 & dynamic impedance for the circuit shown



$$Z_0 = \frac{L}{CR} = \frac{100 \times 10^{-6}}{100 \times 10^{-12} \times 10} = 100\ k\Omega$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = 1.5914 \times 10^6\ Hz$$

$$I_0 = \frac{V}{Z_0} = \frac{100}{100 \times 10^3} = 1\ mA$$

$$I_L = \frac{V}{R + jX_L}$$

$$= \frac{100 \angle 0}{10 + j2\pi \times 1.59 \times 10^6 \times 100 \times 10^{-6}}$$

$$\boxed{I_L = 0.1 \angle -89.42^\circ \text{ A}}$$

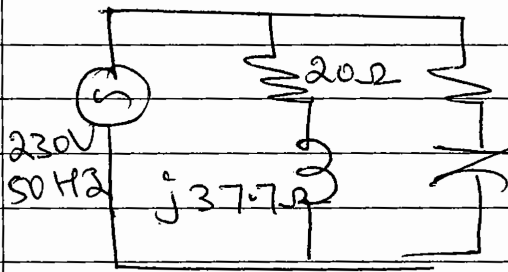
$$I_C = \frac{V}{X_C} = \frac{100 \angle 0 \times 2\pi \times 1.59 \times 10^6 \times 100 \times 10^{-12}}{-j}$$

$$I_C = \frac{0.0999 \angle 0^\circ}{\angle -90}$$

$$\boxed{I_C = 0.0999 \angle 90^\circ \text{ A}}$$

DIVAKAR-B-C

Find the 2 values of C for the circuit shown



$$Y_1 = \frac{1}{20 + 37.7j}$$

$$Y_2 = \frac{1}{10 - jX_C}$$

$$Y_1 = 0.0109 - 0.020j$$

$$Y_1 = 0.0109 - 0.0207j$$

$$Y_2 = \frac{10 + jX_C}{100 + X_C^2}$$

@ resonance imaginary parts of Y_1 & Y_2 must balance each other

$$0.0207 = \frac{X_C}{100 + X_C^2}$$

$$(100 + X_C^2)(0.0207) = X_C$$

$$2.07 + 0.0207\,X_C^2 = X_C$$

$$0.0207\,X_C^2 - X_C = -2.07$$

$$X_C(0.0207\,X_C - 1) = -2.07$$

$$X_C = 46.1419\,\Omega$$
$$X_C = 2.167\,\Omega$$

$$X_C = \frac{1}{2\pi f C} \Rightarrow C_1 = \frac{1}{2\pi \times 50 \times 46.1419}$$

$$C_1 = 68.9\,\mu F$$

$$C_2 = \frac{1}{2\pi \times 50 \times 2.167} = 1.46\,mF$$

$$C_2 = 1.46\,mF$$

Module : 5

TWO PORT NETWORK PARAMETERS.

DIVAKARA.B.C

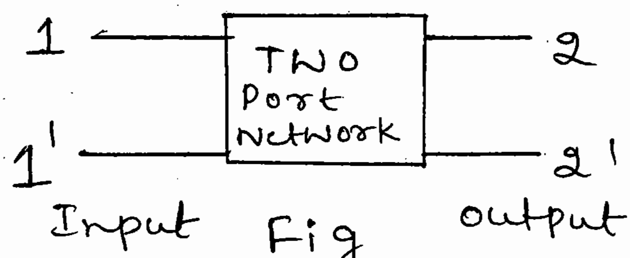
Introduction

A general network having two pairs of terminals forms an important building block in many systems such as communication system, control systems etc.

The pair of terminal at which electrical energy enters is known as the input terminal while the pair of terminal at which the signal leaves is known as the output terminal

A pair of terminals at which a signal may enter or Leave a network is called port

A network having 2 pairs of terminals is known as 2 port network



Fig

Fig represents a two port network with input terminals 1-1' & output terminals 2-2'. A 2-port network has four variables V_1, I_1, V_2 & I_2 of these four variables 2 can be taken as dependent and the other two as independent

Depending on which variables are chosen as independent variables four sets of parameters are defined for a two port network. They are

1. open circuit impedance | Z-parameter
2. Short circuit Admittance parameter (Y)
3. Hybrid or h-parameters
4. Transmission or ABCD parameters

open circuit Impedance parameters [Z-parameters]

V_1 and V_2 are dependent variables where as I_1 & I_2 are independent variables $\therefore$ $V_1 = f_1(I_1, I_2)$

$V_2 = f_2(I_1, I_2)$

The input and output voltages V_1 and V_2 can be expressed in terms of input and output currents I_1 & I_2 as $V_1 = Z_{11}I_1 + Z_{12}I_2 \rightarrow (1)$

$V_2 = Z_{21}I_1 + Z_{22}I_2 \rightarrow (2)$

in matrix form

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

put $I_2 = 0$ in ①

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$ called input impedance (Ω)

from ②

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$ ⇒ forward transfer impedance (Ω)

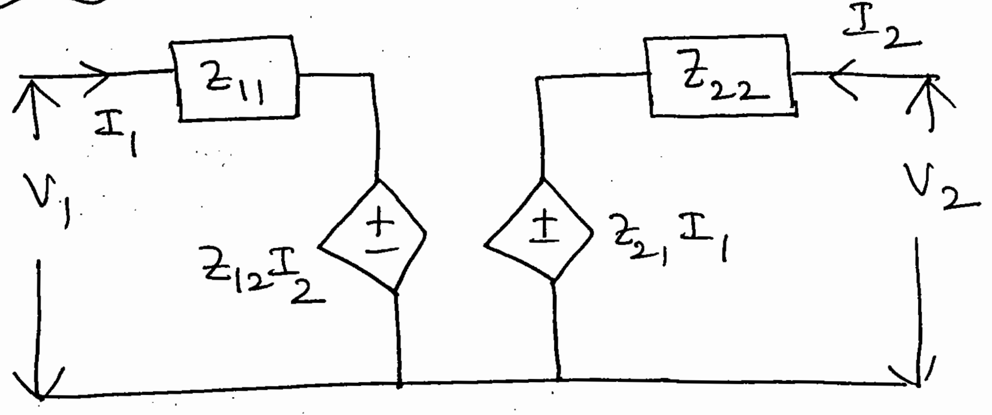
put $I_1 = 0$ from ①

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$ ⇒ Reverse transfer impedance (Ω)

from ② $$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

⇒ output impedance (Ω)

Equivalent Z-parameter model



Y - Parameters [short circuit admittance]

V_1 and V_2 are independent variables where as I_1 and I_2 are dependent variables

$\therefore$ $I_1 = f_1(V_1, V_2)$

$I_2 = f_2(V_1, V_2)$ the input and output currents I_1 & I_2 can be expressed in terms of input and output voltages V_1 & V_2 as

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \rightarrow (a)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \rightarrow (b)$$

In matrix form

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

put $V_2 = 0$ from (1) $Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$

called input admittance ($\mho$)

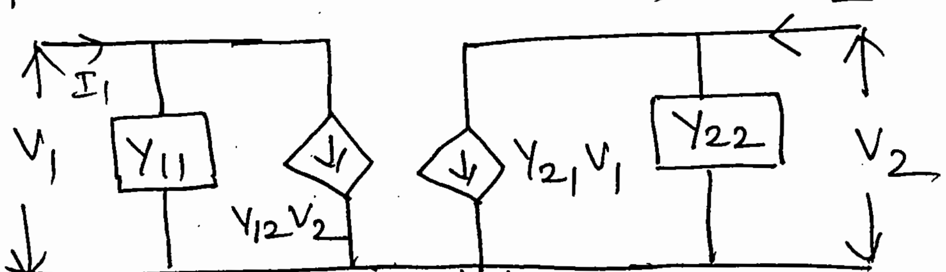
from (2) $Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$ $\Rightarrow$ forward transfer admittance ($\mho$)

Put $V_1 = 0$ from (1)

$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$ $\Rightarrow$ Reverse transfer admittance

from (2) $Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$

output admittance ($\mho$)



model

Hybrid Parameters (h-Parameters)

Hybrid parameters are used in modelling of transistor circuits

I_1 & V_2 ⇒ Dependent Variables

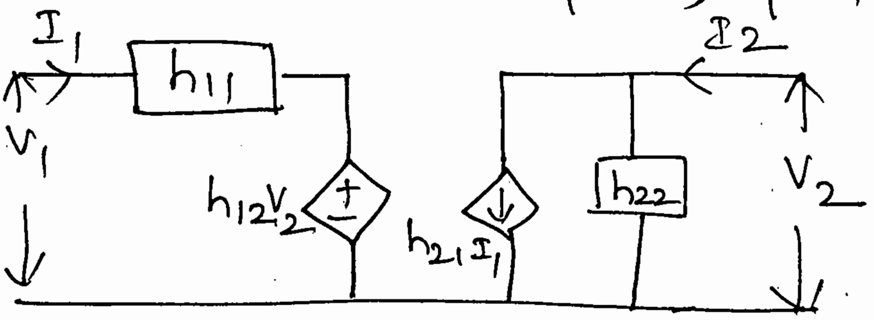
I_2 & V_1 ⇒ Independent Variables

$\therefore\ V_1 = f_1(I_1, V_2)$ & $I_2 = f_2(I_1, V_2)$

$$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (2)$$

In matrix form $\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$



when $V_2 = 0$

$h_{11} = \left.\dfrac{V_1}{I_1}\right|_{V_2=0}$ ⇒ Input impedance (Ω)

$h_{21} = \left.\dfrac{I_2}{I_1}\right|_{V_2=0}$ ⇒ forward current gain

when $I_1 = 0$ ⇒

$h_{12} = \left.\dfrac{V_1}{V_2}\right|_{I_1=0}$ ⇒ Reverse Voltage gain

$h_{22} = \left.\dfrac{I_2}{V_2}\right|_{I_1=0}$ ⇒ output admittance ($\mho$)

h_{11} & h_{22} habe a dimension & no dimension for h_{12} and h_{21}

since these parameters are mixed up they are called hybrid parameters

Transmission | chain | ABCD | General circuit parameters

Transmission parameters are used in the analysis of power transmission Lines

I_2 & V_2 are independent variables where as I_1 and V_1 are dependent variables

$$V_1 = AV_2 - BI_2 \rightarrow (1)$$
$$I_1 = CV_2 - DI_2 \rightarrow (2)$$

In matrix form $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

negative sign is for I_2 is due to fact that I_2 flows in a direction opposite to that assumed

Put $I_2 = 0$ from (1) $A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$

$\Rightarrow$ Reverse Voltage gain

from (2) $C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0}$ $\Rightarrow$ transfer ($\mho$) admittance

Put $V_2 = 0$

from (1) $B = \left.\dfrac{-V_1}{I_2}\right|_{V_2=0}$ $\Rightarrow$ transfer (Ω) impedance

from (2) $D = -\left.\dfrac{I_1}{I_2}\right|_{V_2=0}$ $\Rightarrow$ Reverse current gain

$$[Y][V] = I$$

pre multiplying both sides by $[Y]^{-1}$

$$[Y]^{-1}[Y][V] = [Y]^{-1} I$$

or $[V] = [Y]^{-1} I$

also $[V] = [Z][I]$

$$\therefore [Z] = [Y]^{-1}$$

or

$$[Z][I] = [V]$$

pre multiplying on both sides by $[Z]^{-1}$

$$[Z]^{-1}[Z][I] = [Z]^{-1}[V]$$

or $[I] = [Z]^{-1}[V]$

also $[I] = [Y][V]$

$$\therefore [Z]^{-1} = [Y]$$

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Inter relationship between Z and Y parameters

Express Z-parameters in terms of Y-parameters

W.K.T Z-p's $\Rightarrow$ $V_1 = z_{11} I_1 + z_{12} I_2 \rightarrow (1)$

$V_2 = z_{21} I_1 + z_{22} I_2 \rightarrow (2)$

Y-p's $\Rightarrow$ $I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow (3)$

$I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow (4)$

Representing (3) & (4) in matrix form

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

to find V_1 & V_2

$$V_1 = \frac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{Y_{22}I_1 - Y_{12}I_2}{\Delta Y}$$

$$V_2 = \frac{\begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{Y_{11}I_2 - Y_{21}I_1}{\Delta Y}$$

where $\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$

$$\therefore V_1 = \left(\frac{Y_{22}}{\Delta Y}\right) I_1 + \left(\frac{-Y_{12}}{\Delta Y}\right) I_2 \rightarrow (5)$$

$$V_2 = \left(\frac{-Y_{21}}{\Delta Y}\right) I_1 + \left(\frac{Y_{11}}{\Delta Y}\right) I_2 \rightarrow (6)$$

comparing (5) & (6) with (1) & (2)

$Z_{11} = \frac{Y_{22}}{\Delta Y}$; $Z_{12} = \frac{-Y_{12}}{\Delta Y}$; $Z_{21} = \frac{-Y_{21}}{\Delta Y}$; $Z_{22} = \frac{Y_{11}}{\Delta Y}$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

imp Express Y parameters in terms of Z-parameters 5

w.k.t Y-parameter equations $\Rightarrow$ $I_1 = Y_{11}V_1 + Y_{12}V_2 \rightarrow (1)$

$I_2 = Y_{21}V_1 + Y_{22}V_2 \rightarrow (2)$

Z-p's $\Rightarrow$ $V_1 = Z_{11}I_1 + Z_{12}I_2 \rightarrow (3)$

$V_2 = Z_{21}I_1 + Z_{22}I_2 \rightarrow (4)$

Representing (3) & (4) in matrix form

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}} = \frac{Z_{22}V_1 - Z_{12}V_2}{\Delta Z}$$

$$\therefore I_1 = \left(\frac{Z_{22}}{\Delta Z}\right)V_1 + \left(\frac{-Z_{12}}{\Delta Z}\right)V_2 \Rightarrow (A)$$

$$I_2 = \frac{\begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}} = \frac{Z_{11}V_2 - Z_{21}V_1}{\Delta Z}$$

$\frac{Y_{11}}{\Delta Y}$

$$I_2 = \left(\frac{-Z_{21}}{\Delta Z}\right)V_1 + \left(\frac{Z_{11}}{\Delta Z}\right)V_2 \rightarrow (B)$$

where $\boxed{\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}}$

comparing equations (A) & (B) with (1) & (2)

$$Y_{11} = \frac{z_{22}}{\Delta z} \; ; \quad Y_{12} = -\frac{z_{12}}{\Delta z} \; ; \quad Y_{21} = \frac{-z_{21}}{\Delta z}$$

$$Y_{22} = \frac{z_{11}}{\Delta z}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{z_{22}}{\Delta z} & -\frac{z_{12}}{\Delta z} \\ -\frac{z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{bmatrix}$$

h-parameters in terms of z-parameters

h-p's equations $\Rightarrow$ $V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (1)$

$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (2)$

z-p's equations $\Rightarrow$ $V_1 = z_{11} I_1 + z_{12} I_2 \rightarrow (3)$

$V_2 = z_{21} I_1 + z_{22} I_2 \rightarrow (4)$

from (4) $\quad z_{22} I_2 = V_2 - z_{21} I_1$

$$\therefore \quad I_2 = \left(\frac{1}{z_{22}}\right) V_2 - \left(\frac{z_{21}}{z_{22}}\right) I_1 \rightarrow (5)$$

put (5) in (3)

$$V_1 = z_{11} I_1 + z_{12} \left[\frac{V_2}{z_{22}} - \frac{z_{21}}{z_{22}} I_1 \right]$$

$$V_1 = \left(z_{11} - \frac{z_{12} z_{21}}{z_{22}} \right) I_1 + \left(\frac{z_{12}}{z_{22}} \right) V_2$$

$$V_1 = \left(\frac{z_{11} z_{22} - z_{12} z_{21}}{z_{22}} \right) I_1 + \left(\frac{z_{12}}{z_{22}} \right) V_2$$

$$V_1 = \left(\frac{\Delta z}{z_{22}}\right) I_1 + \left(\frac{z_{12}}{z_{22}}\right) V_2 \rightarrow (6)$$

comparing (5) with (2)

$$\boxed{h_{21} = \frac{-z_{21}}{z_{22}}} \quad \boxed{h_{22} = \frac{1}{z_{22}}}$$

comparing (6) with (1)

$$\boxed{h_{11} = \frac{\Delta z}{z_{22}}}, \quad \boxed{h_{12} = \frac{z_{12}}{z_{22}}}$$

Z-parameters in terms of h-parameters

Z-p's $\Rightarrow$ $V_1 = z_{11} I_1 + z_{12} I_2 \rightarrow (1)$

$V_2 = z_{21} I_1 + z_{22} I_2 \rightarrow (2)$

h-p's $\Rightarrow$ $V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (3)$

$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (4)$

from (4) $h_{22} V_2 = I_2 - h_{21} I_1$

$$\therefore V_2 = \left(\frac{1}{h_{22}}\right) I_2 - \left(\frac{h_{21}}{h_{22}}\right) I_1 \rightarrow (5)$$

Comparing (5) with (2)

$$\boxed{z_{21} = \frac{-h_{21}}{h_{22}}} \quad \boxed{z_{22} = \frac{1}{h_{22}}}$$

Put (5) in (3)

$$V_1 = h_{11} I_1 + h_{12}\left[\left(\frac{1}{h_{22}}\right) I_2 - \left(\frac{h_{21}}{h_{22}}\right) I_1\right]$$

$$= \left[h_{11} - \frac{h_{12} h_{21}}{h_{22}}\right] I_1 + \left(\frac{h_{12}}{h_{22}}\right) I_2$$

$$\Delta h = h_{11} h_{22} - h_{12} h_{21}$$

$$V_1 = \left(\frac{\Delta h}{h_{22}}\right) I_1 + \left(\frac{h_{12}}{h_{22}}\right) I_2 \rightarrow (6)$$

comparing (6) with (1)

$$\boxed{Z_{11} = \frac{\Delta h}{h_{22}}} \qquad \boxed{Z_{12} = \frac{h_{12}}{h_{22}}}$$

h-parameters in terms of Y-Parameters

h-parameter equations $\Rightarrow$ $V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (1)$

$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (2)$

Y-p's $\Rightarrow$ $I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow (3)$

$I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow (4)$

from (3) $Y_{11} V_1 = I_1 - Y_{12} V_2$

$$V_1 = \left(\frac{1}{Y_{11}}\right) I_1 - \left(\frac{Y_{12}}{Y_{11}}\right) V_2 \rightarrow (5)$$

comparing (5) with (1)

$$\boxed{h_{11} = \frac{1}{Y_{11}}}, \qquad \boxed{h_{12} = -\frac{Y_{12}}{Y_{11}}}$$

Put (5) in (4)

$$I_2 = Y_{21}\left[\left(\frac{1}{Y_{11}}\right) I_1 - \left(\frac{Y_{12}}{Y_{11}}\right) V_2\right] + Y_{22} V_2$$

$$= \left(\frac{Y_{21}}{Y_{11}}\right) I_1 + \left(Y_{22} - \frac{Y_{12} Y_{21}}{Y_{11}}\right) V_2$$

$$\therefore \; I_2 = \left(\frac{Y_{21}}{Y_{11}}\right) I_1 + \left(\frac{\Delta Y}{Y_{11}}\right) V_2 \rightarrow (6)$$

comparing (6) with (2)

$$\boxed{h_{21} = \frac{Y_{21}}{Y_{11}}}, \quad \boxed{h_{22} = \frac{\Delta Y}{Y_{11}}}$$

Y-parameters in terms of h-parameters

Y-p's $\Rightarrow$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow (1)$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow (2)$$

h-p's $\Rightarrow$

$$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (3)$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (4)$$

from (3) $\quad h_{11} I_1 = V_1 - h_{12} V_2$

$$I_1 = \left(\frac{1}{h_{11}}\right) V_1 - \left(\frac{h_{12}}{h_{11}}\right) V_2 \rightarrow (5)$$

Comparing (5) with (1)

$$\boxed{Y_{11} = \frac{1}{h_{11}}}, \quad \boxed{Y_{12} = \frac{-h_{12}}{h_{11}}}$$

put (5) in (4)

$$I_2 = h_{21}\left[\left(\frac{1}{h_{11}}\right) V_1 - \left(\frac{h_{22}}{h_{11}}\right) V_2\right] + h_{22} V_2$$

$$= \left(\frac{h_{21}}{h_{11}}\right) V_1 + \left(h_{22} - \frac{h_{12} h_{21}}{h_{11}}\right) V_2$$

$$I_2 = \left(\frac{h_{21}}{h_{11}}\right) V_1 + \left(\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{11}}\right) V_2 \rightarrow (6)$$

comparing equation 6 with 2

$$\boxed{Y_{21} = \frac{h_{21}}{h_{11}}} \qquad \boxed{Y_{22} = \frac{\Delta h}{h_{11}}}$$

Z- parameters in terms of ABCD parameters

Z - p's $\Rightarrow$ $V_1 = z_{11} I_1 + z_{12} I_2 \rightarrow (1)$

$V_2 = z_{21} I_1 + z_{22} I_2 \rightarrow (2)$

ABCD p's $\Rightarrow$ $V_1 = A V_2 - B I_2 \rightarrow (3)$

$I_1 = C V_2 - D I_2 \rightarrow (4)$

from (4) $C V_2 = I_1 + D I_2$

$$V_2 = \left(\frac{1}{C}\right) I_1 + \left(\frac{D}{C}\right) I_2 \rightarrow (5)$$

comparing (5) with (2)

$$\boxed{z_{21} = \frac{1}{C}} \qquad \boxed{z_{22} = \frac{D}{C}}$$

put (5) in (3)

$$V_1 = A\left[\left(\frac{1}{C}\right) I_1 + \left(\frac{D}{C}\right) I_2\right] - B I_2$$

$$= \left(\frac{A}{C}\right) I_1 + \left(\frac{AD}{C} - B\right) I_2$$

$$V_1 = \left(\frac{A}{C}\right) I_1 + \left(\frac{AD - BC}{C}\right) I_2 \rightarrow (6)$$

comparing (6) with (1)

$$\boxed{z_{11} = \frac{A}{C}}, \quad \boxed{z_{12} = \frac{AD - BC}{C}}$$

ABCD in terms of z-parameters

ABCD p's $\Rightarrow$ $V_1 = AV_2 - BI_2 \rightarrow (1)$

$I_1 = CV_2 - DI_2 \rightarrow (2)$

z-p's $\Rightarrow$ $V_1 = z_{11} I_1 + z_{12} I_2 \rightarrow (3)$

$V_2 = z_{21} I_1 + z_{22} I_2 \rightarrow (4)$

from (4) $z_{21} I_1 = V_2 - z_{22} I_2$

$$\therefore I_1 = \left(\frac{1}{z_{21}}\right) V_2 - \left(\frac{z_{22}}{z_{21}}\right) I_2 \rightarrow (5)$$

comparing (5) with (2)

$$\boxed{C = \frac{1}{z_{21}}} \qquad \boxed{D = \frac{z_{22}}{z_{21}}}$$

put (5) in (3)

$$V_1 = z_{11}\left[\left(\frac{1}{z_{21}}\right) V_2 - \left(\frac{z_{22}}{z_{21}}\right) I_1\right] + z_{12} I_2$$

$$= \left(\frac{z_{11}}{z_{21}}\right) V_2 - \left(\frac{z_{22} z_{21}}{z_{21}} - z_{12}\right) I_2$$

$$V_1 = \left(\frac{z_{11}}{z_{21}}\right) V_2 - \left(\frac{\Delta z}{z_{21}}\right) I_2 \rightarrow (6)$$

comparing (6) with (1)

$$\boxed{A = \frac{z_{11}}{z_{21}}} \qquad \boxed{B = \frac{\Delta z}{z_{21}}}$$

Y-parameters in terms of ABCD parameters

Y-p's $\Rightarrow$ $I_1 = Y_{11}V_1 + Y_{12}V_2 \rightarrow (1)$

$I_2 = Y_{21}V_1 + Y_{22}V_2 \rightarrow (2)$

ABCD p's $\Rightarrow$ $V_1 = AV_2 - BI_2 \rightarrow (3)$

$I_1 = CV_2 - DI_2 \rightarrow (4)$

from (3) $BI_2 = -V_1 + AV_2$

$$I_2 = -\left(\frac{1}{B}\right)V_1 + \left(\frac{A}{B}\right)V_2 \rightarrow (5)$$

Comparing (5) with (2)

$$\boxed{Y_{21} = -\left(\frac{1}{B}\right)} \qquad \boxed{Y_{22} = \left(\frac{A}{B}\right)}$$

put (5) in (4)

$$I_1 = CV_2 - D\left[\left(-\frac{1}{B}\right)V_1 + \left(\frac{A}{B}\right)V_2\right]$$

$$= -\left(\frac{AD}{B} - C\right)V_2 + \left(\frac{D}{B}\right)V_1$$

$$I_1 = \left(\frac{D}{B}\right)V_1 - \left(\frac{AD-BC}{B}\right)V_2 \rightarrow (6)$$

comparing (6) with (1)

$$\boxed{Y_{11} = \frac{D}{B}} \qquad \boxed{Y_{12} = -\left(\frac{AD-BC}{B}\right)}$$

ABCD parameters in terms of Y p's :

ABCD p's $\Rightarrow$ $V_1 = AV_2 - BI_2 \rightarrow (1)$

$I_1 = CV_2 - DI_2 \rightarrow (2)$

Y - p's $\Rightarrow$ $I_1 = Y_{11}V_1 + Y_{12}V_2 \rightarrow (3)$

$I_2 = Y_{21}V_1 + Y_{22}V_2 \rightarrow (4)$

from (4) $Y_{21}V_1 = -Y_{22}V_2 + I_2$

$$\therefore V_1 = -\left(\frac{Y_{22}}{Y_{21}}\right)V_2 - \left(\frac{-1}{Y_{21}}\right)I_2 \rightarrow (5)$$

comparing (5) with (1)

$$\boxed{A = -\frac{Y_{22}}{Y_{21}}} \qquad \boxed{B = \frac{-1}{Y_{21}}}$$

put (5) in (3)

$$I_1 = Y_{11}\left[-\left(\frac{Y_{22}}{Y_{21}}\right)V_2 + \left(\frac{1}{Y_{21}}\right)I_2\right] + Y_{12}V_2$$

$$= -\left[\frac{Y_{11}Y_{22}}{Y_{21}} - Y_{12}\right]V_2 + \left(\frac{Y_{11}}{Y_{21}}\right)I_2$$

$$= -\left[\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{21}}\right]V_2 - \left(\frac{-Y_{11}}{Y_{21}}\right)I_2$$

$$I_1 = -\left(\frac{\Delta Y}{Y_{21}}\right)V_2 - \left(-\frac{Y_{11}}{Y_{21}}\right)I_2 \rightarrow (6)$$

comparing (6) with (2)

$$\boxed{C = \frac{-\Delta Y}{Y_{21}}} \qquad \boxed{D = -\frac{Y_{11}}{Y_{21}}}$$

ABCD parameters in terms of h-p's

ABCD p's ⇒ $V_1 = AV_2 - BI_2 \rightarrow (1)$

$I_1 = CV_2 - DI_2 \rightarrow (2)$

h-p's ⇒ $V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (3)$

$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (4)$

from (4) $h_{21} I_1 = -h_{22} V_2 + I_2$

$\therefore \quad I_1 = -\left(\frac{h_{22}}{h_{21}}\right) V_2 - \left(\frac{-1}{h_{21}}\right) I_2 \rightarrow (5)$

comparing (5) with (2)

$\boxed{C = -\frac{h_{22}}{h_{21}}}$ $\boxed{D = \frac{-1}{h_{21}}}$

put (5) in (3)

$$V_1 = h_{11}\left[-\left(\frac{h_{22}}{h_{21}}\right) V_2 + \left(\frac{1}{h_{21}}\right) I_2\right] + h_{12} V_2$$

$$= -\left(\frac{h_{11} h_{22}}{h_{21}} - h_{12}\right) V_2 + \left(\frac{h_{11}}{h_{21}}\right) I_2$$

$$= -\left[\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{21}}\right] V_2 - \left(\frac{-h_{11}}{h_{21}}\right) I_2$$

$$V_1 = -\left(\frac{\Delta h}{h_{21}}\right) V_2 - \left(-\frac{h_{11}}{h_{21}}\right) I_2 \rightarrow (6)$$

Comparing (6) with (1)

$\boxed{A = -\frac{\Delta h}{h_{21}}}$ $\boxed{B = -\frac{h_{11}}{h_{21}}}$

h parameters in terms of ABCD p's

h- p's $\Rightarrow$ $V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (1)$

$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (2)$

ABCD p's $\Rightarrow$ $V_1 = A V_2 - B I_2 \rightarrow (3)$

$I_1 = C V_2 - D I_2 \rightarrow (4)$

from (4) $D I_2 = C V_2 - I_1$

$\therefore I_2 = \left(\frac{C}{D}\right) V_2 - \left(\frac{1}{D}\right) I_1 \rightarrow (5)$

comparing (5) with (2)

$\boxed{h_{21} = \frac{-1}{D}}$ $\boxed{h_{22} = \frac{C}{D}}$

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put (5) in (3)

$V_1 = A V_2 - B \left[\left(\frac{C}{D}\right) V_2 - \left(\frac{1}{D}\right) I_1\right)$

$= \left(\frac{B}{D}\right) I_1 + \left(A - \frac{BC}{D}\right) V_2$

$V_1 = \left(\frac{B}{D}\right) I_1 + \left(\frac{AD - BC}{D}\right) V_2 \rightarrow (6)$

comparing (6) with (1)

$\boxed{h_{11} = \frac{B}{D}}$ $\boxed{h_{12} = \frac{AD - BC}{D}}$

For a Reciprocal network

$Z_{12} = Z_{21}$, $Y_{12} = Y_{21}$, $h_{12} = -h_{21}$

For a symmetrical network

$Z_{11} = Z_{22}$, $Y_{11} = Y_{22}$, $A = D$

Jan 2014 app Following are the hybrid parameters for a network find Y-parameters $\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix}$

$$Y_{11} = \frac{1}{h_{11}} = \frac{1}{5} = 0.2\,\mho$$

$$Y_{12} = \frac{-h_{12}}{h_{11}} = -\frac{2}{5} = -0.4\,\mho$$

$$Y_{21} = \frac{h_{21}}{h_{11}} = \frac{3}{5} = 0.6\,\mho$$

$$Y_{22} = \frac{\Delta h}{h_{11}} = \frac{24}{5} = 4.8\,\mho$$

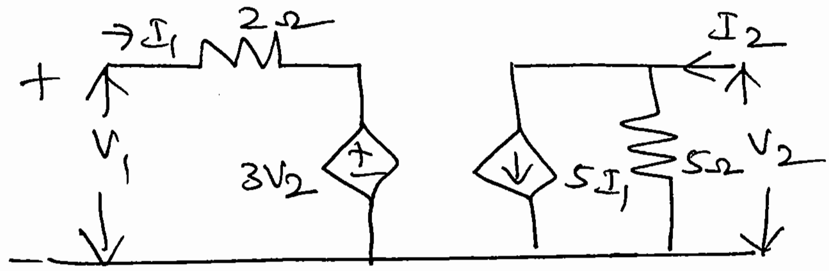
Application of h, Z & T-Parameters

h-parameters ⇒ It is used in the analysis of small signal amplifier using BJT. The equivalent circuit of h-parameters can be used as an ac equivalent circuit of a small signal BJT amplifier to carry out ac analysis of amplifier

Z-Parameter. ⇒ It is used to represent given 2 port network into its equivalent T network which is the most important network in communication systems

ABCD parameters ⇒ are generally used for the analysis of power transmission line where input port is the sending end and output port of the network is the receiving end of the line

Find the transmission parameters for the network shown



KVL @ input side

$$-2I_1 - 3V_2 + V_1 = 0$$

$$\therefore \quad V_1 = 3V_2 + 2I_1 \rightarrow (1)$$

apply KCL @ output side

$$I_2 = 5I_1 + \frac{V_2}{5}$$

$$\therefore \quad 5I_1 = -\frac{V_2}{5} + I_2$$

$$I_1 = -\frac{1}{25}V_2 + \frac{1}{5}I_2 \rightarrow (2)$$

put (2) in (1)

$$V_1 = 3V_2 + 2\left[-\frac{1}{25}V_2 + \frac{1}{5}I_2\right]$$

$$V_1 = \left[3 - \frac{2}{25}\right]V_2 + \frac{2}{5}I_2$$

$$V_1 = \left(\frac{73}{25}\right)V_2 + \left(-\frac{2}{5}\right)(-I_2) \rightarrow (3)$$

Re arranging I_2 in (2)

$$I_1 = \left(-\frac{1}{25}\right)V_2 + \left(-\frac{1}{5}\right)(-I_2) \rightarrow (4)$$

comparing (3) & (4) with ABCD parameters

$$A = \frac{73}{25} = 2.92 \qquad B = -\frac{2}{5} = -0.4\,\Omega$$

$$C = -\frac{1}{25} = 0.04\,\mho \qquad D = -\frac{1}{5} = -0.2\,\mho$$

Prove $AD - BC = 1$ for a reciprocal network

$$A = \frac{z_{11}}{z_{21}}, \quad B = \frac{\Delta z}{z_{21}}, \quad C = \frac{1}{z_{21}} \quad D = \frac{z_{22}}{z_{21}}$$

$$AD - BC = \left(\frac{z_{11}}{z_{21}}\right)\left(\frac{z_{22}}{z_{21}}\right) - \left(\frac{\Delta z}{z_{21}}\right)\left(\frac{1}{z_{21}}\right)$$

$$= \frac{z_{11}z_{22}}{z_{21}^2} - \left(\frac{z_{11}z_{22} - z_{12}z_{21}}{z_{21}^2}\right)$$

$$= \frac{z_{11}z_{22} - z_{11}z_{22} + z_{12}z_{21}}{z_{21}^2}$$

$$= \frac{z_{12}z_{21}}{z_{21}^2} = \frac{z_{12}}{z_{21}}$$

$AD - BC = 1$ since for a reciprocal network $\boxed{z_{12} = z_{21}}$

For a symmetrical network show

$\Delta h = 1$

$$h_{11} = \frac{\Delta z}{z_{22}}, \quad h_{12} = \frac{z_{12}}{z_{22}}, \quad h_{21} = \frac{-z_{21}}{z_{22}}, \quad h_{22} = \frac{1}{z_{22}}$$

$$h_{11}h_{22} - h_{12}h_{21} = \left(\frac{\Delta z}{z_{22}}\right)\left(\frac{1}{z_{22}}\right) - \left(\frac{z_{12}}{z_{22}}\right)\left(-\frac{z_{21}}{z_{22}}\right)$$

$$= \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{22}^2} + \frac{z_{12}z_{21}}{z_{22}^2}$$

$$= \frac{z_{11}z_{22}}{z_{22}^2} = \frac{z_{11}}{z_{22}} = 1$$

since $z_{11} = z_{22}$ for a symmetrical network

The following equations gives the relation ship between voltages and currents of a 2-port network

$$I_1 = 0.25 V_1 - 0.2 V_2$$
$$I_2 = -0.20 V_1 + 0.1 V_2$$

obtain transmission parameters & write the network equations in terms of ABCD parameters

Given $I_1 = 0.25V_1 - 0.2V_2 \rightarrow (1)$

$I_2 = -0.20V_1 + 0.1V_2 \rightarrow (2)$

Y-p's $I_1 = Y_{11}V_1 + Y_{12}V_2 \rightarrow (3)$

$I_2 = Y_{21}V_1 + Y_{22}V_2 \rightarrow (4)$

comparing (1) & (3)

$Y_{11} = 0.25, \quad Y_{12} = -0.2$

comparing (2) & (4) $Y_{21} = -0.20, \; Y_{22} = 0.10$

to find ABCD parameter

$I_1 = 0.25V_1 - 0.2V_2 \rightarrow (5)$

$I_2 = 0.20V_1 + 0.10V_2 \rightarrow (6)$

$V_1 = AV_2 - BI_2 \rightarrow (7)$

$I_1 = CV_2 - DI_2 \rightarrow (8)$

from (6) $0.20V_1 = 0.10V_2 - I_2$

$V_1 = 0.5V_2 - 5I_2 \rightarrow (9)$

put (9) in (5)

$I_1 = 0.25[0.5V_2 - 5I_2] - 0.2V_2$

$I_1 = -0.075V_2 - 1.25I_2 \rightarrow (10)$

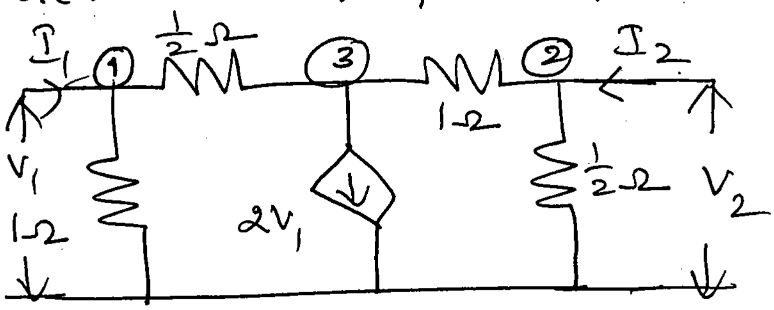
comparing (7) & (9)

$A = 0.5, \; B = 5$

comparing (8) & (10) $C = -0.075, \; D = 1.25$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.5 & 5 \\ -0.075 & 1.25 \end{bmatrix}$$

Determine Y & Z parameters



KCL @ node 1

$$\frac{V_1}{1} + \frac{V_1 - V_3}{\frac{1}{2}} = I_1$$

10

$$3V_1 - 2V_3 = I_1 \rightarrow (1)$$

KCL @ node 2 $\quad \frac{V_2 - V_3}{1} + \frac{V_2}{\frac{1}{2}} = I_2$

$$3V_2 - V_3 = I_2 \rightarrow (2)$$

KCL @ node 3

$$\frac{V_3 - V_1}{\frac{1}{2}} + \frac{V_3 - V_2}{1} + 2V_1 = 0$$

$$3V_3 - V_2 = 0 \quad \text{or} \quad V_3 = \frac{V_2}{3} \rightarrow (3)$$

Put (3) in (1) & (2)

$$3V_1 - 2\left(\frac{V_2}{3}\right) = I_1 \quad \text{or} \quad I_1 = 3V_1 - \frac{2}{3}V_2 \rightarrow (A)$$

from (2) $\quad 3V_2 - \frac{V_2}{3} = I_2$

$$\therefore I_2 = \frac{8}{3}V_2 \rightarrow (B)$$

From (A) & (B) $\quad Y_{11} = 3\,℧, \quad Y_{12} = -\frac{2}{3}\,℧$

$Y_{21} = 0, \quad Y_{22} = \frac{8}{3}\,℧$

to find Z parameters find $[Y]^{-1}$

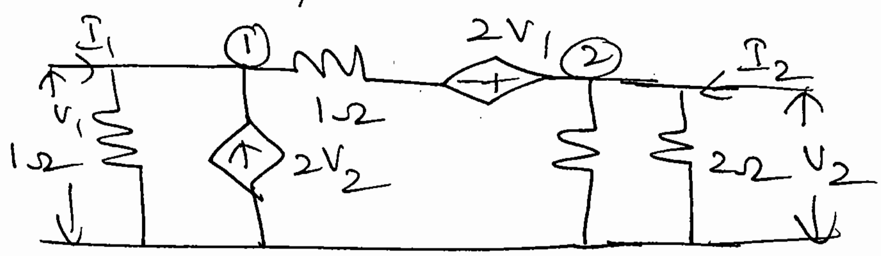
$$[Y] = \begin{bmatrix} 3 & -2/3 \\ 0 & 8/3 \end{bmatrix}$$

$$\Delta Y = \begin{vmatrix} 3 & -\frac{2}{3} \\ 0 & 8/3 \end{vmatrix} = 3 \times \frac{8}{3} + 0 = 8$$

$$[Z] = [Y]^{-1}$$

$$= \frac{\begin{bmatrix} \frac{8}{3} & \frac{2}{3} \\ 0 & 3 \end{bmatrix}}{8} = \begin{bmatrix} \frac{1}{3} & \frac{1}{12} \\ 0 & \frac{3}{8} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

② Find Y & Z parameters



KCL @ node 1

$$\frac{V_1}{1} - 2V_2 + \frac{V_1 + 2V_1 - V_2}{1} = I_1$$

$$I_1 = 4V_1 - 3V_2 \rightarrow ①$$

KCL @ node 2

$$\frac{V_2}{2} + \frac{V_2 - 2V_1 - V_1}{1} = I_2$$

$$I_2 = -3V_1 + 1.5V_2 \rightarrow ②$$

comparing ① & ② with Y-parameters

$Y_{11} = 4\mho$, $Y_{12} = -3\mho$, $Y_{21} = -3\mho$

$Y_{22} = 1.5\mho$

$$Y = \begin{bmatrix} 4 & -3 \\ -3 & 1.5 \end{bmatrix} \quad \Delta Y = 4\times1.5 - 9 = -3$$

$$Z = [Y]^{-1} = \frac{\begin{pmatrix} 1.5 & 3 \\ 3 & 4 \end{pmatrix}}{-3} = \begin{bmatrix} \frac{1}{2} & -1 \\ -1 & \frac{4}{3} \end{bmatrix}$$

$$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -1 \\ -1 & \frac{-4}{3} \end{bmatrix} \Omega$$

Determine Y & ABCD Parameters given z-parameters of a 2 port network are $z_{11} = 20\Omega$, $z_{22} = 30\Omega$, $z_{12} = z_{21} = 10\Omega$

Ans

$$\Delta z = z_{11} z_{22} - z_{12} z_{21}$$
$$= (20)(30) - (10)(10) = 500$$

$$Y_{11} = \frac{z_{22}}{\Delta z} = \frac{30}{500} = \frac{3}{50} = 0.06\mho$$

$$Y_{12} = \frac{-z_{12}}{\Delta z} = \frac{-10}{500} = -0.02\mho$$

$$Y_{21} = \frac{-z_{21}}{\Delta z} = \frac{-10}{500} = -0.02\mho$$

$$Y_{22} = \frac{z_{11}}{\Delta z} = \frac{20}{500} = \frac{2}{50} = 0.04\mho$$

ABCD parameters

$$A = \frac{z_{11}}{z_{21}} = \frac{20}{10} = 2 \; ; \; C = \frac{1}{z_{21}} = \frac{1}{10} = 0.1$$

$$B = \frac{\Delta z}{z_{21}} = \frac{500}{10} = 50 \; ; \; D = \frac{z_{22}}{z_{21}} = \frac{30}{10} = 3$$

Given $I_1 = 0.5V_1 - 0.2V_2$

$I_2 = -0.2V_1 + V_2$ find

Z & ABCD parameters

Given Y- parameters ∴ comparing the given equations with Y-P's

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.5 & -0.2 \\ -0.2 & 1 \end{bmatrix} \mho$$

Z- parameters

$$\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$$

$$= (0.5)(1) - (-0.2)(-0.2) = 0.46$$

$$z_{11} = \frac{Y_{22}}{\Delta Y} = \frac{1}{0.46} = 2.174\Omega$$

$$z_{12} = \frac{-Y_{12}}{\Delta Y} = \frac{-(-0.2)}{0.46} = 0.434\Omega$$

$$z_{21} = \frac{-Y_{21}}{\Delta Y} = \frac{-(-0.2)}{0.46} = 0.434\Omega$$

$$z_{22} = \frac{Y_{11}}{\Delta Y} = \frac{0.5}{0.46} = 1.087\Omega$$

ABCD parameters

$$A = \frac{-Y_{22}}{Y_{21}} = \frac{-1}{-0.2} = 5$$

$$B = \frac{-1}{Y_{21}} = \frac{-1}{-0.2} = 5$$

$$C = -\frac{\Delta Y}{Y_{21}} = \frac{-0.46}{-0.12} = 2.3$$

$$D = \frac{-Y_{11}}{Y_{21}} = \frac{-0.5}{-0.2} = 2.5$$

Jan 2015 06 Following short circuit currents and voltages are obtained experimentally for a 2 port network. determine Y- parameters

i) with short circuited output
$I_1 = 5mA$; $I_2 = -0.3mA$ & $V_1 = 25V$

ii) with input short circuited
$I_1 = -5mA$, $I_2 = 10mA$, $V_2 = 30V$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \rightarrow (1)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \rightarrow (2)$$

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = \frac{5\times10^{-3}}{25} = 0.2m\mho$$

$$Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} = \frac{-5\times10^{-3}}{30} = 0.1667m\mho$$

$$Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} = \frac{-0.3\times10^{-3}}{25} = 0.012m\mho$$

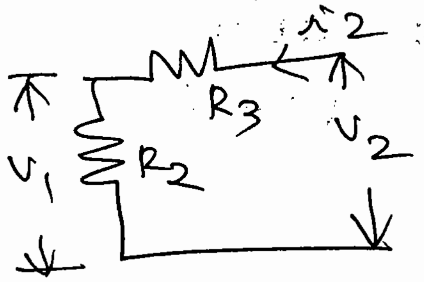
$$Y_{22} = \frac{I_2}{V_2}\Bigg|_{V_1=0} = \frac{10\times10^{-3}}{30} = 0.33\,m\mho$$

Determine h-parameters for the network shown

$$V_1 = h_{11} i_1 + h_{12} V_2$$
$$i_2 = h_{21} i_1 + h_{22} V_2$$

To find h_{12} & h_{22} open circuit input terminals i.e. $I_1 = 0$.

since $i_1 = 0$ $\quad \alpha i_1 = 0$



KVL

$$V_2 = i_2(R_2 + R_3)$$

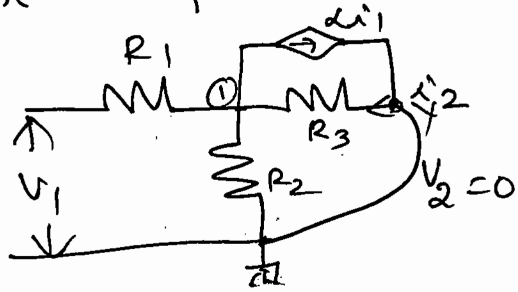
$$h_{22} = \frac{i_2}{V_2}\Bigg|_{i_1=0} = \frac{1}{R_2+R_3}\,\mho$$

Put for $i_2 = \dfrac{V_2}{R_2+R_3}$

$$V_1 = R_2\left(\frac{V_2}{R_2+R_3}\right)$$

$$h_{12} = \frac{V_1}{V_2}\Bigg|_{I_1=0} = \frac{R_2}{R_2+R_3}$$

to find h_{11} & h_{21} short circuit the output terminals $V_2 = 0$



KCL @ node 1

$$\frac{V_x}{R_3} + \frac{V_x}{R_2} + \alpha i_1 = i_1$$

$$V_x = \frac{(1-\alpha) i_1 R_2 R_3}{R_2 + R_3}$$

but $V_1 = V_x + i_1 R_1 \rightarrow$ (A)

put V_x value in (A)

$$V_1 = \frac{(1-\alpha) R_2 R_3 i_1}{R_2 + R_3} + i_1 R_1$$

$$V_1 = \left(R_1 + \frac{(1-\alpha) R_2 R_3}{R_2 + R_3}\right) i_1$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0} = R_1 + \frac{(1-\alpha) R_2 R_3}{R_2 + R_3}$$

KCL @ y

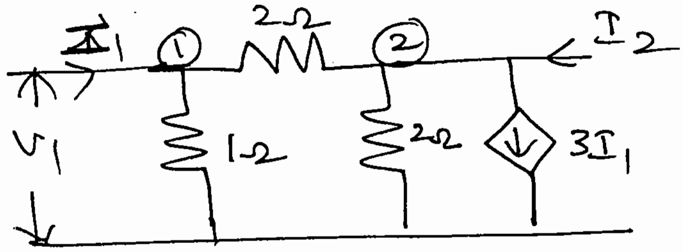
$$i_2 + \alpha i_1 = \frac{-V_x}{R_3}$$

$$i_2 + \alpha i_1 = \frac{-1}{R_3}\left(\frac{(1-\alpha) R_2 R_3}{R_2 + R_3}\right) i_1$$

$$i_2 = -\left(\frac{(1-\alpha) R_2}{R_2 + R_3} + \alpha\right) i_1$$

$$h_{21} = \left.\frac{i_2}{i_1}\right|_{V_2 = 0} = -\frac{(\alpha R_3 + R_2)}{R_2 + R_3}$$

For the network shown find Y & Z parameters



KCL @ node 1

$$\frac{V_1}{1} + \frac{V_1 - V_2}{2} = I_1$$

or $$I_1 = \frac{V_1}{2} + V_1 - \frac{V_2}{2}$$

$$\boxed{I_1 = 1.5 V_1 - 0.5 V_2} \rightarrow (1)$$

KCL @ node 2

$$\frac{V_2 - V_1}{2} + \frac{V_2}{2} = I_2 - 3I_1$$

substitute the value of I_1

$$-0.5 V_1 + V_2 = I_2 - 3(1.5 V_1 - 0.5 V_2)$$

or $$\boxed{I_2 = 4V_1 - 0.5 V_2} \rightarrow (2)$$

comparing (1) & (2)

$Y_{11} = 1.5\,\mho$, $Y_{12} = -0.5\,\mho$, $Y_{21} = 4\,\mho$

$Y_{22} = -0.5\,\mho$ $\therefore [Y] = \begin{bmatrix} 1.5 & -0.5 \\ 4 & -0.5 \end{bmatrix}$

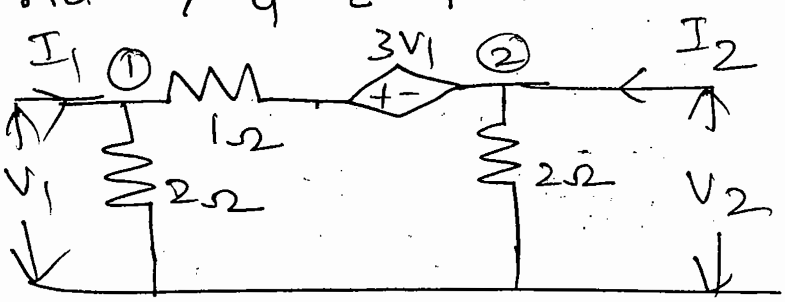
$$\Delta Y = \begin{vmatrix} 1.5 & -0.5 \\ 4 & -0.5 \end{vmatrix} = -0.75 + 2 = 1.25$$

$$[Z] = [Y]^{-1} = \frac{\begin{bmatrix} -0.5 & 0.5 \\ -4 & 1.5 \end{bmatrix}}{1.25}$$

$$[Z] = \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

Find Y & Z parameters

DIVAKARA B.C



apply KCL @ node (1)

$$\frac{V_1}{2} + \frac{V_1 - 3V_1 - V_2}{1} = I_1$$

$$\boxed{I_1 = -1.5\,V_1 - V_2} \Rightarrow (1)$$

KCL @ node 2

$$\frac{V_2}{1} + \frac{V_2 + 3V_1 - V_1}{1} = I_2$$

$$I_2 = 2V_1 + 2V_2 \rightarrow (2)$$

Comparing ① & ② with Y- parameter equations

$$Y_{11} = -1.5\,℧,\quad Y_{12} = -1\,℧,\quad Y_{21} = -2\,℧$$

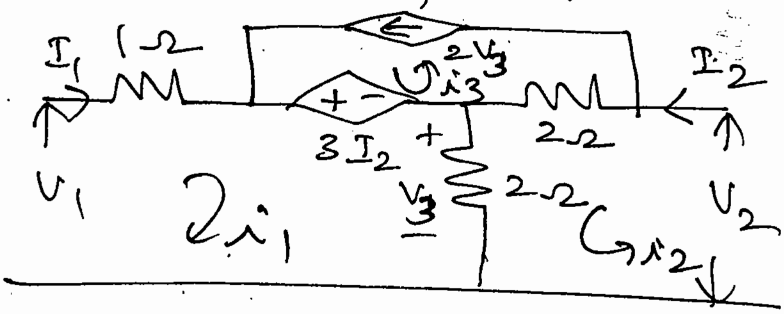
$$Y_{22} = 2\,℧ \quad \therefore \quad Y = \begin{bmatrix} -1.5 & -1 \\ 2 & 2 \end{bmatrix}$$

$$\Delta Y = \begin{bmatrix} -1.5 & -1 \\ 2 & 2 \end{bmatrix} = -3 + 2 = -1$$

$$[Z] = [Y]^{-1} = \frac{\begin{bmatrix} 2 & 1 \\ -2 & -1.5 \end{bmatrix}}{-1}$$

$$Z_{11} = -2\,\Omega,\quad Z_{12} = -1\,\Omega,\quad Z_{21} = 2\,\Omega,\quad Z_{22} = 1.5\,\Omega$$

Determine Y & Z parameters



$$i_3 = 2V_3 \quad \& \quad V_3 = (I_1 + I_2)2$$

KVL to mesh ① $\quad V_3 = (I_1 + I_2)2$

$$V_1 = 1 \cdot I_1 + 3I_2 + V_3$$

$$V_1 = I_1 + 3I_2 + (I_1 + I_2)2$$

$$\boxed{V_1 = 3I_1 + 5I_2}$$

(18)

$V_2 = 2(I_2 - 2V_3) + V_3$

$V_2 = 2I_2 - 8(I_1 + I_2) + 2(I_1 + I_2)$

$V_2 = -6I_1 - 4I_2 \rightarrow (2)$

comparing (1) & (2)

$z_{11} = 3\Omega, \quad z_{12} = 5\Omega, \quad z_{21} = -6\Omega, \quad z_{22} - 4\Omega$

$$[z] = \begin{bmatrix} 3 & 5 \\ -6 & -4 \end{bmatrix}$$

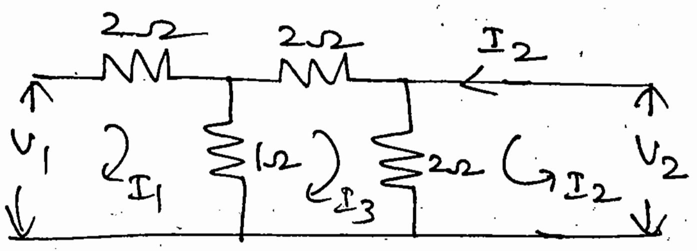
$$\Delta z = \begin{vmatrix} 3 & 5 \\ -6 & -4 \end{vmatrix} = -12 + 30 = 18$$

$$Y = [z]^{-1} = \frac{\begin{bmatrix} -4 & -5 \\ 6 & 3 \end{bmatrix}}{18}$$

$$\Rightarrow [Y] = \begin{bmatrix} -\frac{2}{9} & -\frac{5}{18} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} 0.222 & 0.277 \\ 0.333 & 0.166 \end{bmatrix} \mho$$

2

For the network shown obtain Z-parameters and draw the Z-parameter equivalent circuit



KVL to loop (1)

$$V_1 - 3I_1 + I_3 = 0$$

$$V_1 = 3I_1 - I_3 \Rightarrow (1)$$

KVL to loop (2)

$$V_2 - 2(I_2 + I_3) = 0$$

$$\boxed{V_2 = 2I_2 + 2I_3} \Rightarrow (2)$$

KVL to loop (3)

$$2I_3 + (I_3 - I_1) + 2(I_3 + I_2) = 0$$

$$2I_3 + I_3 - I_1 + 2I_3 + 2I_2 = 0$$

$$-I_1 + 5I_3 + 2I_2 = 0$$

$$5I_3 = I_1 + 2I_2 \rightarrow (3)$$

$$\therefore V_1 = 3I_1 - \frac{(I_1 - 2I_2)}{5}$$

$$V_1 = \frac{15I_1 - I_1 + 2I_2}{5}$$

$$V_1 = \frac{14}{5} I_1 + \frac{2}{5} I_2 \rightarrow (1)$$

$$\therefore \boxed{z_{11} = \frac{14}{5} = 2.8\,\Omega}, \boxed{z_{12} = \frac{2}{5} = 0.4\,\Omega}$$

$$V_2 = 2 I_2 + \frac{2}{5}(I_1 - 2I_2)$$

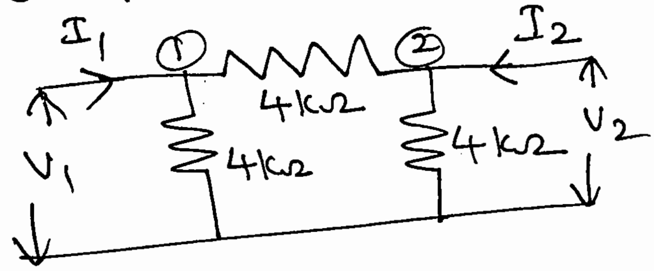
$$V_2 = \frac{10 I_2 + 2I_1 - 4I_2}{5}$$

$$V_2 = \frac{2}{5} I_1 + \frac{6}{5} I_2 \rightarrow (2)$$

comparing with $V_2 = z_{21} I_1 + z_{22} I_2$

$$\boxed{z_{21} = \frac{2}{5} = 0.4\,\Omega}, \boxed{z_{22} = 1.2\,\Omega}$$

Find Y-parameters for the circuit shown and use them to find the Transmission parameters



apply KCL @ node 1

$$\frac{V_1}{4K} + \frac{V_1 - V_2}{4K} = I_1$$

$$I_1 = V_1\left(\frac{1}{4K} + \frac{1}{4K}\right) - \frac{V_2}{4K}$$

$$I_1 = (0.5 \times 10^{-3}) V_1 - (0.25 \times 10^{-3}) V_2 \rightarrow (1)$$

KCL @ node 2

$$\frac{V_2}{4k} + \frac{V_2 - V_1}{4k} = I_2$$

$$I_2 = \frac{-V_1}{4k} + V_2\left[\frac{1}{2k}\right]$$

$$I_2 = (-2.5\times10^{-4})\,V_1 + (0.5\times10^{-3})\,V_2 \rightarrow (2)$$

comparing with Y-parameter equation

$Y_{11} = 0.5\,m\mho$, $Y_{12} = -0.25\,m\mho$

$Y_{21} = -0.25\,m\mho$, $Y_{22} = 0.5\,m\mho$

since $Y_{11} = Y_{22}$ network is symmetrical

& $Y_{12} = Y_{21}$ network is reciprocal.

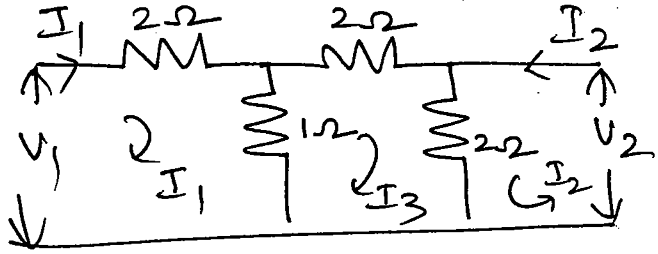
$$A = \frac{-Y_{22}}{Y_{21}} = \frac{-5\times10^{-4}}{-2.5\times10^{-4}} = 2$$

$$B = \frac{-1}{Y_{21}} = \frac{-1}{2.5\times10^{-4}} = 4k\Omega$$

$$C = \frac{-(Y_{11}Y_{22} - Y_{12}Y_{21})}{Y_{21}} = 0.75\,m\mho$$

$$D = \frac{-Y_{11}}{Y_{21}} = \frac{-5\times10^{-4}}{-2.5\times10^{-4}} = 2$$

Determine ABCD parameters



Apply KVL to loop ①

$$V_1 - 2I_1 - (I_1 - I_3) = 0$$

$$V_1 - 2I_1 - I_1 + I_3 = 0$$

$$\boxed{V_1 = 3I_1 - I_3} \rightarrow ①$$

loop ② $2I_3 + 2(I_3 + I_2) + 1(I_3 - I_1) = 0$

$$2I_3 + 2I_3 + I_3 + 2I_2 - I_1 = 0$$

$$-I_1 + 2I_2 + 5I_3 = 0$$

$$\boxed{I_3 = \frac{I_1 - 2I_2}{5}}$$

Loop ③ $V_2 - 2(I_2 + I_3) = 0$

$$V_2 = 2I_2 + 2I_3 \rightarrow ③$$

$$① \Rightarrow V_1 = 3I_1 - \frac{1}{5}(I_1 - 2I_2)$$

$$= 3I_1 - \frac{I_1}{5} + \frac{2}{5}I_2$$

$$\boxed{V_1 = \frac{14}{5}I_1 + \frac{2}{5}I_2}$$

$$V_2 = 2I_2 + \frac{2}{5}(I_1 - 2I_2)$$

$$= 2I_2 - \frac{4}{5}I_2 + \frac{2}{5}I_1$$

$$\boxed{V_2 = \frac{2}{5}I_1 + \frac{6}{5}I_2}$$

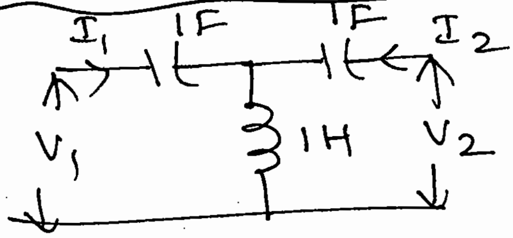
$$A = \frac{z_{11}}{z_{21}} = \frac{14/5}{2/5} = 7$$

$$B = \frac{\Delta z}{z_{21}} = \frac{16/5}{2/5} = 8\,\Omega$$

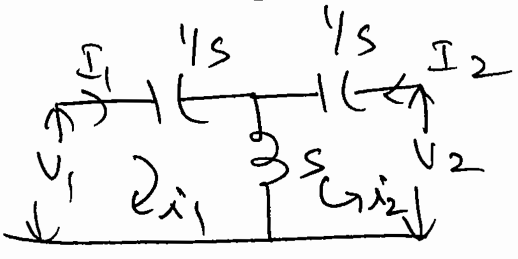
$$C = \frac{1}{z_{21}} = \frac{5}{2} = 2.5\,\mho$$

$$D = \frac{z_{22}}{z_{21}} = \frac{6/5}{2/5} = 3$$

Find h-parameters for the circuit shown



$$L\{1F\} = \frac{1}{s}, \quad L\{1H\} = s$$



KVL to loop 1

$$V_1 - \frac{I_1}{s} - s(I_1 + I_2) = 0$$

$$V_1 = I_1\left[\frac{1}{s} + s\right] + sI_2$$

Loop 2 $V_2 - \frac{1}{s} I_2 - s(I_2 + I_1) = 0$

$$V_2 = sI_1 + \left(\frac{1+s^2}{s}\right) I_2$$

$$\boxed{z_{11} = \frac{1+s^2}{s}\ \Omega}, \quad \boxed{z_{12} = z_{21} = s}$$

$$\boxed{z_{22} = \frac{1+s^2}{s}}$$

$$\Delta z = \left(\frac{1+s^2}{s}\right)\left(\frac{1+s^2}{s}\right) - s^2$$

$$\Delta z = \frac{1 + s^2 + s^2 + \cancel{s^4} - \cancel{s^4}}{s^2} = \frac{1+2s^2}{s^2}$$

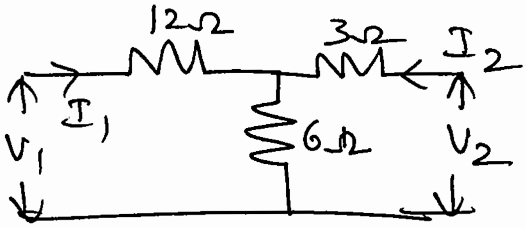
$$\boxed{h_{11} = \frac{\Delta z}{z_{22}} = \frac{1+2s^2}{s^{\cancel{2}}\left(\frac{1+s^2}{\cancel{s}}\right)} = \frac{1+2s^2}{s(1+s^2)}}$$

$$\boxed{h_{12} = \frac{z_{12}}{z_{22}} = \frac{s(s)}{1+s^2} = \frac{s^2}{1+s^2}}$$

$$\boxed{h_{21} = \frac{-z_{21}}{z_{22}} = \frac{-s(s)}{1+s^2} = \frac{-s^2}{1+s^2}}$$

$$\boxed{h_{22} = \frac{1}{z_{22}} = \frac{s}{1+s^2}}$$

Find Z-parameter for the circuit shown



applying KVL $\quad V_1 = 18 I_1 + 6 I_2$

$$V_2 = 6 I_1 + 9 I_2$$

comparing with Z-parameter equations

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

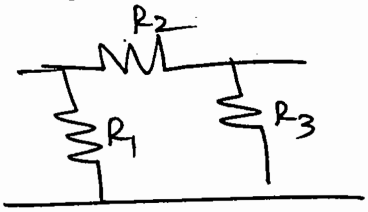
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\boxed{Z_{11} = 18\Omega, \quad Z_{12} = 6\Omega, \quad Z_{21} = 6\Omega, \quad Z_{22} = 9\Omega}$$

Note :-

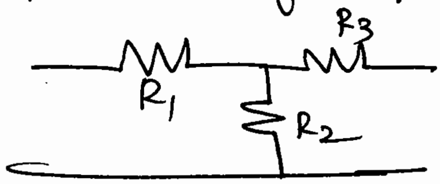
* If there is no dependent source in the given network then $Z_{12} = Z_{21}$ & $Y_{12} = Y_{21}$ the network is said to be Reciprocal

* If the network is of π type

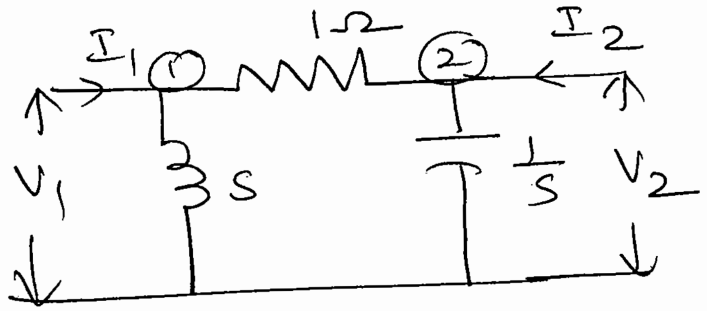


apply KCL

* If the network is of T-type apply KVL



Q) Determine the transmission parameters for the network shown (22)



Apply KCL @ node 1

$$I_1 = \frac{V_1}{s} + (V_1 - V_2)$$

$$= \frac{s+1}{s} V_1 - V_2 \rightarrow (1)$$

KCL @ node 2

$$I_2 = \frac{V_2}{\frac{1}{s}} + (V_2 - V_1)$$

$$I_2 = (s+1) V_2 - V_1$$

$$V_1 = (s+1) V_2 - I_2 \rightarrow (2) \Rightarrow (A)$$

put (2) in (1)

$$I_1 = \frac{s+1}{s}\left[(s+1)V_2 - I_2\right] - V_2$$

$$= \left[\frac{(s+1)^2}{s} - 1\right] V_2 - \frac{s+1}{s} I_2$$

$$I_1 = \left[\frac{s^2+s+1}{s}\right] V_2 - \left(\frac{s+1}{s}\right) I_2$$

$$\Rightarrow (B)$$

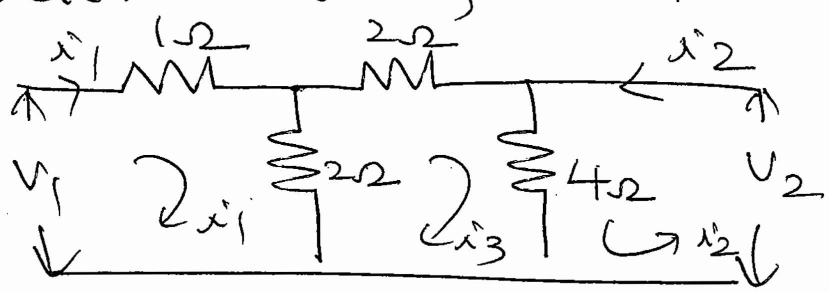
Comparing with ABCD parameters

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{bmatrix} s+1 & -1 \\ \frac{s^2+s+1}{s} & \frac{s+1}{s} \end{bmatrix}$$

Determine hybrid parameters



KVL to loop (1)

$V_1 = 3i_1 - 2i_3 \rightarrow (1)$

KVL to loop (2)

$V_2 = 4i_2 + 4i_3 \rightarrow (2)$

KVL to loop (3)

$2(i_3 - i_1) + 2i_3 + 4(i_3 + i_2) = 0$

$8i_3 = 2i_1 - 4i_2$

$\therefore i_3 = \frac{i_1}{4} - \frac{i_2}{2} \rightarrow (3)$

put (3) in (1)

$V_1 = 3i_1 - 2\left(\frac{i_1}{4} - \frac{i_2}{2}\right)$

$$V_1 = \frac{5}{2} i_1 + i_2$$

& $$V_2 = 4i_2 + 4\left(\frac{i_1}{4} - \frac{i_2}{2}\right)$$

$$= 4i_2 + i_1 - 2i_2$$

$$V_2 = i_1 + 2i_2$$

$$\boxed{i_2 = -\frac{1}{2} i_1 + \frac{1}{2} V_2} \Rightarrow (A)$$

$$\therefore V_1 = \frac{5}{2} i_1 - \frac{1}{2} i_1 + \frac{1}{2} V_2$$

$$\boxed{V_1 = 2i_1 + \frac{1}{2} V_2} \Rightarrow (B)$$

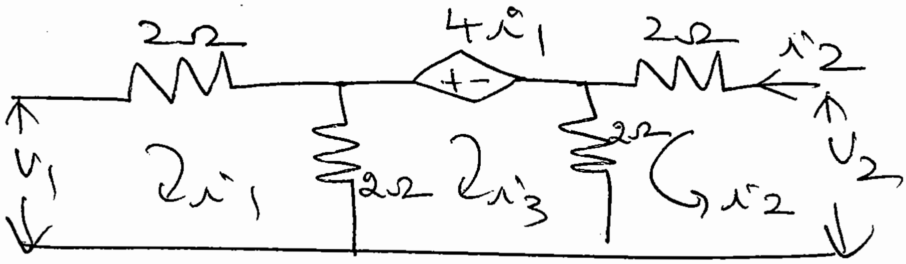
comparing (A) & (B) with h-parameter equations

$$V_1 = h_{11} i_1 + h_{12} V_2$$

$$i_2 = h_{21} i_1 + h_{22} V_2$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 2 & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Find Z & h- parameters for the network shown



KVL to loop ①

$$V_1 = 2i_1 + 2(i_1 - i_3)$$

$$\boxed{V_1 = 4i_1 - 2i_3 \rightarrow ①}$$

KVL to loop ②

$$V_2 = 2i_2 + 2(i_2 + i_3)$$

$$\boxed{V_2 = 4i_2 + 2i_3 \rightarrow ②}$$

KVL to mesh ③

$$2(i_3 - i_1) + 4i_1 + 2(i_3 + i_2) = 0$$

$$i_1 + i_2 = -2i_3$$

$$\therefore \; V_1 = 4i_1 + i_2 + i_1 = 5i_1 + i_2$$

$$V_2 = 4i_2 - i_1 - i_2$$

$$\boxed{V_2 = -i_1 + 3i_2}$$

comparing with z-parameter equation

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix}$$

$$\Delta z = z_{11} z_{22} - z_{12} z_{21}$$
$$= (5)(3) - (1)(-1)$$
$$= 16$$

$$h_{11} = \frac{\Delta z}{z_{22}} = \frac{16}{3}\ \Omega$$

$$h_{12} = \frac{z_{12}}{z_{22}} = \frac{1}{3}$$

$$h_{21} = \frac{-z_{21}}{z_{22}} = \frac{1}{3}$$

$$h_{22} = \frac{1}{z_{22}} = \frac{1}{3}\ \mho$$

$$h = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{pmatrix} \frac{16}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

DIVAKARA BC